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A DYNAMIC MODEL OF THE EFFECT OF DISCREPANT INFORMATION ON UNIDIMENSIONAL ATTITUDE CHANGE[1]

by Stan A. Kaplowitz[2], *Edward L. Fink*[3], *and Connie L. Bauer*[4]

A differential equation model of the attitude change process is proposed which considers an individual's attitude as affected by discrepant messages along a unidimensional continuum. The model posits two dynamic components: *translation* to a new equilibrium due to the impulsive force of the message, and *oscillation* around the moving equilibrium due, in part, to restoring forces resisting displacement from equilibrium. An experiment to test some of the implications of the model is performed, in which 1174 subjects receive one of three messages differing in discrepancy from their attitude, and in which they are randomly assigned to varying times during which they may consider the message; almost all subjects consider the message for a period of not less than 10 and not more than 815 seconds. A structural equation with translation and undamped oscillation is found to significantly but modestly fit the data. Implications for the periodicity and temporal parameters of the attitude change process are discussed, and theoretical and methodological implications of the approach taken are considered.

KEY WORDS: individual, decision making, attitude change, discrepant messages.

IN 1969, MCGUIRE, commenting on the various theoretical approaches to the study of attitudes, stated the following:

> Typically, they [the theories] make, not contradictory predictions, but rather predictions dealing with different independent variables and different mediating processes. Each stimulates research ... by suggesting additional predictions that probably would not have been formulated on the basis of other theories. (p. 271)

Over a decade later we find the same leading theoretical approaches unintegrated; no one approach has clearly been adopted by the research community.

The approach to the study of attitudes to be presented here is to attempt to build a dynamic mathematical model which will allow assumptions to be explicit, implications to be derived symbolically, and the possibility that analogies to other systems may be fruitfully employed. Further, such a model can make relatively precise predictions that may be systematically considered. Models which make such precise predictions can, even in their rejection, sharpen our thinking (cf., Kac, 1969, p. 699).

A number of investigators (Saltiel & Woelfel, 1975; Danes, Hunter, & Woelfel, 1978; Laroche, 1977; Anderson, 1974; Hunter & Cohen, 1972; Fink, Kaplowitz & Bauer, 1983) have proposed mathematical models which predict attitude change as a function of the discrepancy between one's initial position and the position advocated in a message, plus perhaps such other variables as source credibility and prior information about the topic.

The limitation of all of these models is that none of them treat time as a variable. This would not be a problem if, upon receipt of a message, attitudes changed instantaneously and then remained completely fixed until receipt of another relevant external message. Rather, the evidence appears to support McGuire's (1960, pp. 345–346) assertion that "the impact of

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the message on the remote issue occurs only gradually, the opinion ... continuing to change ... for some time after the receipt of the persuasive message." Walster's (1964) study of post-decisional dissonance found some evidence for cognitive oscillations over time. Four minutes after choosing a job, subjects showed clear evidence of regreting their decision. After 15 minutes, she found the usual dissonance reduction effect, and after 90 minutes neither effect was evidenced. Tesser and Conlee (1975) have shown that merely thinking about a topic can lead to attitude change over time. Theories based on models of cognitive structure, such as that by Hunter and his associates (Hunter, Levine, & Sayers, 1976; Poole & Hunter, 1979, 1980), also predict change over time without the receipt of additional messages. Other studies have found that experimentally induced attitude change can decay over time as a result of forgetting of the message and/or the source of a message (see review by Cook & Flay, 1978), or polarize over time as a result of schema-consistent thought (Tesser, 1978).

While a number of studies have demonstrated that even in the absence of new external messages attitudes do change over time, they lack any theoretical apparatus for predicting or explaining these trajectories. By contrast, most of the mathematical models which have been developed simply attempt to predict the end point of the attitude change process without dealing with the trajectory by which this end point is reached; one of the few truly dynamic models (Recker, 1977) is too cumbersome to employ.

Our aim is to develop a dynamic model which deals with the process of attitude change as it proceeds in the absence of further external messages. (Hunter & Cohen's, 1972, "dynamic" models, by contrast, assume that the actor is continuously receiving a particular external message. Hence, these models confound the effects of time with the effect of additional messages.) What can such a dynamic model tell us? First, instead of only being able to predict the equilibrium point of the process, it can make predictions at other points as well. Second, and even more important, to describe the change of an attitude after receipt of a message requires positing various internal forces or processes governing this change. To the extent that we can develop a satisfactory model, these forces will be revealed. Below we present a model in differential equation form; we will then indicate the trajectories and other implications of this model, and report an experiment which estimated and tested aspects of the model.

In order to test such a model one must meet two criteria. First, one must have a very accurate reading of the trajectory, which requires measuring attitudes at many different points in time. Second, it is essential that subjects receive no external messages during the study. No prior study comes close to satisfying these criteria. No study that we are aware of measures attitudes at more than five points in time and many have no control over any external messages received.

Our theory of cognition is aided by the use of a mechanistic metphor. Metaphors are common in the development of scientific theory, as they suggest a limited number of relevant variables and some possible functional relationships among them (Leatherdale, 1974).

A MODEL OF ATTITUDE CHANGE

Our model assumes that:

A1: A cognitive system is a set of concepts; a given concept has both a location and a mass in a cognitive space.

A2: Change in a belief or attitude regarding a particular concept is equivalent to motion of that concept in the cognitive space. (Woelfel & Fink, 1980)

A3: Following McGuire (1969, p. 257), we regard a message as an impulse which disturbs the existing state.

A4: As in Newtonian mechanics, we assume that the amount of acceleration of a concept in the cognitive space will be equal to the amount of force acting upon the concept divided by the mass of that concept.

A5: Moreover, the inertial mass of the

concept is assumed to be a monotonically increasing function of the information the actor possesses about that concept (see Saltiel & Woelfel, 1975 and Danes, Hunter, & Woelfel, 1978 for evidence supporting this assumption).

While an external message is assumed to be a major force for attitude change, we posit the existence of certain additional forces within the actor's cognitive system. First, we assume that some concepts are connected to others and that such connections create resistance to change. This idea is supported by evidence that people will reject, avoid, or distort messages which conflict with cherished beliefs (Festinger, 1957; Aronson, 1969). It is also consistent with research indicating that resistance to persuasion can be increased by anchoring a belief to other beliefs or to positively valenced individuals or groups (Nelson, 1968; Holt, 1970; Watts & Holt, 1970; cf., Leppaluoto, 1972; see McGuire, 1964, pp. 196–197, for many relevant citations). Further, it is consistent with the ability to change beliefs with regard to one concept or set of concepts by providing messages or inducing thoughts about other, related concepts (Leippe, Greenwald, Baumgardner, 1982; Anderson, 1982).

