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THE GENERALIZATION AND DESCRIPTION OF A SET OF EQUATIONS
FOR POINT MOTION IN A MULTIDIMENSIONAL SPACE

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I. Introduction

The measurement of communication is the measurement of process. It is the notion of explanation through the study of change which sets communication research apart from the tributary disciplines of psychology and sociology. However, much of communication research deals only with the symptoms of communication, the static structure which characterizes the communicative act before or after its occurrence. The reason for this is not a lack of motivation on the part of communication researchers, rather it is the weakness of predominant methodological frameworks and the absence of change-oriented analytic tools.

This paper is intended to examine and develop, and extend, the analytic qualities of one communication methodology, multidimensional scaling. Using the Newtonian mechanics paradigm for description and explanation, this paper will present a hierarchy of derived measures to interpret the changes in one variable or a set of variables within a multidimensional framework over time. While the system to be discussed is most often applied to the observation and examination of cognitive displacement (or distance) relative to time, the discussion in this paper might be applied to any set of contemporary social scientific variables. However, for heuristic reasons, and because a clear analogy to physical reality has some compelling consequences, this paper will deal with point (variable) motion in a spatial representation relative to change in time. It will be seen that by treating communication as a force which impinges upon the social or cultural cognitive structure (or space), that changes in aggregate cognition may be mapped systematically according to the Newtonian model.

Cognitive Structure as a Spatial Model

Numerous reports have detailed the representation of cognition as the interrelationship of a set of elements (Woelfel, 1972; Serota, 1974; Saltiel and Woelfel, 1975). Taking this perspective, any thing a person knows is known because it is discriminated from other things. For example, an apple is not a pear because it has differential amounts of various attributes that both share. An apple has more roundness and more redness, and by judging an object on those attributes, one could tell whether the object was an apple or a pear. However, to distinguish the apple from a red rubber ball some other attribute might be necessary. Thus, because the object has more of the attribute edible than a rubber ball, we might judge it to be an apple. This reasoning suggests that the attributes themselves can be things which can be known. Re-stating this more parsimoniously, we could say that each thing is a certain amount different from each other thing. When we measure judgements of these differences, we treat each thing as a certain distance from each other thing.¹

If we accept the notion of distance as the characteristic by which cognitive elements are defined, then implicitly, the cognitive domain can be treated as a space, wherein distance is the characteristic form for all relationships. On this basis, we might choose to represent the set of difference judgements about the things we know as a configuration in an abstract cognitive space.

Since our interest is focused upon communicative acts and knowledge (and consequent behavior), it is necessary to provide an appropriate framework for this study. One way to accomplish this might be to describe the change which occurs in the cognitive space over time. In a Newtonian paradigm, this representation would be achieved through the calculus of motion. Extrapolating from Newton, we might then operationalize communication effects as the description of motion in the cognitive space.

The purpose of this paper is to examine the calculus of motion in multi-dimensional spaces as an analytic tool for communication research. In this section, I have briefly described a context for this presentation. In the second section, the components of mechanics and the Newtonian model necessary for conducting longitudinal analyses of communication processes and effects will be examined. The third section will treat the multidimensional generalization of Newtonian mechanics, identify some persuant complications, and direct attention to the development of analytic procedures for actual application in a communication research context.

II. Essential components from Newtonian mechanics

The Newtonian paradigm is based on the representation of two fundamental variables: distance and time. By observing the interaction of these, several derived measures can be generated which allow us to accurately describe change and relate it to a set of explanatory variables.

The first of these variables, distance, is the discrepancy between points in space. It is described by various terms including position, displacement, and separation. Position is the location of a point in space defined relative to some arbitrary origin point or fixed reference point which may or may not be on the line described by the point while it is relocating in space. Displacement is the difference between old and new spatial positions when a point is relocated in space. Separation describes the interval between two points measured along the line described by those points. We may speak equally well of the fundamental distance measurement as position or displacement or separation.

To provide a context for the measurement and description of distance, it is necessary to conceptually define the idea of a space. French (1971) interprets the Newtonian view of space, describing it as:

" . . . a sort of stationary [three-dimensional] matrix into which one can place objects or through which objects can move without producing any interaction between the object and the space."

Relating objects in the space to the space itself, French continues:

"Each object in the universe exists at a particular point in space and time. An object in motion undergoes a continuous change of its position with time. And although it would not be practicable, one can imagine the charting of positions with the help of a vast network of meter sticks, laid out end to end in a [three-dimensional, cubical] array throughout space. One can conceive of extending such measurements to any point in the universe. In other words, the space is there, and we simply have a practical task of attaching markers to it." (p. 44)

We may note that French is writing about the physical universe with three observed dimensions, however, we may extend his description, mathematically, to a multidimensional space of potentially infinite dimensionality.

