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THE CONSTRUCTION OF A CONTINUOUS MANIFOLD FOR SOCIAL OBJECTS: A PROLEGOMENOUS STUDY

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY GAIL RAE WISAN

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INTRODUCTION

> The half-century from 1637 to 1687 is universally recognized as the fountainhead of modern mathematics. The first date marks the publication of Descartes' <u>Geometrie</u>, the second that of Newton's <u>Principia</u>.[1]

Newton's development of the calculus and his studies of rates of change or motion of physical objects were (along with the accomplishments of Galileo and Keplar) dependent upon Descartes' development of the coordinate system[2] which brought the power of algebra to the study of the geometric relationships (static and dynamic) of physical objects.[3] Any point could be located in the coordinate system by its coordinates. The coordinate system facilitated the systematic analysis and mapping of functional relationships between continuous concepts, defined generally directly or indirectly, in terms of fundamental measurement.

In general, this work is concerned with the possibility of constructing a manifold (analogous to a coordinate system) for social objects. This manifold might provide a continuous common framework from which to analyze discrete social objects (e.g., walking, fighting pollution, laughing). There are a number of possible benefits of this social manifold: 1. constructing a picture of the conceptual space of a similar aggregate and/or social group; 2. uncovering functional relationships directly or indirectly among social objects; 3. contributing to the potential development of a system of fundamental measurement for sociology. These are all potentials which serve to spur on the necessary preliminary work of which this dissertation is a part.

The specific goals and hypotheses of this work are as follows:

A. to analyze the characteristics associated with scientific advance in physics (which most of Chapter One is devoted to);

B. to evaluate (with theoretical and empirical evidence) the available procedures -- in terms of their own validity and in terms of the criteria (continuous, homogeneous, isotropic, linear, and metric) herein established for the construction of the social manifold; (Chapter Two is devoted firstly, to a discussion of the criteria, and secondly, a critical review of technical procedures relevant to the construction of a social manifold)

C. to devise instrumentation, collect preliminary data for use in the system, and utilize the technical procedures decided upon to construct the social manifold; the data collection techniques, underlying theoretical basis, and technical procedures for constructing the desired conceptual space consisting of social objects is referred to all together as the Galileo System; (Chapter Three is specifically concerned with the presentation of the character of the Galileo System.) Hypothesis four specifies the expected dimensional nature of the social manifold.

Hypothesis 4- The dimensions, or axes, required to present the relationships between social objects will be few, perhaps as few as three.

D. to provide a preliminary assessment of the reliability and face validity of the instrument.

Hypothesis 1- The spatial relations between objects in space will be relatively stable over the relatively short time interval between administrations of the questionnaire.

Hypothesis 2- The total amount of movement in the manifold -- measured in terms of the distance between the same object at different points in time (i.e., administrations)-- should not be great.

Hypothesis 3- Movements of the objects in the social space will be orderly, i.e., movement will not be chaotic.

Chapter Four presents the results and a discussion of the results. Finally, Chapter Five provides a summary and conclusions to this work.

FOOTNOTES

[1]E. T. Bell, "The Beginning of Modern Mathematics, 1637-1687," in Robert Marks (ed.), The Growth of Mathematics (New York: Bantam Books, 1964), p. 133.

[2]*Ibid.*, p. 141.

[3]*Ibid.*, pp. 140-47.

CHAPTER I

THEORY AND RESEARCH: SOME DIFFERENCES BETWEEN PHYSICS AND SOCIOLOGY

The disjointed character of theory and research in Sociology represents a basic problem for scientific advancement. That they are disjointed means that sociological theory is often arbitrarily operationalized and that research results do not clearly have a significance for the supposedly operationalized theoretical concept. This dissertation focuses on some specific problems sociology has in contrast to physics as regards the gap between theory and research--and takes some preliminary steps towards developing a possible solution to them.

Theory, Research, and Measurement and the Construction of a Social Manifold

The problem focused on in this dissertation is essentially twofold: 1. the basic concepts of Sociology (e.g. value, norm, act, role) are discrete and qualitative as opposed to continuous and quantitative;[1] and consequently 2. sociology lacks fundamental concepts characterized by fundamental measurement from which other concepts measurements' can be derived; i.e., sociology lacks a system of fundamental measurement. The two problems can be compared to the situation of physical mechanics wherein all concepts can be derived from distance, time and mass (or force) which are quantitative concepts -- both theoretical and empirical at the same time.[2]

Specifically, this work attempts to construct a manifold for sociological variables, i.e., a social space, which will utilize the continuous, quantitative concepts of distance and time as a common framework to analyze the heterogeneous discrete and qualitative concepts of sociology.[3]

The construction of a social space can be approached in a number of ways -- some of which are discussed in Chapter Two. However, the procedure adopted herein was guided by the scientific goals of parsimony and facility for mathematical manipulation. Thus the procedure for constructing a social manifold has been guided by the desire for a spacial manifold which is continuous, homogenous, isotropic, metric and unbounded. (A description of these characteristics and an elaboration of why they are desirable is found at the beginning of Chapter Two.)[4] The social space or manifold is constructed in a manner which will avoid violations of these criteria. The Galileo System[5] is the name used to describe both the procedure for constructing this desired space and the theoretical paradigm underlying the construction of the "desired" paradigm.

It is important to understand that the creation of a manifold with these properties does not ensure or require that the data will conform to these desired properties. The properties are being postulated because of the convenience they provide for mathematical manipulation and for the construction of a continuous quantitative[6] backdrop against which empirical relations may be clearly displayed. In this sense the Galileo System is partly analogous to the Weberian "ideal type." The conformity of the data to the conceptual model is an empirical question (addressed in Chapter Four.)[7] But the basic purpose of the Galileo System is to provide a clear cut baseline against which such empirical relations can be measured. The criteria set down do not require that the relations between the social objects qua numbers in a spacial manifold will correspond exactly to the numerical relations between real numbers, although such an outcome would be highly desirable. As will become clearer later, the Galileo System makes it possible to utilize two of the most fundamental measures of physics - distance and time.

This work does not try to develop a theory for the analysis of the relationships between distance and time and sociological variables; however, some possible concrete interpretations (preliminary as they may be) of the social manifold are suggested for explanatory purposes later in this chapter.

The major tasks undertaken in this work are: (1) establishing the criteria an ideal social manifold should meet, (2) The construction of the social manifold -- utilizing multidimensional scaling techniques, (3) the development of instrumentation for collection of data to use in the construction of the social manifold and (4) the preliminary assessment of the reliability and face validity of the manifold constructed.

The Galileo System and its Expected Character

To reiterate, the Galileo System represents a conceptualization with both theoretical and empirical import. It entails an attempt to construct a conceptual social-psychological space wherein the relations between social phenomena can be represented. Implicitly it also represents an attempt to develop a fundamental system of measurement (with theoretical implications) for sociology.

Just as when the motion of a body changes, there has been an agent of change, the Galileo System provides a continuous space wherein the motion of psychological objects can be observed.

Motion alerts us to look for the change producing phenomena.

Most of the specific hypotheses which are tested in this work concern the reliability of the instrument and the face validity of the preliminary findings from the data. The hypotheses delineated relate to the expected character of the re sulting social space.