There are two different mechanisms by which concepts can be linked. One mechanism is the rigid brace, in which the distances between the various concepts remain fixed. The other is a spring mechanism (see Woelfel & Fink, 1980, p. 159) in which the distances between the concepts can be changed by the presence of oscillations. For small to moderate distortions, the restoring force of the spring is proportional to the stretch or compression of the spring. Such a spring model offers considerable promise because it allows for an initial motion in the direction of the message force to be followed by movement back towards (and even past) the initial position, after counter-arguing and other consistency restoring processes have taken effect. Since a brace can be regarded as a spring whose restoring coefficient is so high that there is no observable oscillation, there is no loss in generality in testing only for the existence of a spring mechanism. Further, as Lorenz (1977, p. 237) suggests, "any self-regulating process in whose mechanisms inertia plays a role tends toward oscillation," and it is not unreasonable to consider, as Lorenz does, cognitive processes within this set.

Second, we assume the existence of a frictional force slowing down the translation (forward motion) of the concepts and slowing down the oscillations resulting from any spring-like connections. An important reason for making such an assumption is that without it, once a message has caused some attitude change, the attitude will keep changing indefinitely. There are, however, three cognitive processes which can be responsible for these frictional and damping forces we have posited. One is the process of *forgetting* the message (or at least its loss from short-term memory). Another may be a person's need to make a decision on an issue so as to stop agonizing over it. Since thought takes energy, we can simply assume that at some point cognitive processing ceases because the expenditure of the energy is not efficient (Berger & Luckmann, 1967, p. 53). Finally, reality constraints and distractions serve to allocate cognitive energies elsewhere, ending indefinite cognitive effort on a given problem.

We now make the assumptions of our model more precise. 1) The concept whose motion we are studying, concept J, has mass m and is located at y, in our unidimensional continuum. 2) J is connected by a spring to an anchoring concept A, with mass m_A and location y_A. (Of course, one concept may be anchored to several others. We are, however, only examining the motion of J, not the motion of these anchor concepts relative to each other, and we are examining motion in only one dimension. Therefore, assuming that the entire mass of all anchor concepts is located at their center of mass is a reasonable simplification.) 3) Prior to the message, the system is in equilibrium. Hence, the restoring force of the spring is zero whenever its length (the distance between J and A) is equal to the initial length, $y_0 - y_{A_0}$. 4) At $t = 0$, an impulse is delivered to J. (Since we are

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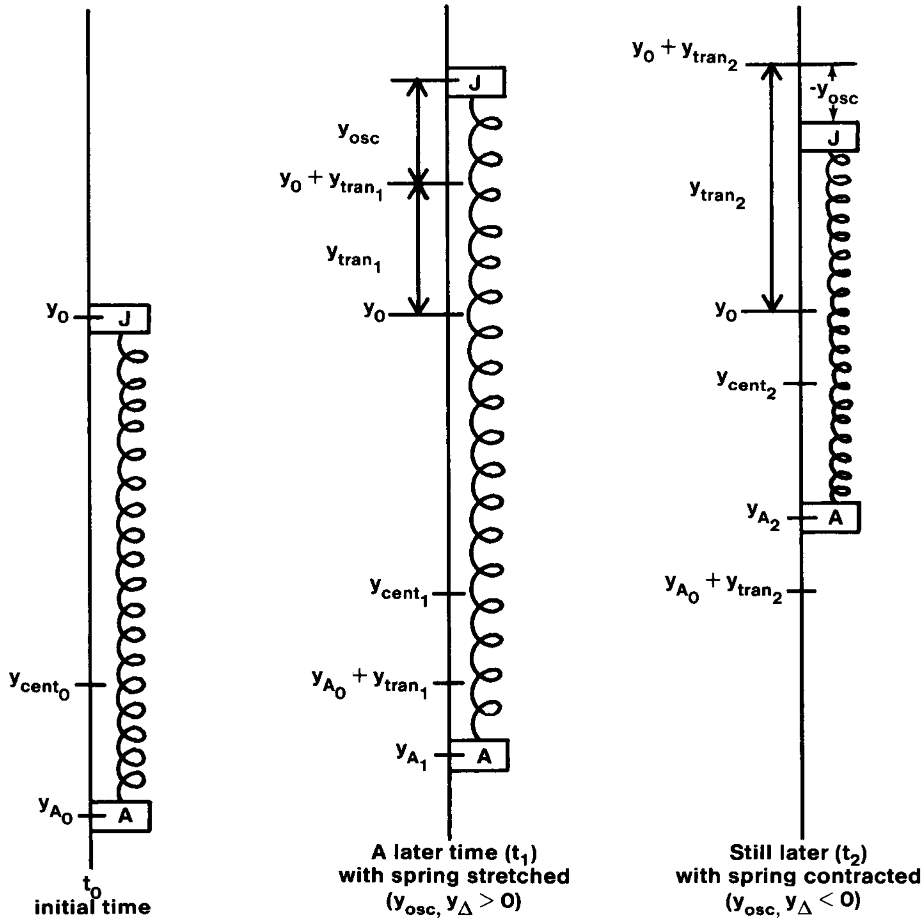

FIG. 1. Spring model of attitude change over time, where A = anchor, J = implicated concept, y_{cent} = location of center of mass, y_A = location of anchor, y = location of implicated concept, y_{tran} = amount of system translation, y_{osc} = amount of oscillation, y_Δ = amount of spring stretch.

considering only one dimension, the effective force is the projection of the force along the vector connecting A and J. Hence, a force which is orthogonal to that vector would have an effective force of zero.) 5) In addition to the restoring force and the frictional force described above, we also assume the existence of a damping force within the spring. As with all springs, this damping force is assumed proportional to the rate of change of the length of the spring. (See Kaplowitz and Fink, 1982, for an explication of other variants of spring models of cognition.)

When the impulse hits J, it produces two kinds of motion: translation of the center of mass of the entire J-A system, and oscillation of both A and J relative to the moving center of mass. The *translational component* of J's motion tells us how much J would have moved if there were only translation and no oscillation. Hence, $y_{tran} = y_{cent.mass} - y_{cent.mass_0}$, and, at $t = 0$, $y_{tran} = 0$.

The *oscillatory component* of J's motion, y_{osc}, is equal to $y - y_0 - y_{tran}$. At $t = 0$, we must have $y_{osc} = 0$. There are two reasons for this. First, if the spring has a damping force, then unless a relevant message was received just a short time before $t = 0$, oscillations from any previous messages should have died out. Second, if there is no damping force and if individuals are oscillating from previous messages, a sufficiently large sample should contain as many people for whom the initial value of y_{osc} is negative as people for whom it is positive. Hence, the mean value of y_{osc} will be zero at $t = 0$ (see Fig. 1).