The other fundamental variable used mechanistically to underlie the descriptive paradigm for motion is time. Newton could offer no definition for time, but attempted this description:

"Absolute, true, and mathematical time, of itself, and from its own nature flows equally without relation to anything external, and by another name is called duration." (French, 1971:44)

Newton considered time an entity with the characteristic of uniformity, a standard against which events could be judged. Reichenbach (1958) makes this idea more specific. He suggests that time can be treated as one spatial dimension of a spatial manifold. The solution to the problem of understanding time is thus made analogous to that of understanding space. According to Reichenbach, this is accomplished through the use of coordinative definition, or definition achieved by relating a concept to some particular thing. For distance, this suggested that an arbitrary unit of length could be chosen and used as the standard for measuring all other lengths, and again arbitrarily (or, by coordinative definition), this standard would be congruent to an equivalent standard at

any place in space. For time, this means that an arbitrary interval can be coordinated to a periodic process such as the rotation of the earth. As Reichenbach illustrates:

"Whether two distant line-segments are equal is not a matter of knowledge but of definition; and this definition consists ultimately in reference to a physical object coordinated to the concept of a unit . . .

"Similar considerations must be carried through for the problem of time . . . for time, too, there is a comparison of length. Before we enter into an epistemological investigation, let us first examine what time intervals physics considers to be equal in length. The rotation of the earth is the most important example; we say that the time intervals which the earth requires for one complete rotation are equal. For the subdivision of such time intervals, we use a different method, namely, the measurement of angles. We accept time intervals as equal if they correspond to equal angles of the earth's rotation. Through the combination of these two methods, we obtain the measure of time, and the flow of time we have thus obtained is called uniform." (1958:113-4)

While Reichenbach treats time mathematically as a spatial dimension, in accord with Kantian epistemological arguments, he does suggest that it has qualities which set it apart from the notion of space. The measurement of time is made through the observation of the duration of the earth's rotation. The uniformity of this process provides a necessary congruity for further comparative measurement while subdividing the diurnal period provides the measure of interval magnitude. Reichenbach argues that counting periodic processes does not involve the measurement of distance, and is therefore an indication of the uniqueness of time. In this way, time may be considered independent of spatial dimensions even though it may be easily represented as such.

Within the realm of time and space measurement, we must accept the assumptions of fundamental measurement. Time and space have been considered as extensive properties for which no more basic measureable properties exist (Campbell, 1928). As Newton suggests, and Reichenbach and Einstein support, both time and distance are measured against themselves with regard to an

arbitrary standard. For this reason, and because we accept the idea of congruence among fundamental measures, they are considered epistemologically suited to the description of events and processes. Also, as indicated earlier, the consequences of the choice of this framework is compelling for the representation of a changing knowledge structure. Using this groundwork, we can now move to the set of derived measures which allow us to describe how this system works. We will examine motion as the characteristic form of time-space interaction.

Velocity and acceleration: the basic relationship

The central concept in a quantitative description of the relationship of space to time is velocity. Velocity is the vector representation of motion, or change in spatial position relative to a consequent change in time. The measurement of velocity begins with the observation of two (or more) points in time. With these measures (denoted by r_1, t_1 and r_2, t_2) we can deduce the magnitude and direction of the average velocity between two points:

$$V_{\text{average}} = \frac{r_2 - r_1}{t_2 - t_1}$$

For the purposes of explanation and prediction, however, it is necessary to derive a more accurate measure for velocity at any point along the continuum of change. A measure of change for the smallest possible duration of time would yield a quantity of instantaneous velocity, or the magnitude and direction of velocity of an infinitesimally small occurrence of change. This is given by

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \quad [1]$$

The quantity dr/dt is synonymous with the limit of $\Delta r/\Delta t$ and is referred to as the first derivative of r with respect to t . Geometrically, this is the

slope of a tangent to the graph of r versus t at a specific value of t . This is the general vector description of instantaneous velocity. While it is possible to represent velocity specifically as change in separation

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} ,$$

this is the limiting case of the more general dr/dt . We would be likely to apply the more specific representation when the origin of our representation lies on the line along which we are measuring change. In this unidimensional case Δs would be equal to Δr . However, in the multidimensional paradigm (or any other multivariate scheme) the origin will often be disassociated with the line of motion, and velocity relative to the origin will be measured as a change in displacement (see Figure 1). In the decomposition to orthogonal components, we may be more interested in the change, Δs , due to the linearization of the component factors.