Hypothesis 1 - The spatial relations between objects in space will be relatively stable over the relatively short time interval between administrations of the questionnaire (which was designed to furnish data for the construction of the social space.)

Hypothesis 2 - The total amount of movement in the manifold -- measured in terms of the distance between the same object at different points in time (i.e., different administrations) -- should not be great.

Hypothesis 3 - Movements of the objects in the social space will be orderly, i.e., movement will not be chaotic.

Hypothesis 4 - The dimensions, or axes, required to present the relations between the social objects will be few, perhaps as few as three.

Failure of empirical data to conform to these hypotheses would lessen the utility of the Galileo System for the description of social phenomena.

<u>The Use of Fundamental Continuous Concepts</u>

Fundamental and derived measurement characterize physics whereas measurement in the social sciences is essentially "by fiat." While this does not signify that all social science measurement is meaningless, it does mean that results of empirical sociological investigations cannot be assumed to have implications for the discrete qualitative theoretical concept which presumably was the source for the investigation. The gap between theory and research represent a major problem for the development of sociology through the close interconnection of theory and research. It is difficult to put to test concepts which have no clear operational meaning. By clear is meant to signify the precision of description possible with continuous mathematics.

A major advantage that physical measurement has over social measurement is the existence of a few fundamental measures that are measured in terms of their own internal relationships. Pfanzagl defines fundamental measurement as "the construction of scales by mapping an empirical relational system isomorphically into a numerical relational system."[8] Distance and time are the most fundamental concepts of physical mechanics. The unit of any quantity in mechanics can be expressed in terms of any combination of distance, time, and mass. (Only temperature and electric current are needed as additions to derive all concepts in physics. These five variables are generally considered the five fundamental variables of physics.) The units of mass and force can be treated in terms of distance and time, if the other (mass and force respectively) is held constant.[9]

While it is true that qualitative terms exist in physics, it is also true that if of any experimental use, they are derivable from the five fundamental concepts. For example, while atom may be conceived of as a discrete term, it is also true that it can be derived from the fundamental concepts. Moreover it is exactly those terms which were not derivable from the fundamental measures, and hence were not open to operationalization -- e.g., phlogisten, ether -- which had to be discarded for scientific advance.[10]

It is important to understand that in the physical sciences and particularly physics all basic or fundamental theoretical concepts are themselves equally like research language--quantitative terms like mass, time, velocity, distance -- all implying in their very statement ratio-type quantitative measurement and research; to say that $M = \frac{F}{A}$ is itself to cite a ratio. The theoretical and research language of physics share a fundamentally common character.

The problem of sociology may well be the unbridgeable gap between a theoretical language that is discrete and qualitative (e.g., action, norm, role) and the continuous and quantitative and commensurate with the character of scientific research. It is imperative to understand that such a strategy is appealing not because it seems to be in some sense "true" in its description of human social behavior as we know it, but because of its potential utility. Nunnally points out that theories consist of collections of words (statements about the nature of events,) and "though such theories may suggest interesting investigations of cross structures among sets of observables, the evidence obtained is not so much of the <u>truth</u> of the theories as it is proof of their <u>usefulness</u>."[11] Social scientists may be so sure of the "truth" of their discrete qualitative construction[12] that they ignore the possible usefulness of a continuous and quantitative construction.[13]

<u>Measurement</u>

Torgerson relates the problem as he sees it for the social sciences:

> The concepts of theoretical interests tend to lack empirical meaning, whereas the corresponding concepts with precise empirical meaning often lack theoretical import. <u>One of the great problems in the development of science</u> is the discovery or invention of constructs that have, or are likely to have, both.[14]

Torgerson, believes that the problem of developing rules of correspondence between theoretical constructs and observable data "reduces to the problem of devising rules for the measurement of the construct."[15]

Typical of the problem of contemporary sociology is the following statement concerning theory and research. "Sociologists seem to have two distinct languages, one which is in some sense more complete than the other. The first is a theoretical language in which we do our thinking. The second is an operational language involving explicit instructions for classifying or measuring."[16] The requisite process of measurement in research suffers from its separation from its theoretical concepts. Since qualitative measurements of the theoretical concepts is difficult to attain, the results of quantitative research and the relevance of quantitative findings tend to be of tenuous relevance to qualitative theory. Herein lies much of the problem of "measurement by fiat." Measurement by fiat or what Torgerson believes to be an "arbitrary definition" of a "presumed relationship between observations and the concept of interest" characterizes social sciences -- (e.g., Socioeconomic Status, Intelligence.)[17] Physics is characterized by a system of measurement based on three fundamental quantitative concepts -- distance, time, and mass. The measurement of all other concepts can be derived.

> Derived measurements obtain their meaning from laws which relate properties to other properties; e.g., mass is force divided by acceleration. Density is the ratio of mass to volume;[18]

For example: $\overline{V} = \frac{D}{T}$ average velocity = $\frac{\text{Distance}}{\text{Time}}$

Derived measurement leads easily into testable theoretical relationships. This dissertation represents some preliminary work in the direction of developing a fundamental system of measurement for sociology from which the measurement of other concepts can be derived.

Velocity, acceleration--as well as force and mass sometimes--derive their theoretical and research conceptualization from ratio relationship between other properties. The following description of how other concepts can be derived from distance and time is included because it is suggestive as to the character of measurement in Newtonian mechanics--a successful science.[19]

The meaning of these concepts is closely constrained in terms of their ratio relationships with other fundamentally defined concepts. Following are illustrations of derived measurement:

1. Velocity, a term with both theoretical and operational significance, is described in terms of the quantitative ratio $\frac{D}{T} = \bar{V}$ (D = distance covered, T = time passed, and $\bar{V}$ = average velocity);

2. Acceleration can be conceptualized in terms of the change in velocity/time elapsed i.e., $A = \frac{V}{T}$ or in terms of distance and time, as:

$$A = \text{distance}/(\text{time})^2;$$

3. The magnitude of a force can be determined by comparing the relative accelerating effect upon the same object of different forces. The ratio of the forces is the same as the ratio of the accelerations (which it has been just pointed out is derivable from distance and time):

$$\frac{F_1}{F_2} = \frac{A_1}{A_2}$$

4. The definition of mass as a quantity of matter has been considered, by some physicists, an antiquated and unfortunate use by Newton. A "resistance to acceleration" has been suggested as an intuitive alternative definition:

$$\frac{F}{A} = M. \qquad M = \text{mass}$$

Mass as a derived measurement has been used to contribute to the development of Einsteinian relativity theory.[20]

Measurement at its highest level entails the assignment of numbers to properties of objects in a manner which corresponds to the relationship between numbers in the real number system. (That real numbers are continuous can be recognized by their description as "the set of all endess decimals.")[21] Real numbers are characterized by order, distance and origin.

In the physical sciences, most measurements of phenomena are on ratio scales (i.e., absolute zero point) so that mathematical manipulations such as multiplication, division, addition, and subtraction as well as algebra, analytic geometry, calculus and the more stringent statistics are possible.

These ratio level measurement capabilities are a direct result of the fact that the fundamental theoretical concepts of physics are themselves quantities and defined in terms of their ratio relationships to themselves and/or each other.