We now consider the forces and equations of motion for these two components separately. The equation of force and acceleration for the *translational* motion is:

$$M\ddot{y}_{tran} + C\dot{y}_{tran} = 0, \tag{1}$$

where $M = m + m_A$ (the total system mass), C is the coefficient of translational friction, and where the dots indicate the first and second derivatives of y_{tran} with respect to time. The solution of Eq. (1) for $C \neq 0$ is:

$$y_{tran} = a_1 + a_2e^{-(C/M)t}. \tag{2}$$

In this equation and all other differential equation solutions to be presented, the a_i reflect initial conditions. When $t = 0$, we expect no translation to have occurred; hence, at $t = 0$,

$$a_1 + a_2 = 0.$$

Therefore,

$$a_1 = -a_2, \quad \text{and}$$

$$y_{tran} = a_1(1 - e^{-(C/M)t}). \tag{3}$$

If $-C/M < 0$, then as $t \rightarrow \infty$, $\hat{y}_{tran} \rightarrow a_1$, a new, stable equilibrium position. If $-C/M > 0$, then as $t \rightarrow \infty$, $\hat{y}_{tran}$ increases without limit and there is no equilibrium. Since we expect C (the linear damping coefficient) to be positive, and M (the mass of the entire system) to be positive, it follows that $-C/M < 0$, and the translating component is expected to reach equilibrium.

We now consider the *oscillating* component of J's motion. The force-acceleration equation is:

$$m\ddot{y}_{osc} + c\dot{y}_\Delta + ky_\Delta = 0, \tag{4}$$

where c is the spring's linear damping coefficient and k is its linear restoring coefficient.

$$y_\Delta = y_{osc} - y_{A,osc} \tag{5}$$

is the expansion ($y_\Delta > 0$) or contraction ($y_\Delta < 0$) of the spring from its equilibrium length. Because the oscillations of A and J do not affect the center of mass (motion of the center of mass is translation) and since at the spring's equilibrium length, $y_{A,osc} = y_{osc} = 0$, it must always be the case that

$$-m_A y_{A,osc} = my_{osc}. \tag{6}$$

Solving for $y_{A,osc}$ in Eq. (6) and substituting for y_Δ in Eq. (4) changes Eq. (4) to

$$m\ddot{y}_{osc} + c(1 + m/m_A)\dot{y}_{osc} + k(1 + (m/m_A))y_{osc} = 0. \tag{7}$$

Letting $c^* = c(1 + m/m_A)$ and $k^* = k(1 + m/m_A)$, we can rewrite equation (7) as

$$m\ddot{y}_{osc} + c^*\dot{y}_{osc} + k^*y_{osc} = 0, \tag{8}$$

where we call c^* the weighted linear damping coefficient and k^* the weighted linear restoring coefficient. Analysis of this equation leads to the characteritics equation

$$m\lambda^2 + c^*\lambda + k^* = 0,$$

which is a quadratic equation. If we set m equal to 1, we find

$$\lambda = -c^*/2 \pm \frac{\sqrt{c^{*2} - 4k^*}}{2}.$$

This equation results in three distinct structural solutions for the oscillation component of the differential equation model (for demonstration, see, e.g., Haberman, 1977, Petrovskii, 1969; Greenberg, 1980).

1. If $c^{*2} > 4k^*$, the λs are real and unequal. This is referred to as the *overdamped case.* In this case,

$$\hat{y}_{osc} \equiv \hat{y}_{over} = a_3e^{\lambda_1 t} + a_4e^{\lambda_2 t};$$

since at $t = 0$, $y_{osc} = 0$, $a_3 = -a_4$. The equilibrium of the oscillating component in this case depends on the values of λ_1 and λ_2. If either $\lambda > 0$, the system is unstable. It may be demonstrated that if either λ is

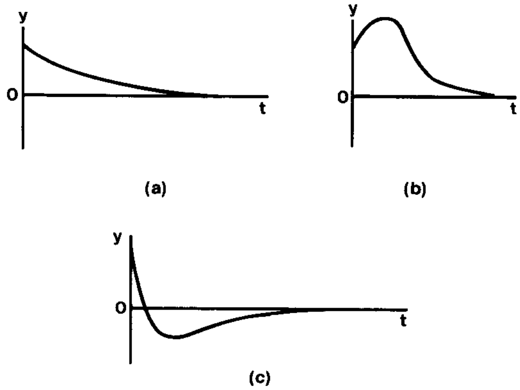

FIG. 2. Trajectories of overdamped or critically damped oscillating component. (From Haberman, 1977, p. 40).

positive, then both λs are positive and c^* and k^* must be nonzero and of opposite sign, while if either λ is negative, then both are negative and c^* and k^* must be nonzero and of the same sign. We believe that the restoring coefficient and the damping coefficient are nonnegative; therefore $\lambda_1 < 0$ and $\lambda_2 < 0$. The oscillating component $\hat{y}_{osc} \equiv \hat{y}_{over}$ is then stable. As $t \rightarrow \infty$, $\hat{y}_{over} \rightarrow 0$. The trajectories of stable overdamped systems appear as in Fig. 2.

2. If $c^{*2} = 4k^*$, the λs are real and equal. This is referred to as the *critically damped case.* (In practice, since this requires an exact equality estimated from empirical data, it is usually considered not to occur in nonidealized systems.) In this case,

$$\hat{y}_{osc} \equiv \hat{y}_{crit} = e^{\lambda t}(a_3 + a_4 t);$$

since at $t = 0$, $y_{osc} = 0$, a_3 must be 0, and $\lambda = -c^*/2$. If the damping coefficient is positive (i.e., there is "friction" in the system, or energy which is lost to the environment), then, as $t \rightarrow \infty$, $y_{crit} \rightarrow 0$. If there is no damping force or a negative damping force, then as $t \rightarrow \infty$, $y_{crit} \rightarrow \infty$; the oscillating system cannot achieve an equilibrium. We assume that $c^* > 0$ and, therefore, the equilibrium value for $\hat{y}_{crit}$ is 0. The stable trajectories for $\hat{y}_{crit}$ are indistinguishable from the stable trajectories of $\hat{y}_{over}$ (see Fig. 2).