A further aspect of the measurement of velocity, which helps us understand simultaneously changing relationships, is the observation of relative velocity. This component of the descriptive system gives the rate of change of one object relative to another object in its own reference frame. If, for example, object 1 is located at r_1 and object 2 is at r_2 , the vector distance R is give by

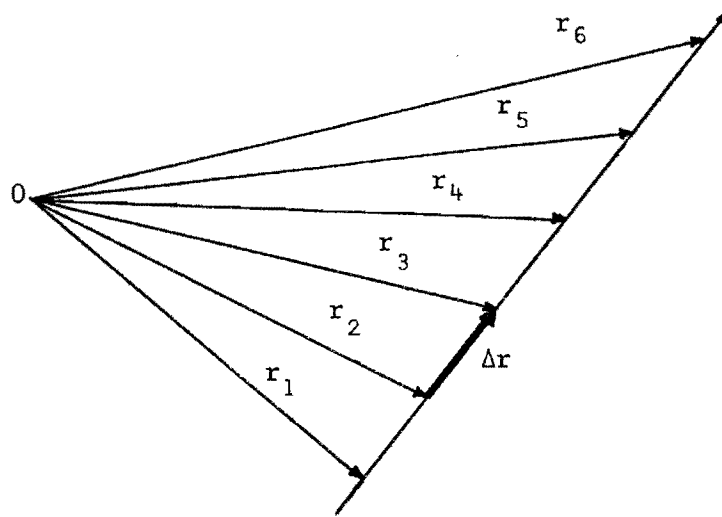
$$R = r_2 - r_1$$

The rate of change of R , since r_1 and r_2 may both be in motion, is given as V , where

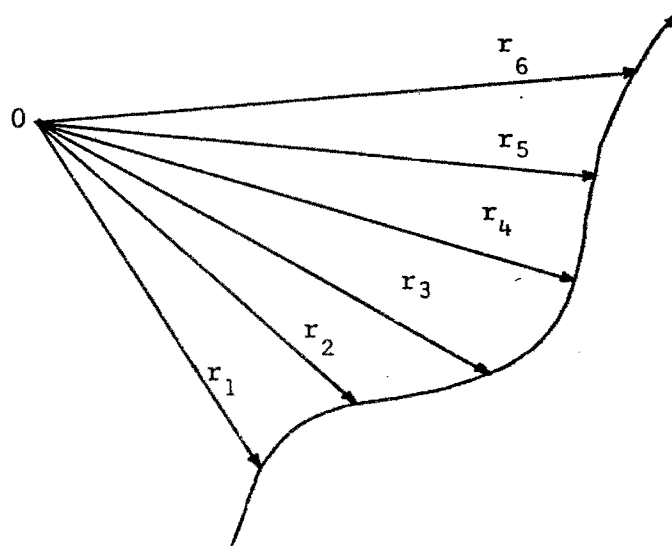
$$V = \frac{dR}{dt} = \frac{dr_2}{dt} - \frac{dr_1}{dt}$$

[2]

or



(a)



(b)

Figure 1. Linear (a) and curvilinear (b) motion with vector positions referred to an origin not on the line of motion.

$$V = v_2 - v_1$$

Thus, the relative velocity V is the velocity of object 2 in the moving reference frame of object 1. The important implication of this principle for measuring cognitive change is that our observation of change will be relative to the subset of cognitive elements chosen as a reference frame for the measurement.

While velocity is the central descriptive concept of motion, a more informative concept for developing explanation of the relationships between motion and the forces which govern that motion is acceleration. Acceleration is the rate of change of velocity with regard to time and is represented as

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad [3]$$

Thus, acceleration is the first derivative of velocity with respect to time. Alternatively, it may be thought of as a function of the rate of position change against time, the second derivative of s with respect to t

$$a = \frac{d^2s}{dt^2}$$

or more generally, taking into account changes in direction of velocity as well as magnitude of velocity,

$$a = \frac{dv}{dt} = \frac{dr^2}{dt^2} \quad [4]$$

The analysis of straight-line motion allows us to generate solutions for specific instances of change. By finding the mathematically defined way in which position and velocity vary with time along any one dimension, we can lay the groundwork for the multidimensional variation present in a longitudinal analysis.

The solution of derivatives is achieved by the process of differentiation.