This dissertation is a step towards the development of a theoretical concept--an underlying paradigm--that is quantitative and commensurate with scientific research. It is likewise, at the same time, attempting--in a very preliminary way--to develop a system of fundamental measurement for sociology.

This strategy, it is necessary to understand, is appealing not because it seems "true" in its description of human social behavior as we know it, but in the sense of Nunnally, because of its potential utility. Quantitative and continuous representations are open to verification in a manner which has proved fruitful in other sciences and seems worth pursuing in sociology for that reason.

Some Examples of Fundamental Physical Measurement--Distance and Time in Physics

Distance and time can be measured in terms of ratio relationships to themselves. These terms have both constitutive and operational meaning. They are both fundamental measurements.

Both the unit and the number of times the unit appears in the distance measurement must be specified for fundamental measurements. A standardized unit must be specified if the measurement is to have significance. Time is based on a clock system or clock which is "any mechanism which gives us a set of signals such that the duration between any two adjacent signals is always the same."[22]

"Good clocks are the class of physical phenomena which repeat themselves over and over again in such a way that the number of repetitions of any one of the members of a class has always a constant ratio to the number of repetitions of any member of that class."[23] Both time and distance measurement entails establishing a standard of measurement which serves as a basis for comparison for any particular distance or time. "We now see that the problem of observing a change in shape of an object, or a lack of change depends on our ability to construct measuring devices which, to the best of our knowledge, do not change in shape."[24] Because units of measurement are so crucial to measurement of motion among physical objects, it is plausible to suggest that social motion or change in social behavior may also benefit from a standardized system of measurement.

Motion: The Significance of the Spatial Manifold in Physics and Sociology

Physics deals with motion and the forces which bring about motion. Thus, physics needs besides scalars, i.e., just magnitudes (e.g., the mass of an object) also vectors to describe the direction associated with motion. Thus angle is sometimes considered a fundamental measurement in terms of motion and location.[25]

The analysis of location and motion (i.e. directed distance) for physics (and perhaps also social psychological objects) requires a spatial manifold. Both the description and explanation of motion (a change of position) of objects entails the use of the concept of directed distance (often called displacement -- a vector quantity). Direction (e.g., + or -, North or South, East or West, up or down, forward or backward) can initially be an arbitrary designation. The original choice of a positive direction is arbitrary though future designations within an analysis must be consistent with the original choice if confusion is not to result.[26] Calculation of a distance involves at least two points --d(X,Y). Direction between two points can be conceptualized only in terms of an arbitrary point, i.e., an origin in space.[27] The change of position of a physical or social psychological object implies a change of position relative to some frame of reference. For example, in the Galileo System lying down and sitting are <u>given</u> as one galileo apart. Some sort of spatial manifold is required to systematically represent the motion of physical or social psychological objects when there are two or more dimensions to consider. Positive or negative movement (i.e. displacement or directed movement on a line) is conceptualized in terms of an arbitrary point (or origin.) A system of time permits the description of a sequence of positions of an object (physical or social psychological) in motion and thus potentially an analysis of the causes of its motion. Measures of distance (or length) and angles enable description of the relative arrangements of objects at a point in time.[28]

The Desirability of Constructing a Manifold for Social Objects

A spatial manifold for discrete social objects might permit the location of these objects in terms of the quantitative, continuous concepts of distance and time, as well as the relationship angle. As was indicated above, description of spatial locations over time might reflect conceptual (and in some manner behavioral) patterns. Object motion (i.e., social change?) might reflect functional relationships between social objects which were otherwise not obvious.

The Galileo spatial manifold is both a graphic representation and a theoretical paradigm for social science wherein the relationships between social objects might be expressed in terms of fundamental continuous, quantitative variables--e.g., distance and time. This spatial model can be sketched as follows: The dimensions of the manifold are wholly determined by its content of social objects (conceived of in term of behavioral significance.) Some examples of the behaviors and social-objects are walking, sitting, studying, war, fighting pollution, making love, me. In all, this dissertation reports the results of research based on thirty-three pairs of behaviors and objects were scaled for use in constructing a multidimensional space. As was pointed out above for physical objects, it is analogously suggested that change of position (motion) of the social objects might be systematically described in terms of a coordinate system, a relative frame of reference. Calculations of physical motion are made in terms of an object's change of position relative to an arbitrary origin within a specified time period. Concepts such as force, mass, velocity, and acceleration can be derived from the more primitive concepts of distance and time. If social objects can be located in a similar arbitrary coordinate system, these same variables may have some significance for social phenomena.

Mass has been defined as "resistance to acceleration." If a given force is being applied to a number of related objects (e.g., studying mathematics, studying physics) with one associated behavioral object moving more closely into an area of behavioral objects that are frequently being performed (measured as a continuous, quantitative variable), one might derive the mass in terms of the resistance to acceleration. Thus an object which is more massive (e.g., based on more information) might move less--or even not at all--given the same application of force; attitude (following Woelfel usage) has been conceived of in this work as a vector representing a proposed rate of behavior which is a reflection of the mean aggregate of all the incoming information (also as vectors of proposed rates of behavior.)[29] The use of spatial model based most fundamentally on distance and time (but from which mass, force, velocity, and acceleration can be derived or perhaps other completely sociological concepts) offers the possibility of constructing a theory for social behavior which is closely alligned to research: A system of measurement and a theory might thus emerge simultaneously. Distance and time are not really inherently explanatory concepts only for physical objects but may generally be conceived as such because of past uses. Future utilities should determine the extent to which concepts are used in the social sciences and not past conceptual frameworks. Distance and time might offer a common framework from which to view objects. Their potential social applications should not be constricted merely on the basis of the seeming "truth" of "more social" approaches.[30]

Some Illustrative Hypothetical Descriptions of the Galileo System

For illustrative purposes, preliminary though this statement may be, the significance of distance and time in the Galileo System will be elaborated while recognizing that theory construction is not directly this work's purpose.

The product of the Galileo System will be a multimulti-dimensional (i.e., at least two axes required) configuration of points which represent social objects. Some examples of behaviors used in the Galileo System are smoking marihuana, going to college, me, revolution and swimming, as well as more prosaic behaviors such as walking, sitting and eating. Thirty-three behavioral objects were included in all.

Behavioral objects which are clustered near (i.e., a relatively small distance) the object "me" for the aggregate are expected to be performed more regularly than behaviors associated with objects located far from "me". For example, if "smoking marijuana" is a relatively small distance from "me" while another aggregate or group yields a pattern of marijuana being very far from its "me," it would be expected that the former group smokes marijuana regularly while the latter does not. In the summer (given accessibility), it would be predicted that the points representing swimming and sun-bathing would be closer to objects associated with behaviors being acted out, while in the winter they would be more distant from me and the behaviors which are regularly performed.

Time, as well as distance, has also been described as a significant concept for the Galileo System. To calculate the rate of change of position for a psychological object (e.g., women's liberation) in the behavioral manifold requires a time measurement as well as the measurement of the distance (from the origin--with its significance for direction). Thus if the point representing fighting pollution moves rapidly in towards me or some frequently performed objects it would signify pollution fighting behavior is on the increase rapidly.