3. If $c^{*2} < 4k^*$, the λs are complex conjugates; defining

$$r = -c^*/2$$

and $\omega = \sqrt{c^{*2} - 4k^*}/2i$,

$$\lambda_1 = r + \omega i$$

and $$\lambda_2 = r - \omega i.$$

In this case,

$$\hat{y}_{osc} \equiv \hat{y}_{under} = e^{rt}(a_3 \sin(\omega t) + a_4 \cos(\omega t));$$

since at $t = 0$, $y_{osc} = 0$, $a_4 = 0$. r indicates how quickly the oscillation dies out: if $r = 0$, the oscillation around the equilibrium value continues indefinitely. If $r < 0$, the oscillation damps out over time and the equilibrium of the oscillating component is stable. If $r > 0$, the amplitude of the oscillation increases without limit, and the oscillating component is unstable. Since c^* is assumed ≥ 0, then $r \leq 0$, and the equilibrium value of the oscillating component is 0. Fig. 3 presents graphs of the oscillating

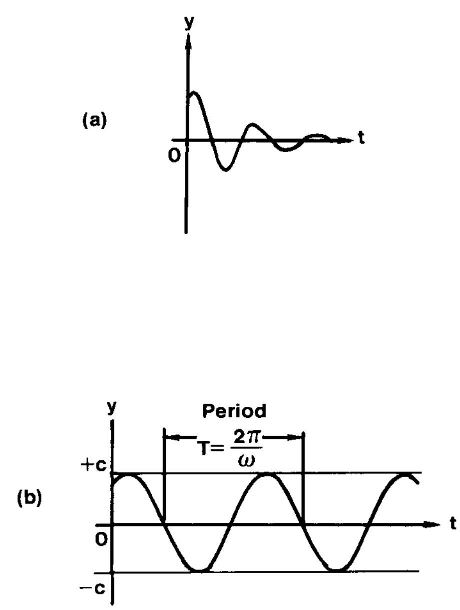

FIG. 3. Trajectory of underdamped oscillating component: (a) with damping; (b) without damping. (From Thomas, 1972, p. 922 and p. 924).

TABLE 1
STRUCTURAL VERSIONS OF THE COMPLETE DIFFERENTIAL EQUATION MODEL, ASSUMPTIONS FOR EQUILIBRIUM, AND THEIR DESCRIPTIVE PARAMETERS.

General Structural Version of the Complete Differential Equation	Structural Versions after Imposing Initial Conditions ($\hat{y}_{tran_0} = \hat{y}_{osc_0} = 0$)			
	Equation	c^*	k^*	y_∞
Overdamped Case:				
$\hat{y} = a_3e^{\lambda_1 t} + a_4e^{\lambda_2 t} + (a_1 + a_2e^{-(C/M)t}) + a_5$	$\hat{y} = a_3(e^{\lambda_1 t} - e^{\lambda_2 t}) + a_1(1 - e^{-(C/M)t}) + a_5$	$-(\lambda_1 + \lambda_2)$	$\lambda_1\lambda_2$	$a_1 + a_5$
Critically Damped Case:				
$\hat{y} = e^{\lambda t}(a_3 + a_4t) + (a_1 + a_2e^{-(C/M)t}) + a_5$	$\hat{y} = e^{\lambda t}(a_4t) + a_1(1 - e^{-(C/M)t}) + a_5$	-2λ	λ^2	$a_1 + a_5$
Underdamped Case:				
$\hat{y} = e^{-(c^*/2)t}(a_3\sin(\omega t) + a_4\cos(\omega t)) + (a_1 + a_2e^{-(C/M)t}) + a_5$	$\hat{y} = e^{-(c^*/2)t}(a_3\sin(\omega t)) + a_1(1 - e^{-(C/M)t}) + a_5$	$-2(-c^*/2)$	$(-c^*/2)^2 + \omega^2$	$a_1 + a_5$

m (mass of impacted concept) $\equiv 1$. For stable equilibrium, we assume $c, C, k \geq 0$, $m, M > 0$, $\lambda_1, \lambda_2, \lambda < 0$. $a_5 = y_0$.

component with damping (Fig. 3a) and without damping (Fig. 3b).

The above analysis indicated three possible structural solutions for the hypothesized differential equation model of the oscillations, with the following assumptions and implications summarized in Table 1.

As indicated earlier, many of the mathematical models of attitude change predict the change as a function of the discrepancy between one's initial position and the position advocated. By contrast, this model, while specifying the process of attitude change, does not predict attitude change as a function of discrepancy. Rather, messages of different discrepancies are assumed to exert different forces, resulting in different impulses. This sets up different initial conditions in the solution equation and different trajectories. In the experiment to be presented, we will evaluate the adequacy of this model.

METHOD

Experimental data collection

Overview. Subjects were approached by an experimenter who gave them one of three small booklets dealing with the Student Health Service Fee at Michigan State University (MSU). One booklet advocated increasing the current rate of \$18 per term (quarter) by \$20 per term. Another advocated an increase of \$40 per term. The third one stated that while some advocated increasing the fee, others thought this unnecessary. The experimenter then asked the subject to think about the issue. After a randomly selected time interval, he/she returned to ask the subject to indicate his/her recommendation regarding the Health Service Fee. Hence, we have two independent variables: discrepancy of message received and time lag from receipt of message to response; our one dependent variable is the health service fee increase recommended.

Topic. Using the topic of the health service fee has the following advantages. First, a change proposed or adopted in this fee may be measured on a theoretically continuous and unbounded scale (dollars). Further, the scale unit (one dollar) is quite meaningful to each subject and has high consensual meaning. Thus, we hope to minimize measurement problems that come about from discrete, bounded, and novel scales: our reliability on this measure should be quite high, and data transformations to meet distributional assumptions should be simple to operationalize if they are needed.

Second, this topic is of extreme interest to the students of Michigan State University (our subject pool). This is evidenced by articles on this topic that appeared reasonably frequently in the student newspaper over the school year (1979–80).

Third, we wanted a topic for which there was a relatively high premessage consensus. From pretests we found that most students are opposed to a large increase in the health service fee, and that they view "extreme" positions as positions that advocate a large increase (rather than a decrease) in that fee. Pretests took place in which the authors asked students in their classes (N = 76) for the students' own views of what a health service fee increase should be, and what a "maximum" increase should be. The mean increase advocated (sometimes with conditions for change in service) was $5.18 (range: −18 to 25); the mean "maximum increase" was $9.49 (range: 0 to 32).

Based on the pretests, two levels of discrepant position were chosen for the experimental messages: a moderately discrepant position (a $20 increase in the quarterly fee), and an extremely discrepant position (a $40 increase in the quarterly fee).

Experimenters. Experimenters were students enrolled in the Winter, 1980 undergraduate course on persuasion taught in the Communication Department at MSU by the second author (N = 67), and the students enrolled in the undergraduate course on social psychology taught in the Sociology Department at MSU that same term (N = 80) by the first author. The students gathered the data as part of the requirement in both courses for research participation.

Subjects. The subjects were sampled from students in the main library at MSU between February 14–27, 1980. To be included in the sample, a subject must have been unacquainted with the experimenter and not adjacent to another student. The total number of subjects in the study was 1,228.