From the differential calculus, we know that the derivative of any variable with regard to any other variable can be expressed as a linear or curvilinear function describing the graph of the one against the other.² Any particular function could represent either the theoretical expression of some relationship or the result of deduction from some empirical data using curve-fitting techniques such as the method of least-squares (Draper and Smith, 1966) or other computational procedure (Daniel and Wood, 1971). To arrive at the derivative, we apply a simple rule. Given a known function, the derivative will be equal to the exponent multiplied by the base with a new exponent equal to the original reduced by a factor of one (assuming that the exponent is a rational number). Thus, for the line function

$$y = mx^n$$

the derivative for any power of x is

$$\frac{dy}{dx} = mn x^{n-1}$$

For example, the derivative of

$$y = x^4$$

would be

$$\frac{dy}{dx} = 4x^3$$

This procedure, and the power rule, can be extended to functions with multiple terms and coefficients such that a line described by

$$y = 2x^4 + 6x^3 - x^2 + 4x - 2$$

would have the derivative

$$\frac{dy}{dx} = 8x^3 + 18x^2 - 2x + 4$$

Note that this procedure may be applied for higher order derivatives since d^2y/dx^2 is simply dy/dx with regard to x . Thus, for the above example

$$\frac{d^2y}{dx^2} = 24x^2 + 36x - 2$$

With respect to time and position, we have the line function

$$s = mt^n$$

and the resultant derivatives

$$\frac{ds}{dt} = mnt^{n-1} \quad [5]$$

or

$$v = mnt^{n-1} \quad [6]$$

and

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = mn(n-1)t^{n-2} \quad [7]$$

It is obvious from Eq. 5, Eq. 6, and Eq. 7 that the resolution of descriptive motion variables is based upon the time dependence of separation, velocity, and acceleration. This time dependence is represented geometrically as area under a time curve. Change in velocity is given as the area under a curve of acceleration against time and change in separation is given as the area under the curve of velocity against time. These relationships are represented in Figure 2.

The use of velocity and acceleration as change descriptors serves two purposes. The first of these is to describe or explain changes which have occurred and for which data already exists. This is achieved by the procedures described above. The second is to facilitate prediction of future positions. This is accomplished when a higher order derivative such as

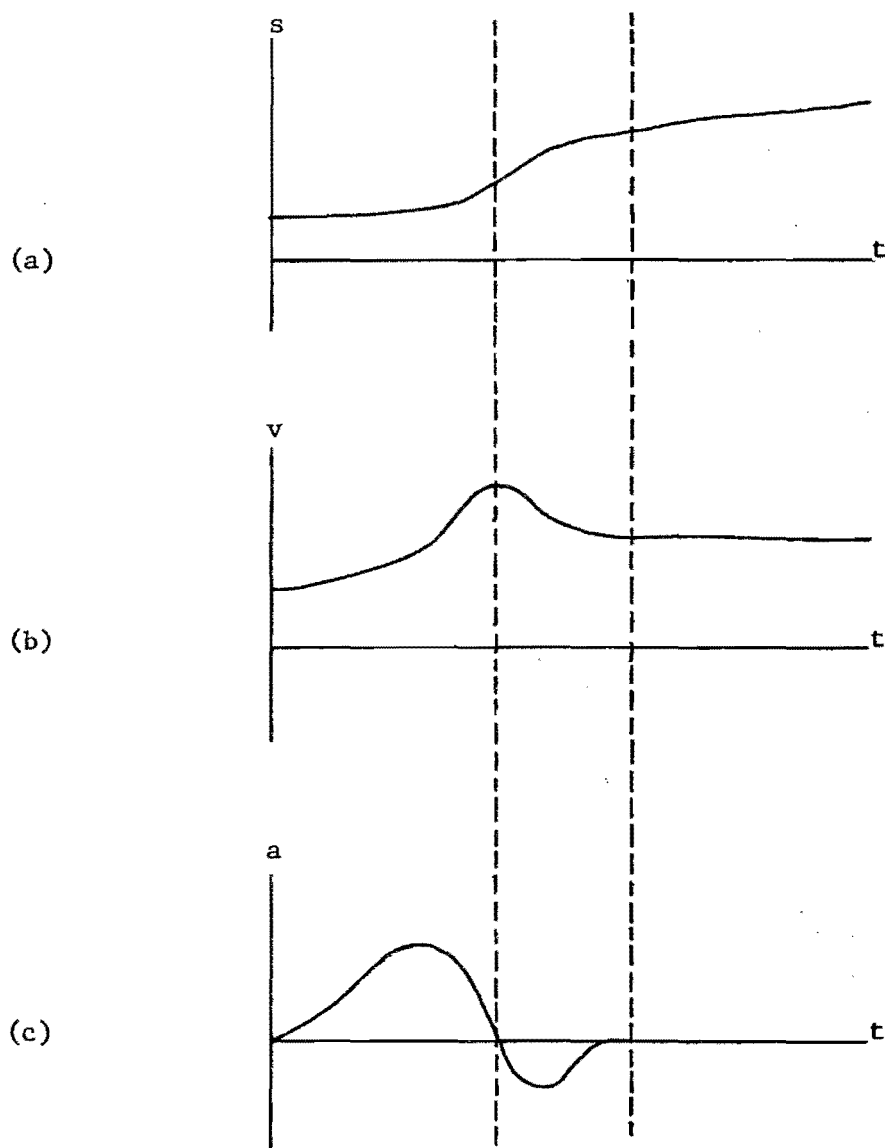


Figure 2. A set of graphs for a hypothetical change in position against time. Concurrent changes in (a) position, (b) velocity, and (c) acceleration are illustrated.