The above examples are merely meant to be suggestive. It is important to understand that this work is more of a preliminary attempt to construct and evaluate (reliability and face validity) a social manifold than an attempt to predict, construct or verify a theory.

The Specific Aims of the Dissertation

To reiterate, the specific goals of this dissertation are fourfold:

1. to analyze the nature of the characteristics associated with scientific advance in physics;

2. to decide upon the initial mathematical algorithms for the establishment of a spatial coordinate system which has as contents meaningful sociological objects and which meets the criteria for physical science theory established in Chapter 1. This involves a discussion of prior related work in the psychometric literature and a critique based on the above criteria;

3. to devise instrumentation and collect preliminary data for use in the system;

4. to provide a preliminary assessment of the reliability and face validity of the instrument.

Thus, in summary, this work represents only a preliminary step--involving the exploration of some technical, methodological problems--towards the development of a system of measurement based on quantitative fundamental concepts (both theoretical and empirical) for the social sciences.

Footnotes

[1]This does not mean the concepts are inherently qualitative. However, typically, they are treated as such. The situation could possibly be compared to that of the historical circumstances of the concept atom. The concept atom was conceived by the ancient Greek Democritus but its scientific value came only much later, i.e., with its quantitative development.

[2]The development of modern chemistry involved rejecting the four elements theory (i.e., fire, water, earth, and air) and the phlogiston theory. The scientific revolution in chemistry involved weighing, i.e., measuring. It was the added weight of an object after burning which was significant. It was the heaviest part of the air which was oxygen.

[3]Perhaps, for discrete and qualitative concepts to be useful (in terms of explanation and prediction) in a precise way, a continuous, quantitative backdrop is needed.

[4]By continuous, is meant the technical mathematical meaning of infinitely divisible. The Galileo questionnaire is constructed so as to yield magnitude estimations. Numbers are continuous in the resulting scale in the sense that there is no break in the possible continuum of numbers.

[5]The name Galileo is used because of his significance in the breakthrough to modern science in physics. His breakthrough has been partly associated with his conceiving of motion in a new way (i.e., the "impetus theory" as opposed to the Aristotelian "ever present force" view of motion). Moreover, quantitative analysis was very important in his work. See Herbert Butterfield, Origins of Modern Science (New York: Free Press, 1957).

[6]Qualitative analysis in chemistry tells you the number and kinds of elements. Quantitative analysis tells you the amount of the element and their quantitative relationship. Quantitative analysis gives significance to the qualitative, nominal distinctions.

[7]For example, there are large negative roots, and triangle inequalities in the resulting data. Moreover, other geometries may eventually be found which better represent the empirical relationships found among the data.

[8]J. Pfanzagl. Theory of Measurement. (Wurzburg, Germany: Physica-verlag, 1968).

[9]Frances Sears and Mark Zemansky, University Physics, Second Edition (Reading, Massachusetts: Addison-Wesley, 1955).

[10]See Butterfield, op. cit. and see final chapter of this work.

[11]Jum Nunnally, Psychometric Theory (New York: McGraw Hill, 1967), p. 98.

[12]For example, the phenomenological approach in Sociology attempts to recreate human interaction in its full richness. Maurice Natanson in affirming the phenomenological approach says the following:

> it fulfills the method of Verstehende since it offers a philosophy of the social world, rather than techniques or devices in the narrower methodological sense. . . When the naturalist approaches social reality in terms of the methods of natural science, he forfeits his philosophical concern with a crucial dimension of reality and indeed reduces himself to limbo. Phenomenology claims to reconstruct social action providing a fundamental clarification of its intentional structure within the framework of a comprehensive philosophy. It claims to return us to the social world in its full richness and urgent complexity.

The problem with this goal for science qua science (perhaps not the goal of Natanson) is that analysis of only essential characteristics is the goal. Thus Galileo paid no attention to the color of the ball and board in his experiments on gravity. Richness of description is sacrificed to verifiability (and its other side -- disproof.) See Maurice Natanson, "A Study in Philosophy and the Social Sciences," Philosophy of Social Sciences, Edited by M. Natanson. (New York: Random House, 1963), pp. 284-285.

[13]See for a relevant discussion of physics, A. D'Abro, The Evolution of Scientific Thought (New York: Dover, 1950.)

D'Abro, in describing the compatability of Einstein's view with that of mathematicians says that pure mathematicians:

> more than all others, have been led to realize how cautious we must be of dictates of intuition and so-called common sense. They know that the fact that we can conceive or imagine a certain thing only in a certain way is no criterion of the correctness of our judgment. . . . At all events, mathematicians, as a whole, refused to question the soundness of Einstein's theory on the sole plea (of some non-mathematicians) that it conflicted with our traditional intuitional

Concepts of space and time..." (p. xvi)

According to D'Abro, . . . Berkeley's view of space -- as arising from experience -- has been supported over Kant's who believed concept of three dimensional Euclidean space was "antecedent to all reason and experience and was essentially a priori, a form of pure sensibility." (p. xix)

Moreover, in discussing the evolution of scientific thought, he says, "As soon as we recognize that the fundamental continuum of the universe and its geometry cannot be posited a priori and can only be disclosed to us from place to place by experiment and measurement, a vast number of possibilities are thrown open. Among these the four dimensional space-time of relativity, with its varying degrees of non-Euclidianism, finds a ready place." (p. xx)

[14] Warren Torgerson, Theory and Method of Scaling (New York: Wiley, 1958), p. 8.

[15] *Ibid.*, pp. 9-10; see also Nunally, *op. cit.*, pp. 2-3.

[16] Franz Adler, "Operational Definitions in Sociology," American Journal of Sociology, 52 (March, 1947) 438-444, cited by Hubert Blalock, "The Measurement Problem: A Gap Between the Language of Theory and Research," Methodology in Social Research, Edited by Blalock and Blalock (New York: McGraw Hill, 1968), p. 14.

[17] Torgerson, *op. cit.*, pp. 21-22. His discussion was taken from N. R. Campbell, An Account of the Principles of Measurement and Calculation, (1928).

[18] *Ibid.*

[19] That is a science with explanatory laws, predictive ability and technical achievements.

[20] See Henry Semat, Fundamentals of Physics (New York: Holt, Rinehart and Winston, Inc., 1945), pp. 618-619.

M - the mass of a body in motion - is given by:

$$M = \frac{M_o}{(1 - V^2/C^2)^{\frac{1}{2}}}$$

This formula only becomes significant for particles moving at speeds which are great relative to the speed of light. Greater force is required to accelerate the same particle when its velocity is comparable to the speed of light than when its velocity is comparatively small. However, M_o is never more than M.

[21]Israel Rose, A Modern Introduction to College Mathematics (New York: Wiley, 1959), p. 59.

[22]Walter Michels, et. al., Foundations of Physics (Princeton, New Jersey: D. Van Nostrand Company, 1968), p. 23.

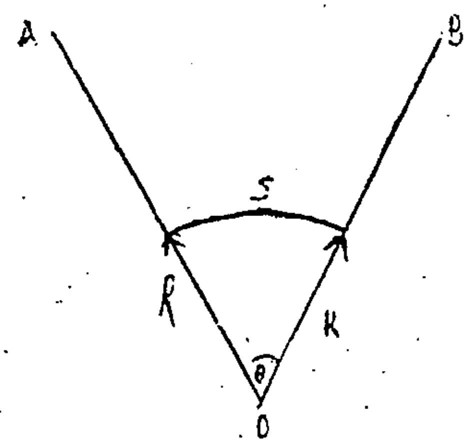
[23]Ibid., p. 25.