Procedures. Experimenters worked in pairs on one side of the main library. Each experimenter was responsible for gathering data from nine subjects. To accomplish this, each experimenter received index cards which indicated the three time lags to be utilized for each of three messages. Each experimenter had a watch that indicated seconds, booklets with the messages concerning the health service fee, and index cards previously assigned to experimental conditions for the subject's response.

Experimenters randomized the index cards and approached library users whom they did not know and who were working alone. It was thought that those who were alone in the library would be unlikely to talk about this issue to others during the time between presentation of the message and their response. The experimenters asked each potential subject if he/she was a student at MSU and if he/she would agree to participate in a survey. If the student agreed to participate, he/she was given one of three booklets, ostensibly from "MSU Students for Better Campus Health." In the *no position advocated* condition, the booklet began:

> MSU Students for Better Campus Health is a recently formed group who want to improve the medical and psychological help available to students at MSU. Some people have proposed to our committee that the best way to improve the care available to students is to increase the Student Health service fee, to provide extra funds for more and better personnel. Others believe that any increase in this fee is not needed.
>
> Before we make any formal recommendation to the President's Special Committee on Health Care Services, we want to know how members of the student body feel about this proposal. We would like you to think about this for a little while and then we will ask you whether you favor increasing the student health service fee and, if so, by how much.

In the $20 and $40 conditions, the booklet began with the same first sentence, and then went on to state the following:

> Studying the situation has led us to conclude that the current health care situation on campus needs substantial improvement. We have also come to the conclusion that the only way to improve things is to spend more money on these services. Unfortunately, the financial situation is very tight for both the University and the State of Michigan, so the only way to get additional funding is through increasing the student health service fee.
>
> After careful investigation, *we strongly recommend increasing the student Health Service fee by $20 ($40) per term, from the current fee of $18 per term to a total of $38 ($58) per term.* Since there are

> about 44,000 students at MSU, most of whom attend school for three terms each year, this increase in the student health service fee would provide an additional $2.75 ($5.5) million per year. This money will be used to greatly improve the quantity and quality of personnel available to serve the medical and psychological needs of students at MSU, as well as to provide all necessary medical supplies and equipment.
>
> Before we make any formal recommendation to the President's Special Committee on Health Care Services, we want to know how members of the student body feel about this proposal. We would like you to think about this for a little while and then we will ask you whether you favor increasing the student health service fee and, if so, by how much.

The message in all booklets concluded as follows:

> Since we want your own candid view, free from any other influences, we would like to request that you not discuss this issue with anyone, including the person surveying you, until after you have given us your response.
>
> Thank you very much for your cooperation.

The experimenter recorded the time at which the subject finished reading the booklet, and then either gave the subject the index card to fill out (the "immediate" time lag condition), or said, "I'd like you to consider this. I'll be back in a short while." By pre-assignment, the index card indicated to which of 21 time-lag conditions the subject was assigned. These conditions were "immediate," 30 seconds, 1 minute, and so on every thirty seconds until ten minutes. If the subject was not in the "immediate" condition, the experimenter returned to get the subject's response at the time lag indicated. (Of course, some subjects were unavailable at that time. The experimenter was to return until the subject was available.) When the experimenter returned, the subject filled out an index card which said:

> I favor increasing the student health service fee
>
> of $18.00 per term
> by $_____ per term
> for a total of $_____ per term.

In addition, the card asked for the subject's sex, the number of terms the subject had been at MSU, and the number of times the subject used the services of the Health Center that academic year. The experimenter recorded the time at which the subject answered the first question (increase in health service fee).

Following this, the experimenter gave the subject a debriefing form. This stated that the survey was being done for a class studying attitude change, and asked the subject not to discuss the study for two weeks. All subjects agreed to this. Any questions the subject had were then answered, and the subject was thanked for participating.

Operational assumptions. We now make explicit the assumptions that allow us to test a model of individual cognitive dynamics by using many people at different times. We assume that for all subjects, c, C, m, M, and k are equal; that the probability of a subject processing the information from the message in a given specific time interval t to $t + \Delta t$ is equal across subjects; and that the subjects are equal in their initial value y_0.

Data analysis and strategy

Data to be analyzed. Subjects for whom both the subject's position on increasing the health service fee and the resulting total health service fee were missing were eliminated from analysis. Subjects who provided only their proposed total fee had their increase fee score estimated as TOTAL FEE−18 (since the correct fee is $18.00). Subjects whose proposed total fee and proposed increase fee were inconsistent (i.e., INCREASE + 18 ≠ TOTAL) had their increase fee estimated by giving equal weight to the two variables we measured. This was done by setting INCREASE = (INCREASE RAW + TOTAL − 18)/2. Subjects for whom the time lag between presenting the message and obtaining the subjects' response was estimated as less than 10 seconds had their time-lag score converted to 10 seconds, since it was not possible to respond in significantly less time that this. For all other subjects, the actual time, in seconds, was used in the analysis.

It should be noted that the proposed increase in the health service fee by the subjects is not being transformed or smoothed in any way. (Smoothing is the use of a measure of central tendency for the dependent variable based, in part, on values of the dependent variable nearby in terms of the independent variable). To find harmonic components, smoothing is often employed, especially for data at equal time intervals (see, e.g., Mayer & Arney, 1974, p. 326). On the other hand, "... smoothing the data smooths the function being estimated as well. ... Smoothing, then can smooth away important effects" (Mosteller & Tukey, 1977, p. 61). We have chosen to use untransformed data except to correct for inconsistent scores.

Choosing the structural solution. To evaluate the differential equation model, three processes must be engaged in: first, a structural version must be chosen as the correct functional form; second, the parameters of the structural version must be estimated; third, the estimated parameters of the structural equation must be converted to estimates of the differential equation model.

Determining the structural version. To determine the structural version, data from each message condition will be used separately. If one of the three versions results in a better fit to the data (by a smaller sum of squared residuals), then this version will be retained as the version to utilize to fit the data for all conditions simultaneously.

Parameter estimation. To estimate the parameters, the data from the three message conditions will be utilized simultaneously. To utilize all the data simultaneously, parameters that are expected to have certain values regardless of message condition will be constrained to these values, while those parameters for which this is not expected will be free parameters.

The constrained values for the parameters come from three sources. First, by design (i.e., random assignment to message condition), we expect the initial value y_0 (i.e., the attitude prior to receiving a message) to be the same, regardless of message condition.

Second, we expect the attitudinal trajectory to have a stable equilibrium. Thus, the assumptions indicated in Table 1 for the structural equations will be imposed. (It should be noted that parameter estimation via programs such as the SPSS NONLINEAR routine required these same assumptions so that "overflows" can be avoided and the minimization function can iterate to solution; see the Appendix for technical information concerning the procedures used here).