acceleration has been deduced from some other set of conditions such as a known influence (such influences might be force or mass, as discussed later in this paper). The prediction is made for some future point by reducing acceleration to its primatives, first velocity and then position. The equation for position can then be solved for some specific value of t .

Two procedures exist for finding position from acceleration or velocity, integration and antidifferentiation. These procedures are again based upon the time dependence of separation, velocity, and acceleration. The interrelationships of their functions are expressed by equations for the area under the curves of these functions. Mathematically, the area under a curve is written as a definite integral. For example, change in velocity takes the form

$$v_2 - v_1 = \int_{t_1}^t a(t) dt \quad [8]$$

where $a(t)$ indicates that acceleration is to be considered as a function of time. Often this evaluation is applied to data for which some set of initial conditions (v_0) are known for an indefinite period of time. This is written

$$v - v_0 = \int_0^t a(t) dt \quad [9]$$

In the case of Eq. 8, we are limited to a simple description of the change in velocity. Using Eq. 9, it is possible to identify some subsequent point on the time line for which we may derive a measure of the instantaneous velocity. Similarly, we may represent the distance traveled by finding the area under the velocity-time curve

$$s_2 - s_1 = \int_{t_1}^t v(t) dt \quad [10]$$

By evaluating the curve from $t = 0$ with some initial condition or value for s_0 , it is possible to find the resultant position of s for any arbitrary value

of t on the line of change according to

$$s - s_0 = \int_0^t v(t)dt \quad [11]$$

For most composite functions, the practical solution of an integral involves substitution of the functional form from a standard table of integrals (or by computer algorithm). For less complicated forms such as linear equations, monotonic curvilinear equations, and certain common trigonometric function, it is often desirable to use the computational equivalent of integration, antidifferentiation. This procedure simply reverses the differentiation technique and power rule. Thus, with a given curve of acceleration

$$a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = mt^n \quad [12]$$

we find velocity by

$$v = \frac{ds}{dt} = \left(\frac{m}{n+1}\right)t^{n+1} \quad [13]$$

and position by

$$s = \left(\frac{m}{n+1}\right)/(n+2)t^{n+2} \quad [14]$$

Thus, for the hypothetical condition

$$a = 2t^2 + 4t - 10$$

exponents would be raised by a factor of one and the coefficients divided by the new exponents to yield

$$v = 2/3t^3 + 2t^2 - 10t + c$$

and similarly

$$s = 1/6t^4 + 2/3t^3 - 5t^2 + ct + d$$

where c and d are constants which become ordinate intercepts for the equations and which are determined on the basis of initial conditions for the integral.

To generalize the discussion of motion descriptors, it is necessary to

develop kinematic equations for the hyperspatial case often found in multi-dimensional data. This will be facilitated by replacing the univariate scalar functions with vector functions of the form

$$F = (f_1, f_2, \dots, f_n)$$

where F has n independent components with the limit

$$\lim_{h \rightarrow 0} F(h)$$

and $F(h)$ is the vector function of h . For F to be differentiable, each component must have a limit $h \rightarrow 0$. Thus, we define the limit of $F(h)$ as

$$\lim_{h \rightarrow 0} F(h) = (L_1, L_2, \dots, L_n)$$

where $L_1 = \lim_{h \rightarrow 0} f_1(h)$, $L_2 = \lim_{h \rightarrow 0} f_2(h)$, \dots , and $L_n = \lim_{h \rightarrow 0} f_n(h)$.

For any point P moving along a curve in a n -dimensional space, we can describe n functions, $x_i = f_i(t)$, where $i = 1, 2, \dots, n$. The position vector, a vector from the origin to P is given by

$$R = \sum_{i=1}^n k_i x_i \quad [15]$$

or

$$R = \sum_{i=1}^n k_i f_i(t) \quad [16]$$

where k_i ($i = 1, 2, \dots, n$) are the orthogonal unit vectors which define the underlying coordinate system of the n -space.