[24]Ibid., p. 9.

[25]Ibid., p. 30.

(a) Fractions of revolutions around a point can be used to measure the angle, i.e., amount of rotation. Degrees (o) and radians (rad) are both used to measure angles: 360^o = 1 revolution
2 rod = 1 revolution

(b) The size of an angle can be expressed as $\theta = S/R$; S = the distance measured along the arc in the same length unit as R.
If S = R, then θ = 1 radian



[26]Ibid., pp. 19-20.

[27]Ibid.

[28]Ibid., p. 40.

[29]Woelfel et. al., "A Theory of Force Aggregation in Attitude Formation and Change," (unpublished paper in progress, 1970).

[30] See Footnote 13 for a relevant discussion.

[31] See Woelfel, *op. cit.*

CHAPTER II

CRITICAL REVIEW OF TECHNIQUES RELEVANT TO THE CONSTRUCTION OF A SOCIAL MANIFOLD

Introduction

In chapter one, an analysis was conducted regarding the relationship between theory and research in sociology as contrasted with the relationship between theory and research in physics. The problem of measurement was specifically focused upon. It was pointed out that two characteristics which distinguish physics are 1.) that its basic concepts are jointly theoretical and empirical (continuous variables), and 2.) that its basic theoretical concepts are characterized by having fundamental measurement -- with the measurement of all other theoretical concepts being derivable from those measured fundamentally. The significant role that coordinate systems play (both theoretically and empirically) in physics was also explained. For coordinate systems to be of use, however, requires continuous variables. And if these continuous variables are to be significant, they must have or develop theoretical ties; relating concepts through their joint derivation from a few fundamental concepts would seem to facillitate recognition of lawful functional relationships.

This work is specifically concerned with the construction and preliminary evaluation of a manifold for social objects (analogous to a manifold for physical objects) wherein continuous variables can be functionally related. Implicit also in the development of a continuous social manifold is the potential for the development of a system of fundamental measurement.

This chapter is divided into two major sections. The first part describes the criteria for the construction of a social manifold and explains the value of these criteria. The second section presents a review of available techniques relevant to the construction of a social manifold. These techniques are evaluated both in terms of themselves as valid procedures and also in terms of the criteria delineated for the construction of the social manifold. The techniques evaluated herein include: factor analysis of cross products, factor analysis of scalar products, the Semantic Differential technique, metric multidimensional scaling, and nonmetric multidimensional scaling.

Criteria for Constructing the Social Manifold

Theoretical and methodological considerations favor conceptualizing the social manifold as a space that has the following characteristics: continuous, linear homogeneous, unbounded, isotropic, and metric.

These assumptions facilitate mathematical manipulation by utilizing a spatial paradigm wherein descriptive and explanatory concepts are quantitative, continuous and expressable as ratios. The social manifold as so specified would be accessible to high powered mathematical analysis.

By homogeneous is meant that the space is characterized by a uniform composition or structure. Variations in measurement will thus be due to differences among the objects being measured or among the properties of objects as opposed to variations in the characteristics of the space per se. Likewise, the characteristic of isotropy means that the properties of the space, qua a space, do not vary according to direction or, in other words, according to the axes. The space is continuous and thus, in its technical meaning the separation between any two points in the space is infinitely divisible. Likewise though, continuity also signifies that the space, qua space, is composed of a set of points which corresponds to the same set of transfinite cardinal numbers as does the set of real numbers.[2] Along with these more mathematically technical meanings of continuous is the idea of a continuum: "something continuous or homogeneous of which no distinction of content can be affirmed except by reference to something else (as duration.)"[3]

This continuous, homogeneous, and isotropic character of the space provides something of a fundamentally common character from which to observe discrete variations, which perhaps are seemingly unpatterned and unintelligible.

Linearity is defined by the additive function

$$y = mx + b$$

This expression states that the change in any given variable is directly (or inversely) proportional to the forces impinging upon it. It is important to understand, however, that the assumption underlying linearity in this dissertation is not that all relationships are linear, but that the spatial coordinate system in which these relations are arrayed is itself linear. Any deviation from linearity can thus be clearly displayed against the linearity of the space rather than being confounded by non linearities in the space itself.

By unbounded is meant that the space as a space is not temporally limited. While the particular configuration of points representing behavioral objects is a product of a particular sample at a particular time, this configuration can be compared across time for the same sample. Also - as was pointed out above - the spatial continuum has cardinal numbers associated with the transfinite set of real numbers.[5] Thus, the term unbounded has technical meaning.[6] By characterizing the space as metric is meant that the spatial model entails a standard of measurement. The comparison of objects, properties, or constructs quantitatively implies some common standard. Furthermore, ratio measurement implies the necessity of a zero point, i.e., an origin in a spatial model.

To summarize, the behavioral space is absolutely continuous and homogeneous. The space qua space has no distinctions of content except by reference to something else, e.g. duration, or a metric measure from an arbitrarily constructed reference origin. The advantage of a construction of this type of space is that it provides something of a fundamental common character-- a basic underlying structure, a theoretical paradigm -- from which seemingly insensible and indefinite variations can be analyzed. The construction of a social manifold is suggestive for the future development of a system of fundamental measurement which at the same time has theoretical significance. This spatial paradigm provides a generic structure through which quantitative description and analysis of relationships among the social objects in the behavioral manifold is possible.[7]

Distance, time and the other derived constructs of velocity, acceleration, mass and force, are--as has been pointed out above--continua which means each represents a quantitative set which is isomorphic with the set of real numbers. Given these concepts' isomorphism with the set of real numbers, the tools of high powered mathematical analysis (with all its advantages of precision of research findings and thus precision of theoretical feedback for further theoretical construction) is available.

The goal of course is not mathematical manipulation as a fancy research technique. These tools can represent mere sophistry if the aim of representation of lawful human behavior is forgotten. The point is that the language of everyday life is loose and vague--and inappropriate to science. Concepts which have exact quantitative meaning (i.e., continuous variables) have clarity and much potential for uncovering functional relationships between social variables.[8]

Harvey's discovery of the circulation of the blood followed his attention to the amount of blood coming into and being pumped out of the heart per unit of time -- i.e., the rate of a particular quantity of blood being pumped. Thus again we see that this loose concept of circulation which is used in everyday language could only be of use to the scientist when it was translated into a fundamental quantitative measure. Circulation represents a derived measure. Sociologists--both those who oppose the "empiricists" and those who represent the American empirical tradition--have generally not confronted the problem of the absence of a system of fundamental measurement in sociology from which the measurement of other social concepts could be derived.

The Coordinate System was important in the development of modern physics because it facilitated the measurement of motion through its capacity to locate any point in space with reference to the point's coordinates. (All of the coordinates in the system have been defined relative to a zero point of origin.) Thus the Coordinate System is both a tool of measurement and a theoretical basis for procedure for clearly working out the implication of relationships among variables underlying physical laws. This dissertation is concerned with the development of a social manifold wherein there would also be a reference for the social objects and coordinates from which to locate the object in the social manifold.