Third, we expect some parameters to be equal regardless of message condition. These are the values directly derivable from the differential equation model, and are functions of m (the mass of J), M (the combined mass of A and J), c^* (the weighted linear damping coefficient), and k^* (the weighted linear restoring coefficient). In the overdamped case, these parameters are λ_1, λ_2, and $-C/M$. In the critically damped case, these parameters are λ and $-C/M$. In the underdamped case, these parameters are $-c^*/2$, ω, and $-C/M$. Note that the parameters indicated by as are not functions of m, M, c, or k, and are, in this sense, arbitrary; these parameters are not constrained to be equal across the three message conditions. The results of the analysis will be parameter estimates from one equation utilizing all the data to estimate these parameters. This should provide us with relatively efficient estimates; for a fuller discussion of strategies to be employed in nonlinear estimation, see Daniel and Wood (1971, pp. 193–225) and also Beck and Arnold (1977).

Structural versions of the differential equation model will be analyzed via the Nonlinear Regression procedure in the Statistical Package for the Social Sciences (Robinson, 1977). Nonlinear routines yield only approximate standard errors. The user specifies tolerance levels for relative changes in parameter estimates and the sum of squares, and the procedure iterates to a solution. This results in the following

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potential difficulties:

> We are never guaranteed that a given procedure will converge to a desired solution. . . .
> A given procedure starting at a specified point may converge to a *local* minimum that is not the *absolute* minimum sought. . . . The convergence procedure may not be monotonic, that is, the *i*th iteration may sometimes be inferior to the $(i-1)$th. . . . It sometimes happens that two *different* sets of parameter values, c_{I} and c_{II}, will have the *same* (minimum) value for Σ. Mathematically, either of these is acceptable. Which solution is obtained often depends on the starting values. (Meyer 1975 p. 400)

In addition to these difficulties, parameters may be underidentified or severely multicollinear. In this situation, to obtain precise estimates for the parameters requires some parameters being dropped from the model.

RESULTS

Means. For the *no position advocated* condition, $\bar{y} = -.119$, sd = 7.253, $n = 391$. For the *$20* condition, $\bar{y} = 3.029$, sd = 8.929, $n = 391$. For the *$40* condition, $\bar{y} = 6.764$, sd = 13.496, $n = 390$. Thus, we see that there are differences induced by our experimental messages, and we will now proceed to estimate the parameters of our model and test its goodness of fit.

Choosing the structural model. The data within each message condition were analyzed to determine which of the three structural solutions of the differential equation provided the best fit. In each case, the underdamped version was clearly superior; as a matter of fact, the overdamped and critically damped versions provided essentially no improvement in fit relative to a model predicting that each response, regardless of time lag, should be the mean response for that message condition. On the basis of these analyses, the underdamped version was chosen as the solution to test utilizing the entire data set.

The underdamped structural equation is:

$$\hat{y} = e^{(-c^*/2)t}(a_3 \sin(\omega t)) + a_1(1 - e^{-(C/M)t}) + a_5.$$

As explained in the section on data analysis, some parameters are constrained to equality across the three message conditions, because of assumed identical initial conditions or because they are functions of the coefficients of the differential equation model. Since a_1 and a_3 are not expected to be the same across conditions, we may replace them as follows:

$$a_3 = b_2 + b_3 D_1 + b_4 D_2$$

$$a_1 = b_6 + b_7 D_1 + b_8 D_2,$$

where

$$D_1 = \begin{cases} 1 \text{ for the \$20 and \$40 conditions} \\ 0 \text{ for the ``no position advocated'' condition,} \end{cases}$$

and

$$D_2 = \begin{cases} 1 \text{ for the \$40 condition} \\ 0 \text{ for the \$20 and ``no position advocated'' conditions.} \end{cases}$$

In this way, differences across message conditions are possible. For example, assuming our model has stable equilibria, $\hat{y}_\infty$ (the predicted equilibrium value) is

$b_6 + y_0$ for the "no position advocated,"

$b_6 + b_7 + y_0$ for the $20 message condition,

and

$b_6 + b_7 + b_8 + y_0$ for the $40 condition.

In full, *the general structural model* is

$$\hat{y} = e^{b_1 t}((b_2 + b_3 D_1 + b_4 D_2)\sin(b_5 t)) + (b_6 + b_7 D_1 + b_8 D_2)(1 - e^{b_9 t}) + b_{10},$$

where

$$b_1 = -c^*/2$$

$$b_5 = \omega$$

$$b_9 = -C/M$$

$$b_{10} = a_5 = y_0.$$

Stepwise analysis of the general structural model. The general structural model has ten parameters. We begin our analysis of this general model by testing, in sequence, the contributions of the compo-

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nents of our model. Our strategy is conservative, in that we will first test the hypothesis that seems to have the greatest a priori probability, and then test the incremental contribution of the remaining parameters, as they are theoretically separable.

Our first hypothesis tests whether there is significant translation due to the message. Since many studies have posited and found that discrepant messages induce attitude change, this hypothesis is most consistent with previous studies. However, by including a time dependency in the translation, we have a kinematic model; by explicating this model in terms of forces, we have a dynamic model. The *translation only* model is as follows:

$$\hat{y} = (b_6 + b_7 D_1 + b_8 D_2)(1 - e^{b_9 t}) + b_{10}.$$

The hypothesis to be tested is (H_1):

The translation model explains more variation than could be expected by chance.

The estimated parameters of this model may be found in Table 2. The test of this variant of the general structural equation is found in Table 3. Technical information concerning the tests of this and subsequent models may be found in the Appendix. Table 3 indicates that the "translation only" model explains a statistically significant amount of variation in the data. (The ANOVA F test assumes that (population) residuals are normal, homoscedastic, and nonautocorrelated. The nonautocorrelation is expected as a result of the experimental design. To evaluate homoscedasticity, a residual scattergram was examined, resulting in the exclusion of two outliers; the data appeared homoscedastic except for these outliers. To evaluate the normality of the residuals, the skewness of the residuals was computed (see Table 2). The assumption of normal residuals seems plausible.) We reject the null hypothesis of no translation component.