The velocity of P in an n -space is defined by the derivative of R as

$$v = \frac{dR}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta R}{\Delta t}$$

where R is given as

$$R + \Delta R = \sum_{i=1}^n k_i (x_i + \Delta x_i) \quad [17]$$

as ΔR approaches zero. By subtracting Eq. 15 from Eq. 17 we get

$$\Delta R = \sum_{i=1}^n k_i \Delta x_i$$

Subsequent evaluation of ΔR with respect to Δt gives

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \frac{\Delta R}{\Delta t} &= \lim_{\Delta t \rightarrow 0} \left(\sum_{i=1}^n k_i \frac{\Delta x_i}{\Delta t} \right) \\ &= k_1 \lim_{\Delta t \rightarrow 0} \frac{\Delta x_1}{\Delta t} + k_2 \lim_{\Delta t \rightarrow 0} \frac{\Delta x_2}{\Delta t} + \dots + k_n \lim_{\Delta t \rightarrow 0} \frac{\Delta x_n}{\Delta t} \end{aligned}$$

In its most compact form

$$v = \frac{dR}{dt} = \sum_{i=1}^n k_i \frac{dx_i}{dt} \quad [18]$$

The result obtained by solving Eq. 18 is equivalent to the obtained solution from differentiation both sides of Eq. 15, holding k_i ($i = 1, 2, \dots, n$) constant. That is, the resultant is the sum of all $k_i f_i(t)$, since evaluation of $f_i(t)$ for any individual value of k_i produces a vector function with all other values of k_i equal to zero.³

The acceleration vector in generalized form is obtained from the velocity vector, again by subsequent differentiation

$$a = \frac{dv}{dt} = \sum_{i=1}^n k_i \frac{d^2 x_i}{dt^2} \quad [19]$$

Mass and force: Influences on motion

In the preceeding section, the kinematic equations for velocity and acceleration were developed as a descriptive system for motion, or change, of conceptual positions in a multidimensional space. This section will discuss those components of the Newtonian paradigm which may be adopted for

explanatory purposes with models of information influence and cognitive change. The two principal elements of this explanatory scheme are force and mass; these can be related to the kinematic elements through the equations for dynamics.

Force can be conceived of as a causal component of acceleration. As it has been shown, a point in motion moves at some velocity, or rate of change. If an interaction between the concept represented by a point and the external system occurs, a change in the state of motion occurs. This resultant of the influence of the external system is a change in the rate of change, or acceleration. In general, given the formulation of Newton's model of physical motion, acceleration is produced by a force⁴ acting on a body such that it is proportional to the mass of that body. In restatement, acceleration is directly proportional to the total force exerted. This is described by the equation

$$a = kF$$

where k is a proportionality constant. Alternatively, we may state the force-acceleration relationship as

$$F = k'a$$

where k' is the inverse of k .

Conceptually, physicists and engineers have used the constant k' as a factor of resistance to force. It was found (by Newton) that applying a known force to different bodies produced differential amounts of acceleration and change. Since force was held constant, it was posited that bodies which accelerated rapidly had a small resistance to change while bodies which accelerated slowly had a higher value of resistance. Newton inferred from this the existence of a quantity m , mass, and indicated

$$F\Delta t = m\Delta v$$

which can be reformulated (given $\Delta v \rightarrow 0$) as

$$F = ma \quad [20]$$

or

$$m = F/a$$

As a model of cognitive change, the relationship $F = ma$ may or may not be appropriate. It is an empirical question as to whether or not changes in associational distance are proportional to some influence on cognition. Yet, as the general systems theorists suggest, that while the analogy may not serve as "proof" of a theory, it may aid significantly in the formulation of that theory. Early evidence (Woelfel and Haller, 1971; Woelfel and Saltiel, 1973; Barnett, Serota, and Taylor, 1974) suggests that, as concepts become "known," as information about concepts is accumulated, they become more resistant to external influence or force. Thus, mass may be a viable as well as heuristic construct for this treatment.

Several types of force-mass-acceleration relationships exist. These include systems acting under constant force and constant mass, systems acting under variable force and constant mass, systems acting under constant force and variable mass, and so forth.