In the first section of this Chapter a number of characteristics were cited as criteria in constructing the social manifold. Why these characteristics are considered desirable was also discussed. It must be reiterated that the model used in constructing the social manifold did not and does not constrain the data to conform to the characteristics of the model.

Available Technical Procedures[9] Relevant to the Construction of the Desired Social Manifold

While there are no predecessors who have attempted the construction of a manifold with the jointly theoretical and empirical interests of this dissertation, there have been those who have utilized physics in some manner as a model for social science[10] and there has also developed a body of literature concerning multidimensional scaling which is the basic tool for the construction of a social manifold. It is that latter work which is most relevant to the construction of a social manifold.

Cross Products - Scalar Products Controversy

Factor Analysis is an integral part of multidimensional scaling. However, the raw data is often in the form of distances in multidimensional scaling.

Factor analysis attempts to express n variables in terms of (n-1), or preferably less, factors. This goal can be thought of geometrically in terms of trying to express n vectors in terms of k vectors which represent a "basis" for that space (k - n). If a set of vectors form a basis for a space V, then every vector in that space is expressible uniquely as a linear combination of the basis vectors. Basis vectors are linearly independent. For example, if x, y and z are the basis vectors of a space, then Vector A in that space could be expressed as follows:

$$A = a_1x + a_2y + a_3z.$$

In other words a_1, a_2, and a_3 represent the coordinates of a on the three independent axes of the space. The requirement of linear independence is that the uniqueness of any other vector in the space is assuredly determined in terms of the basis vectors. A linearly dependent set might be represented in an infinite number of representations.[11]

Let us return now to the question of scalar products and cross products. Nunnally, a psychometrician, acknowledges the nonorthogonality or at least questionable orthogonality of a matrix of cross products of raw distance matrices. However, he denies the significance of this "convention."

> There is no mathematical necessity for restricting the use of D and of cross-products analysis to those situations where variables are uncorrelated. For the mathematical analyses, the orthogonal space is constructed, and there is no need to make the angles among vectors proportional to their natural correlations. The real issue is the interpretability of the analyses.[12]

I will present both theoretical and empirical evidence which makes Nunnally's position questionable.

Mathematical Explanation of the Relationship Between Cross Products and Scalar Products

"With any 2 vectors we associate a Scalar a.b which is defined by the equation

$$a.b = \tfrac{1}{2}\,(a + b)^2 - |a|^2 - |b|^2,$$

$$a.b = \tfrac{1}{2}\,(\Sigma(a_i + b_i)^2 - \Sigma a_i^2 - \Sigma b_i^2,$$

Expanding, $$a.b = \tfrac{1}{2}\Sigma a_i^2 + \Sigma 2ab_i^2 + \Sigma b_i^2 - \Sigma a_i^2 - \Sigma b_i^2,$$

$$a.b = \Sigma a_i b_i$$

$$a.b = a_1b_1 + a_2b_2 + a_3b_3$$

"This equation defines a Scalar Product in terms of the coordinates in a rectangular coordinate system..."[13] The coordinates on axis 1 of point a is a_1. If the system is not rectilinear,[14] then

$$a.b = \sum \lambda a_i b_i$$

$$\lambda = \text{cosine } \theta ab$$

Cosine θ drops out in a rectilinear coordinate system because:

$$e_i e_j = \lambda_{ij} \quad (e = \text{basis vector})$$

$$\lambda_{ij} = \begin{vmatrix} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{vmatrix}$$

It can be seen then, that strictly mathematically, cross products are equivalent to scalar products only under the special case of the cosine being equal to zero or one, i.e., the two vectors being orthogonal or superimposed. In the case of orthogonal vectors, this describes the situation whereby any point in space can be located in terms of the basis vectors of the space.

Vectors a and b's coordinates are given by $a = \sum a_i e_i$ and $b = \sum b_i e_i$.

> The vectors e_1, e_2, e_3 are called a set of basis - vectors and a_1 a_2 a_3 are the coordinates of vector a relative to these basis vectors. This is abbreviated as: $a = a_i e_i$. Any point in space can be located relative to the basis vectors - e_1, e_2, e_3 (or axes).[15]

P = a point in space

OP = position vector of P relative to 0 the origin.

$OP = x_1 e_1 + x_2 e_2 + x_3 e_3$.

x_i = the coordinates of x on the basis vectors (e_i).[16]

Thus the Scalar Product of a vector A and a basis vector gives the projection of that vector A on the axis.

If you had the coordinates of 2 points on basis vectors, by taking the cross products of the coordinates you would get their Scalar Products: the projection of the vectors on each other.

Factor Analysis, Metric Multidimensional Scaling, and Scalar Products

It may be useful to stop momentarily and put in perspective this presentation of the relationship between cross products and scalar products. Much of this explanation and discussion has been in terms of the coordinates of points on axes (or basis vectors). The primary problem of multidimensional scaling is, however, to construct from given distances a multidimensional space and thus answer the following: (1) What are the number of axes or dimensions? (2) What are the coordinates of the points (or social objects or stimulus objects) on the axes (or dimensions)?

It would thus seem that the purpose of metric multidimensional scaling, in a technical sense, would be to transform the data into some form upon which factor analysis could be performed. Factor analysis answers the question concerning dimensionality.[17] Remember also that it is the object's factor loading which is the projection of that point in space (i.e., the object) on the axis.

Thus the question remains as to how to convert distances between pairs of objects into a spacial configuration. Multidimensional scaling is a technique for the presentation of proximity relations among stimuli or stimulus objects (depending upon your perspective) as points located in space. A similar way to conceptualize it is that multidimensional scaling entails assigning a set of numbers to represent the similarities between a set of points.[18]

Young and Householder derived, using Euclidean geometry, a means of determining the dimensionality and imposing a coordinate system on a set of points (stimuli) when only the distances between pairs of stimuli were given. They showed how any element in a symmetric distance matrix (n-1) x (n-1) could be converted into the Scalar Product of a pair of points by utilizing a third point and geometric laws and thus locate points in space. From the law of cosines, it can be seen that for any three points:

$$d^2_{jk} = d_{ij}^{\,2} + d_{ik}^{\,2} - 2d_{ij}d_{ik}\text{Cos } \theta_{jik}$$

Moving terms yields:

$$d_{ij}d_{ik} \text{ Cos } \theta_{jik} = \tfrac{1}{2}\,(d_{ij}^{\,2} + d_{ik}^{\,2} - d_{jk}^{\,2}).$$

$d_{ij}\ d_{ik}$ Cos θ_{jik} represents the Scalar Products of vectors d_{ij} and d_{ik}. Young and Householder formed a matrix B_1 wherein each element b_{jk} was the Scalar Product of vectors d_{ij} and d_{ik}. It should be apparent that all the information required to solve the above equation are distances. Young and Householder reduced the problem of finding a matrix A "which specifies the configuration of points" to the trigonometric problem of determining a triangle when the length of its sides are given. The following are Young and Householder's theorems and apply to any B_i matrix whose elements are Scalar Products.