Our second hypothesis concerns the oscillation component. We have already found that the underdamped version of the differential equation model appears to be superior to the other two versions when the message conditions were examined separately. Hence, we now test H_2:

The addition of an oscillation component which is undamped (as in Fig. 3(b)) to the "translation only" model incrementally

TABLE 2
PARAMETER ESTIMATES (APPROXIMATE STANDARD ERRORS), AND ESTIMATES OF DESCRIPTIVE PARAMETERS FOR THREE STRUCTURAL MODELS OF THE UNDERDAMPED CASE.*

Parameter	Model: Translation Only (1)	Translation Plus Oscillation (2)	Translation Plus Damped Oscillation (3)
b_1	—	—	− .003 (.002)
b_2	—	− .400 (.742)	−1.363 (1.505)
b_3	—	.108 (1.036)	.431 (2.009)
b_4	—	2.782 (1.035)	6.076 (2.674)
b_5	—	.466 (.001)	.464 (.001)
b_6	−4.026 (1.972)	−3.879 (1.946)	−3.784 (1.880)
b_7	3.254 (.805)	3.247 (.807)	3.277 (.812)
b_8	4.136 (.818)	4.089 (.819)	4.106 (.827)
b_9	− .017 (.013)	− .017 (.012)	− .016 (.011)
b_{10}	3.666 (1.817)	3.545 (1.780)	3.416 (1.700)
c^* (weighted linear damping coefficient)	0**	0**	.006
k^* (weighted linear restoring coefficient)	0**	.217	.215
Period of oscillation (sec.)	0**	13.483	13.541
Time to 90% translation (sec.)	135.446	135.446	143.912
$\%R^2$	7.2	8.1	8.3
Skewness of residuals	.716	.674	.666

* m(mass of impacted concept) = 1; N = 1172.
** fixed value.

TABLE 3

STEPWISE ANALYSIS OF VARIANCE FOR THREE STRUCTURAL MODELS OF THE UNDERDAMPED CASE ($N = 1172$).*

Source of Variation	Sum of Squared Deviations	Degree of Freedom	Mean Square	F
Total (deviations from the mean)	131,670.75	1171		
Explained by translation	9,452.69	4	2363.17	22.56**
Unexplained by model with translation	122,218.06	1167	104.73	
Increment explained by oscillation	1,225.74	4	306.44	2.95†
Unexplained by oscillation	120,992.32	1163	104.03	
Increment explained by damping	278.42	1	278.42	2.68
Still unexplained	120,713.90	1162	103.88	

* The second and third F statistics sequentially test the statistical significance of incremental explained variance.
** $p < .001$.
† $p < .025$.

explains more variation than could be expected by chance.

The equation to be compared to the "translation only" is as follows:

$$\hat{y} = (b_2 + b_3D_1 + b_4D_2)\sin(b_5t) + (b_6 + b_7D_1 + b_8D_2)(1 - e^{b_9t}) + b_{10}.$$

Examination of Tables 2 and 3 shows that the oscillation is statistically significant, even though an additional four parameters were added to the model. Furthermore, the oscillation model requires b_5, the frequency of oscillation, be nonzero. Table 2, column 2, shows not only that the estimated value of b_5 is nonzero, but that this value is extremely statistically significant.

Lastly, we test the hypothesis that the oscillations die out over time, which requires $b_1 < 0$. This is contrasted with the hypothesis that the oscillations continue with the same amplitude, which results from $b_1 = 0$ (assumed by H_2). The hypothesis to be tested is (H_3):

The inclusion of a damping term to the oscillation component, resulting in damped oscillations (as in Fig. 3 (a)) in the general structural model incrementally explains more variation than could be expected by chance.

The results in Tables 2 and 3 clearly indicate that H_3 is not supported. The damping parameter b_1 is estimated as −.003, which is not significantly different from 0 (Table 2) and the inclusion of this parameter does not give a statistically significant incremental F value (Table 3). Table 4 provides the correlations between the estimated parameters for the translation plus undamped oscillation model.

Other estimated values

Table 2 indicates the values of parameters of the differential equation model and other estimated values. Looking only at column 2 (corresponding to the translation plus undamped oscillation model), we see that c^* (the weighted linear damping coef-

TABLE 4

CORRELATION OF ESTIMATES FOR TRANSLATION PLUS OSCILLATION STRUCTURAL MODEL OF THE UNDERDAMPED CASE ($N = 1172$).

	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
b_3	−.716	1.0						
b_4	.000	−.488	1.0					
b_5	−.003	.001	−.003	1.0				
b_6	−.029	.014	.018	.011	1.0			
b_7	.073	−.080	.030	−.008	−.197	1.0		
b_8	.010	.020	−.036	.043	−0.18	−.387	1.0	
b_9	.034	−.024	.009	.011	−.031	.249	.302	1.0
b_{10}	−.002	.008	−.020	−.009	−.951	−.030	−.020	−.096

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ficient) is 0, while k^* (the weighted linear restoring coefficient) is estimated as .217, relative to the mass of "tuition increase" which is defined as 1.0. The period of oscillation is about 13.5 seconds, or its frequency (the reciprocal)) is .07 Hertz. Table 5 indicates, for each message condition, the timing and amplitude of the oscillations. The amplitude of the oscillations are greatest for the most discrepant of the messages, as one would expect if the more discrepant messages corresponded to greater impulsive forces.

The translation is estimated to be 90% completed by about 135 seconds after the message. Equilibrium values for the messages are found in Table 5. Table 5 also reports the estimated initial velocity and acceleration for the three message conditions. Finally, we may also provide results which have implications for assessing how subjects "average" incoming discrepant information with their existing attitudes. In the present study, the initial position (y_0) is estimated as 3.545 (Table 2), while the equilibrium values for the three conditions are estimated as −.334 (No Position Advocated), 2.913 ($20), and 7.002 ($40) (Table 5). This shows there merely thinking about the issue resulted in a change of −3.879 (cf. Tesser, 1978). The $20 message was almost sufficient to overcome the effect of thought, and the $40 message induced change in the direction advocated which surpassed the initial position.

DISCUSSION

The model: Sources of invalidity

We found empirical support for a mathematical model evaluated with statistical techniques. The model that is supported is a nine parameter model that treats attitude change as a dynamic process of translation to a new equilibrium plus constant oscillation around this translating equilibrium.

While the model is supported by the data, the amount of variation explained by the model is not very much (R^2 = 8.1%). Assuming the model is correct, several explanations for its limited success are possible. First, recall our operational assumptions: We assumed c, C, m, M, k, y_o and the probability of processing the information in a specific interval were equal for all subjects. To the extent that this is incorrect, we should find our model leaves variance in the dependent variable unexplained. In addition, (1) The reliability of the subject's attitude may be quite low; (2) The precision and reliability of the measurement of the time lag between the receipt of the message and the attitude measurement may have been low, since experimenters differed in how carefully they measured the time, and also in their operationalization of when the message was received and when the attitude was recorded; and (3) A few experimenters may have provided fraudulent data. According to Roth (1966), this is an unacknowledged but real possibility whenever "hired hands" are used in research.