Under the conditions of continuing and constant force, motion can be described according to the elementary kinematic equations

$$v = at$$

and

$$x = \frac{1}{2} at^2 \quad (\text{total distance travelled})$$

The total effect of a force acting on a body at rest is given by the

equation

$$Ft = mat = mv \quad [21]$$

and

$$Fx = (ma)(1/2 at^2) = 1/2mv^2 \quad [22]$$

where F is the momentum of the system and Fx is the kinetic energy. For a period t , momentum is the impulse, or change in the rate of motion imparted to a body, and is expressed as the mass-distance ratio over time. For the same period t , Fx is the work done by the force expressed as the mass-distance ratio over distance. For a body in motion, and for which the initial conditions are known, Eq. 21 becomes

$$Ft = mat = m(v_2 - v_1) \quad [23]$$

while Eq. 22 becomes

$$\begin{aligned} Fx = max &= ma \left(\frac{v_1 + v_2}{2} \right) t \\ &= 1/2m(a)(v_1 + v_2) \\ &= 1/2(mv_2^2) - 1/2(mv_1^2) \end{aligned} \quad [24]$$

To eliminate the conditions of constant force and acceleration, and broaden the scope of application for these central dynamical equations, we restate Eq. 20 as

$$F = m \frac{dv}{dt}$$

multiply by dt and integrate to get

$$\int_{t_1}^t F dt = m \int_{v_1}^v dv = m(v_2 - v_1) \quad [25]$$

Similarly, multiplying by dx and integrating, we get

$$\int_{x_1}^{x_2} F dx = m \int_{x_1}^{x_2} \frac{dv}{dt} dx$$

with

$$\frac{dv}{dt} dx = \frac{dx}{dt} dv = v dv$$

therefore

$$\int_{x_1}^{x_2} F dx = m \int_{v_1}^{v_2} v dv = 1/2 m (v_2^2 - v_1^2) \quad [26]$$

In this way, we describe the effect of a force acting on a body along the line of motion. Under the condition of variable mass we would simply substitute dm/dt for m in the preceeding series of equations.

Of course force, taken as a vector quantity, may be decomposed into orthogonal components for solution according to the equations above. The vector F is given be

$$F = \sum_{i=1}^n k_i f_i$$

where f_i is the orthogonal component of F acting in the direction of k_i .

The basic force-acceleration relationship then becomes

$$F = \sum_{i=1}^n k_i f_i = ma = \sum_{i=1}^n m k_i a_i$$

or simply

$$F = \sum_{i=1}^n m k_i a_i \quad [27]$$

Reformulating Eq. 27 to solve for the orthogonal component of acceleration, we get

$$a_i = f_i/m \quad (i = 1, 2, \dots, n) \quad [28]$$

which similar to Eq. 19, combines as

$$a = \sum_{i=1}^n a_i = \sum_{i=1}^n f_i/m \quad [29]$$

Solving Eq. 28 according to some known or predicted conditions for force and mass, we arrive at the necessary information to solve Eq. 8 to Eq. 14 of the kinematic set. This allows us to move from a known dynamic solution to the description of motion state change and the prediction of future states.

In the final section, this procedure will be reviewed, some problems will be considered, and subsequent directions for development will be proposed.

III. Considerations for further development

The first two sections of this paper have dealt with the cognitive spatial model as a context for longitudinal communication theory and with the development of a calculus for such a mathematical theory. This treatment should, however, be recognized as rudimentary and incomplete because of several convergent problems. First, the components of a science of mechanics are extensive and go far beyond the scope of this work. Second, this presentation has considered only a few of the possible variables and equation structures for this type of analysis (however, note that solutions for many other variables can be achieved by simple transformations on the equations presented). Finally, theoretic and analytic developments lack the empirical support necessary to direct discussion beyond this point. Applications to the assessment and understanding of communication processes can only be conjectured without a test of the Newtonian relationships presented. A brief review of the suggested procedures will put these problems into perspective.

The series of equations in the second section can be divided into two main groups. The first set belong within a deductive framework for the solution of higher order descriptive models and the derivation of explanatory variables. The other set produce an inductive framework for predicting future system states

on the basis of existing or theoretically prescribed antecedents.

The deductive framework allows us to begin with multidimensional longitudinal data and from it fabricate useful interpretations of the data structure. The procedures of such an approach might be:

- a) The data matrices are computed and decomposed for description of simple motion along each of the independent dimensions. Usually, this will involve techniques of curve-fitting or regression to find the best linear or curvilinear form.
- b) These change equations are then differentiated to find the first and second derivatives of motion: velocity and acceleration. Accordingly, this provides the basic kinematic descriptions for the occurrence of changes in the data.
- c) By applying known or hypothetical values for force or mass we then solve the dynamic equations for impulse and work. This yields the basic dynamic descriptions of the changes which have occurred and allows us to relate those changes to some specific set of endogenous influences. It is at this level that we structure tests of the force and mass components so that values for these variables may be applied within an inductive framework for subsequent predictions of change.