1. If a matrix B_i is positive semi-definite, the distances between the stimuli may be considered as distances between points lying in a real Euclidean space. For B_i to be positive semi-definite the triangle law must be fulfilled: the sum of two sides of a triangle cannot be less than the third side.

2. The rank of any positive semi-definite matrix B_i is equal to the dimensionality of a set of points.[1] The rank of a matrix for a given set of points will remain constant over variations in the choice of a point i to be the origin.

3. Any positive semi-definite matrix B_i may be factored to obtain a matrix A, where

$$B_i = AA'$$

If the rank of matrix B_i is equal to r, where r (n-1), then matrix A is an (n-1) x r rectangular matrix whose elements are the projections of the points on r orthogonal axes with origin at the ith point of their dimensional, real Euclidean space.[19]

Torgerson improved on the Young and Householder formulation by deriving from it an equation which computes the Scalar Products plus imposes an axis on the center of the configuration of points so that the origin of the Scalar Products Matrix would be at the centroid of all the stimuli (or social psychological objects).[20]

$$b^*_{jk} = \tfrac{1}{2} \left(\frac{1}{n} \sum_j^n d_{jk}^2 + \frac{1}{n} \sum_k^n d_{jk}^2 - \frac{1}{n^2} \sum_j^n \sum_k^n d_{jk}^2 - d_{jk}^2 \right)$$

Torgerson's formula can be made more comprehensible by changing some of the subscripts without changing any of the formula's substantive meaning:

$$b^*_{gh} = \tfrac{1}{2} \left(\frac{1}{n} \sum_k^n d_{kg}^2 + \frac{1}{n} \sum_k^n d_{kh}^2 - \frac{1}{n^2} \sum_k^n \sum_j^n d_{jk}^2 - d_{gh}^2 \right).$$

$k = 1 \qquad j = 1$

It is easier to see from this that Torgerson's formula is based on the Young and Householder usage of the law of triangle inequality: two sides of a triangle are always equal to or greater than a third side. (f. n. on straight line). However, Torgerson has substituted the mean distance from all other points, $k = 1 \ldots n$, for Young and Householder's third point i. Moreover, Torgerson also subtracts the midpoint of the distances and in a sense imposes a central axis on all the points. Note axis coordinates have not yet been derived. All we have is a matrix B with elements b^*_{jk} --scalar products-- spatially located around a rather centrally located origin.[21]

Resolution of the "Scalar Products versus Cross Products" Debate

The scalar products are V_1V_2 cosine $\theta v_1\ v_2$ (where v_1 = vector 1). This is consistent with the earlier discussion of cross products where $a.b = a_1b_1$ in a rectalinear system (cosine equals 0 or 1) but in other cases $a.b = \lambda a_1b_1$, where λ = cosine θab. It is thus incorrect to use the sum of cross products -i.e., in the sense of multiplying a pair of rows across columns, or a pair of columns across rows,[22] -- as a substitution for scalar products when the items subject to the cross products transformation are not independent and orthogonal. Otherwise, a crucial element of information will be missing as to the spatial relations between the items.

The sufficiency of factor analysis of cross products as opposed to scalar products was put to an empirical test. A perfect data matrix for an oblong solid was submitted to the following procedure. A distance matrix of an oblong solid's eight corner points was constructed. The matrix was transformed (via SSUPAC) to one of cross products (i.e., $\sum_k^m a_k b_k$). A Principal Axis factor analysis was then performed. The results were inadequate. The oblong solid could not be reconstructed from the resulting factor loadings. A whole dimension was left out. Even when data is perfect, the cross products of dependent (i.e., not orthogonal) items does not reproduce a configuration of points perfectly.[23]

The relationship between cross products and scalar products has to some degree been confused in the psychometric literature. Part of the reason for the confusion is because of the not untypical procedure of psychologists of treating either persons as orthogonal -- even though their test scores are not uncorrelated -- or of treating psychological tests as orthogonal and independent. These assumptions are not usually empirically true. However, as has been pointed out above, the location of a pair of vectors in space can only be accurately determined in terms of the cross products of their values on the basis vectors -- i.e., the orthogonal vectors defining the space. Nunnally uses the following derivation to show the relationship between the distance and cross products measure:[24]

$$(1) \quad D_{ab}^2 = \Sigma(X_{aj} - X_{bj})^2$$

$$= \Sigma(X_{aj}^2 + X_{bj}^2 - 2\, X_{aj}\, X_{bj})$$

$$= \Sigma X_{aj}^2 + \Sigma X_{bj}^2 - 2\Sigma X_{aj}\, X_{bj}$$

$$(2) \quad \sum X_{aj} X_{bj} = \frac{\sum X_{aj}^2 + \sum X_{bj}^2 - D_{ab}^2}{2}$$

Nunnally does not point out that the initial equation represents the distance formula for deriving the distance between two points from their coordinates on the axes, i.e., $j = 1..r$. It cannot be assumed that all the , for example, test variables used as j's (in transforming a raw data matrix - e.g., distances - to a cross products matrix) would be orthogonal, i.e., independent, in a geometric representation.

Nunnally says that "the major problem in multidimensional scaling is to establish an origin for the space." He says that once the origin is determined "the squared distance of each point from the origin can be found, and this supplies the information necessary to compute the sums of cross products."[25] Nunnally's description is inconsistent with the actual procedure generally followed in multidimensional scaling. In the example Nunnally cites to demonstrate the procedure, he has as given the "score" of a point a on dimension A and dimension B. He then derives the distance of the point from the origin by the Pythagorean theorem, although he does not identify his mathematical reasoning. But, it is necessary to remember, that neither the dimensionality, nor the "scores" on the dimensions are known. The problem for Nunnally may be that as a psychometrician, he generally works with scores of people on either tests defined as the dimensions or else people assumed to be orthogonal at the outset. He seems to be trying to redefine the problem in terms of the cross products some psychometricians use, -- perhaps illegitimately -- for factor analysis. Nunnally even lifts an example of multidimensional scaling directly from Torgerson and describes it as the "sum of cross products." Nunnally is implicitly suggesting that Torgerson means the same thing as the former does by his derivation described above (equation two) -- rather than the scalar products which it actually is. Note also that the scalar product is closely alligned with geometric representation; also note that the normal procedure of many psychometricians, including Nunnally, entails assuming as given the geometric relationship of orthogonality. This might indicate some of the reasons underlying Nunnally's somewhat misleading analysis of multidimensional scaling and cross products.