The effect of any of the above sources of error would be to produce noise (random error). Therefore, the fact that the model performed considerably better than chance is not *because* of, but *despite* these potential problems. Several procedures may be employed to correct these possible problems. First, the amount of time to consider the message should be more carefully controlled, and more precisely measured. Second, we may consider individually calibrating the discrepancy of the message, rather

TABLE 5
DESCRIPTION OF THE MESSAGE CONDITIONS AS DERIVED FROM THE TRANSLATION PLUS OSCILLATION STRUCTURAL MODEL OF THE UNDERDAMPED CASE ($\ddot{y}_0$ = 1172).*

Descriptive Aspect	Message Condition		
	No Position Advocated	$20	$40
$\dot{y}_0$ (initial velocity)	−.252	−.147	1.219
$\ddot{y}_0$ (initial acceleration)	.0001	−.0008	−.0020
y_∞ (equilibrium value)	−.334	2.913	7.002
first extremum is:	minimum	minimum	maximum
amplitude of oscillation	±.400	±.292	±2.490

* Time between extrema = 6.742 sec. = ½ of the period of oscillation. First extremum occurs at 3.371 sec. (¼ period).

than assuming homogeneous aggregates. Third, we may attempt to manipulate m, M, k, and c; if successful, this would not only control for heterogeneity of the subjects, but also provide a clearer understanding of the factors that cause these parameters, and a stronger validation of the model. We propose that factors such as the amount of information the subject has about the cognitive molecule, the level of association of the cognition with other cognitions, the amount of distraction present, the amount of counterarguing the subject engages in, and the degree to which the cognitive configuration as a whole is altered by the message may all be related to the parameters of the differential equation model. If the model is correct, these parameters are the fundamental process parameters.

An alternative to manipulating the differential equation parameters is to predict them or their structural equation counterparts on the basis of other variables. For example, b_9 represents the friction of translation. If distraction affects b_9, then, assuming linearity,

$$\hat{b}_9 = b_{11} + b_{12}(\text{Distraction}).$$

If we had a measure of distraction, we may replace b_9 by the terms on the right in our model, and see how well this does. This adds a parameter to be estimated, and thus violates the principle of parsimony, but it may add further theoretical understanding to the dynamics that we propose.

Methodological implications

We have some evidence that the attitude change induced by a discrepant message takes time; it takes about 135 seconds to be 90% completed, and 271 seconds to be 99% completed. In addition, oscillations have been found with a period of about 13.5 seconds. This suggests that studies which measure attitudes at only a few time points, several minutes apart, can not ordinarily detect the dynamic process we are studying. We now have some initial evidence for the temporal parameters of the attitude change process.

A second methodological implication concerns the relevance of data analysis over time in the absence of dynamic formulations. If we consider observations that produce a perfect sinusoidal pattern and group them into categories that coincide with their period, it is clear that we would find absolutely no "main effect" of time. While thoroughly sampling the time domain has difficulties (Arundale, 1980), we should realize that measuring data over time does not necessarily mean that we are sensitive to processes that may generate the trajectories in our data. To evaluate a trajectory, a process must be posited which requires that a specific family of trajectories be examined.

Perhaps the most important methodological implication is that studies of attitude change must give more attention to the time which has elapsed between the persuasive stimulus and the attitude measurement. If messages of different discrepancies cause oscillations which have different frequencies, or are not in phase, one's conclusion as to which message was most effective may be determined by the time interval from message to measurement. (In this study, the statistically derived trajectories for the $20 and $40 messages do, in fact, cross each other). Moreover, if the procedures of different experimental conditions take different amounts of time, it is possible that the effect of the treatment and of time will be confounded.

Theoretical implications

The possibility of cognitive harmonics, while only modestly supported by the evidence here, is an exciting one. Humans seem to create or impose rhythm on all sorts of endeavors, as witnessed by the songs of laborers and the nursery rhymes of children (Burling, 1966; Jusczyk, 1977). (We are indebted to W. O. Hagstrom for alerting us to the research in this area.) The cadence of what is said is often as important as its content: to the comedian and the orator, good timing is fundamental (see Goldstein, 1970; Wilson, 1979, chapter 4). It is as if a message may be sent creating resonances in the recipient's cognitive system. Resonances occur when the frequency of an applied harmonic force is the same as the natural frequency of the system to which it is applied. In the study of attitudes, we have hypothesized the existence of such a frequency, which we estimated here at about .07 Hz. This extremely low frequency oscillation is lower than the frequency of waves recorded for humans via electroencephalography, but not dramatically lower (e.g., δ waves have a frequency of .5 to 3 Hz; see Vasiliev, 1976, p. 20). The cognitive oscillations, assuming no new impacts on the cognitive system, would seem to correspond to the incessant consideration and reconsideration of Tevya in *Fiddler on the Roof*, but here the period is 13.5 seconds. This is just enough time to say "on the one hand . . . and on the other hand . . . ."

The fact that the oscillations show no clear tendency toward damping may seem quite surprising, as it seems that people do eventually make a decision and stick with it. We have two possible explanations for this. One possibility is that while the damping coefficient was not significantly different from zero, our estimated value (−.003) does reflect genuine damping. If this value is correct, it takes almost four minutes (231 seconds) for the amplitude to be reduced to half. It may be, given the large variance in the data, that the experiment would have to continue for much longer than 15 minutes for the damping coefficient to be significant. The other possibility is that the damping which appears to exist results from committing oneself to a decision, and that until such a commitment is made, the actor's view continues to oscillate.

While the results provide modest support for the processes of attitude translation and oscillation, we believe the approach to theory construction and testing which the study demonstrates, is as important as the results. We started with a verbal theory of the effects of a message on cognitive change. We made this into a precise mathematical theory, and tested it, using appropriate statistical techniques. (Since the theory was expressed as a differential equation, it was necessary to integrate the equation before testing the theory.) Further development of this theory should come through improved procedures. In addition, from a theoretical standpoint, we should relate the process parameters to other cognitive and social variables, and explore the relationship between cognitive oscillations and any thought processes which may be associated with them (see Petty, Ostrom & Brock, 1981, for a review of recent research in this area). We present such a theoretical elaboration in another paper (Kaplowitz & Fink, 1982).

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APPENDIX

Technical Information for Nonlinear Analyses.*

Item	Value
b_1 upper bound	0
b_2 lower bound	−10
b_3 lower bound	−10
b_4 lower bound	−10
b_5 lower bound	0
b_6 lower bound	−15
b_6 upper bound	15
b_7 lower bound	0
b_7 upper bound	20
b_8 lower bound	0
b_8 upper bound	20
b_9 upper bound	0
b_{10} lower bound	−4.5
b_{10} upper bound	4.5
relative change in a parameter for solution	1.0×10^{-6}
relative change in sum of squared residuals for solution	3×10^{-6}

* Ratio to initial sum of squares and pivot tolerance were the default values of the SPSS Nonlinear program. Note that none of the estimated values were driven to its upper or lower bound.

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