For example, we might choose to develop a scale of mass. Using data in which the same force (perhaps a message given under rigid experimental controls) is applied to each cognitive element, we arrive at accelerations for each of the objects. We then use these results to define a mass scale under the condition of Eq. 20, where

$$F = m_1 a_1 = m_2 a_2 = m_3 a_3 = \dots$$

And, therefore

$$m_1/m_2 = a_2/a_1, m_1/m_3 = a_3/a_1, \text{ etc.}$$

Choosing any particular object (such as the one defined by $m_1 a_1$), we can arbitrarily define its mass as one standard unit. Against m_1 , a quantitative measure of all other masses (for a constant value t) may be obtained. Similar procedures may be applied to develop scales of force.

The inductive framework allows us to start with known or posited explanatory variables to arrive at a set of kinematic equations. From these equations, solutions for some future and indefinite value of time may be found. The general procedures involved here are:

- a) Values for the explanatory variables of force and mass are predicted from some theory or they are solved using experimental data and the equations for motion dynamics. Values from the scale development procedure suggested in (c) of the deductive scheme, might be applied in this instance.
- b) The acceleration function(s) derived from the solution of the dynamic equations are then integrated. This provides the researcher with functions for velocity and motion along orthogonal components of the multi-dimensional space.
- c) Given initial conditions for the system being analyzed, the orthogonal motions can then be solved for any future point in time under the assumption of the continuance of the conditions of force and mass for which the system was expressed (or under prescribed changing values of force and mass taken from empirical observation).
- d) The orthogonal components of the solution are resolved to find the predicted variable values given by the position vectors. The set of these position vectors for all possible concepts is the spatial matrix

at the prescribed point in time for which the orthogonal motion equations were solved.

As suggested, several problems exist with these analytic procedures. The first of these is a lack of parsimony resulting from the deletion of numerous components of mechanical analysis. This paper can only suggest the development of such methods as the vector analysis of momentum, the analysis of motion under the influence of collisions or (hypothetical, cognitive) gravitational force, or the analysis of such special cases as Coriolis force or harmonic motion. A treatment of the analysis of momentum, for example, would provide a more streamlined description of change and offer potentially faster and more complex solutions to the types of problems we are trying to solve. Exposition on these topics is, however, the objective of numerous, substantial works within the physical sciences. Further, given the current status of communication research technology, these issues remain solely within the realm of theory.

A second problem is the selection or generation of starting values and data from which to solve the system of kinematic and dynamic equations. We have assumed the availability of real or theoretically prescribed values for time, position, force, and mass. In fact, our procedures dictate that some part of this set exist in a combination which allows us to solve for the remaining unknowns. However, situations may arise in which we have to begin with some other variable such as kinetic energy. In instances such as this, it will be necessary to insert steps into our procedure and derive new equations from those presented so that the system may be solved for its more basic components.

The final major problem or concern of this paper is the decision to include and exclude variables. We should ask which components are missing from the discussion, which are unnecessary, and which should be modified or deleted from the system of equations. This is the question of isomorphism between

analytic scheme and the processes which we seek to describe. The solution of this problem provides a key to the direction in which our efforts should proceed.

At one level, the treatment of descriptive and explanatory variables has been elementary. The principal components of the extant Newtonian model have been proposed as a practical tool for studying communication and its effects on cognition. They are assumed to provide a satisfactory scheme for analyzing multidimensional data. At a second level, this treatment is a highly complex proposal of theoretical interrelationships between messages and cognition. Either perspective dictates the necessity for testing. If the procedures described above can be sustained under the tests of internal and external validity, then the continuing adaptation from mechanics will be an important contribution to the study of communication. If the present paradigm and its elementary analogies withstand this test, the next steps will have already been provided in the advanced treatment of mechanics. If they fail, then it will be necessary to seek reformulations which better fit the description and representation of human information processes.

FOOTNOTES

1. Several extensive treatments of this issue are available. In addition to articles by Woelfel and by Serota (preparatory to this work) which formulate arguments in specific terms of multidimensional scaling theory, see philosophical discussions by Einstein (1961), Torgerson (1958), Suppes (1951), and Stevens (1951).
2. We will exclude from this discussion the set of discontinuous functions for two reasons. First, they are mathematically nondifferentiable and thus difficult to describe in terms of velocity and acceleration. Second, since multidimensional cognitive space models are being used in this context to describe a continuous phenomena (e.g., communication processes), it is not necessary to consider discrete functions at this time.
3. The metric tensor for solving an orthogonal coordinate system is Kronecker delta. Therefore, a value of one for any single element k_i will cause all other values in the set of k to be equal to zero.
4. While we speak of a singular force acting on a body, we are actually referring to the resultant of a system of forces. Since force is a vector quantity, the resultant is simply the vector sum of all forces acting. The total effect of a system of forces will be in the direction of the resultant.

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