The superiority of scalar products over cross products for the transformation of raw distances has been demonstrated both in terms of mathematical analysis and in terms of empirical evidence. The inadequacy of the factor analysis of the cross products of the distance matrix of eight corner points of an oblong solid. Factor analysis of the scalar products of the oblong solid's distances resulted in three factors -- length, width, and height. Plotting the factor loadings (or coordinates) produced a scale version of the oblong solid.[26]

Nonmetric Multidimensional Scaling

Multidimensional scaling began with unfolding methods. The goal was to find the required number of dimensions and the rank order of scaling on the dimensions. However, multidimensional unfolding was fully nonmetric and thus one could not reproduce the configuration; only the rank order of projections on dimensions could be constructed. As Green says Coombs provided the "conceptual ground" for later developments but later techniques provided more information for the same assumptions i.e., only ordinal estimations were required. Shepard provided the real beginnings of nonmetric multidimensional scaling.[27]

Nonmetric scaling has origins in unidimensional scaling. Pioneering work was done by Abelson who used ordinal ranking of pairs of points to determine spatial relations between points on a line. This type of scale, while often not interval, is superior to scales of only rank order.[28]

Two major contributors to the development of nonmetric multidimensional scaling have been Kruskal and Shepard. Kruskal's work represents a direct extension of Shepard's work which was pioneering in the field of nonmetric multidimensional scaling. Multidimensional scaling is a relatively new field and nonmetric multidimensional scaling is even newer. Shepard said the goal of multidimensional scaling should be the monotonicity between the experimental similarities or dissimilarities and the distances between points in the derived configuration. Kruskal also believed that the requirement of order was enough to tightly constrain the solution and reproduce the configuration. Kruskal's major contribution was to attempt to define how dissimilarities and distances should be related and to develop a statistic for evaluation of the "goodness of fit" of the derived configuration to the original dissimilarities.

Kruskal's basic position is also that dissimilarities and distances should be monotonically related. Kruskal says he views the problem of multidimensional scaling as one of "statistical fitting" wherein distances must be found which best fit the given dissimilarities. Basically Kruskal's technique entails a monotonic regression of distance upon dissimilarity and then the use of the normalized residual variance as the quantitative measure of error.[29]

Nonmetric multidimensional scaling's goal is summarized by Forrest Young. "The purpose . . .is to find a set of numbers representing the separations (or dissimilarities) between a set of points. That is, the method attempts to provide a scaling solution with ratio properties which is based on data without ratio properties."[30]

Kruskal's qualitative error measure is called (Kruskal's) Stress. The best fitting solution is defined as the solution with the lowest stress. He defines the level of acceptability of the normalized residual variance (Stress) as follows:

Stress	Goodness of Fit
20%	poor
10%	fair
5%	good
2½%	excellent
0%	"perfect"

Kruskal says that by perfect he means a solution wherein there is a "perfect monotone relationship between the distances and dissimilarities."[31]

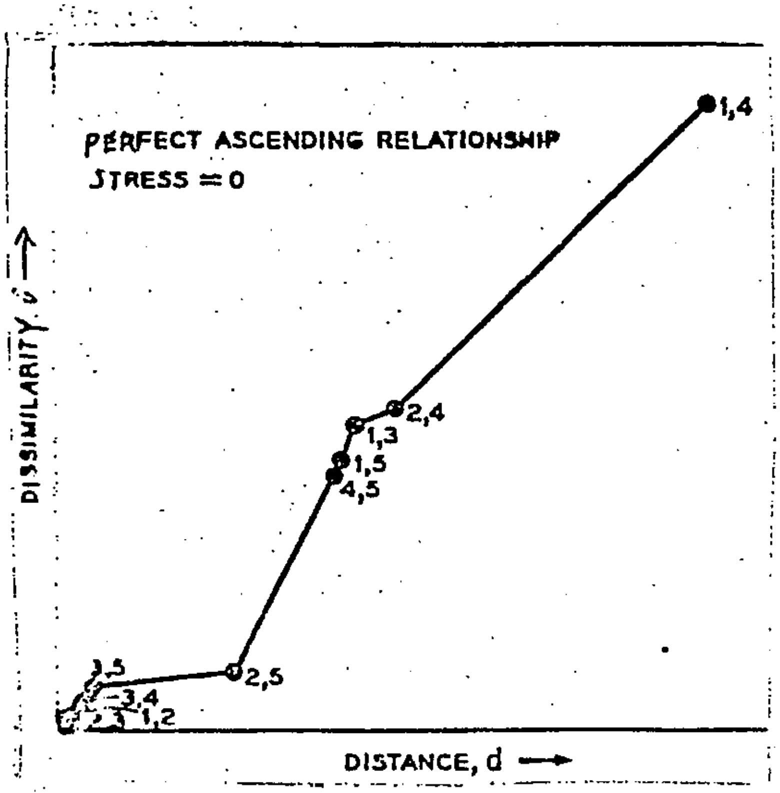


Figure 1. Kruskal's Stress=Zero.

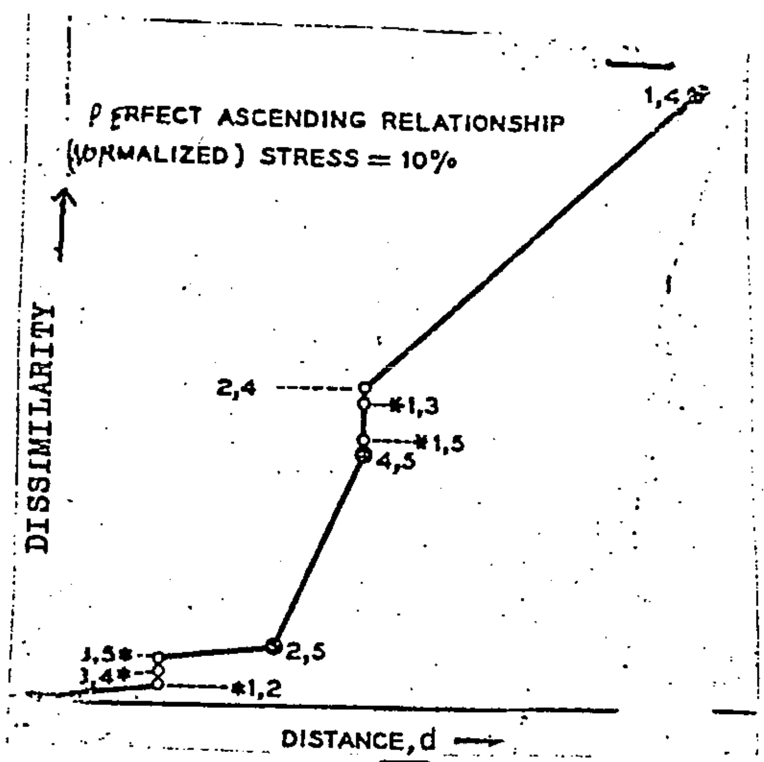


Figure 2. Kruskal's Stress=10%

$$\text{raw stress} = S^* = \sum_{i\,j}(dij - \hat{d}ij)^2$$

dij represents the values fitted to the monotonically ascending line relating the distances to the dissimilarities. Thus each dij is greater than the one before.

The horizontal deviations are represented by dij - dij. Shepard notes his statistics close relationship to the "residual sum of squares" type of fitting technique.

Kruskal standardizes his statistic as follows:

$$\frac{S^*}{T^*} = \frac{\sum_{i\,j}(dij - \hat{d}ij)^2}{\sum_{i\,j} dij^2}$$

With both Kruskal's stress and Young-Torgerson's index of fit the derived distances are compared to the monotonically transformed disparities (e.g., Kruskal's dissimilarities). This procedure is followed because, nonmetric Scaling assumes no more than ordinal information in the original similarities. Thus, data information provided initially may be disregarded; e.g., data collected with ratio estimations - as in the Galileo System - may lose some of its potency.

Young's definition of metric determinacy (labeled M) "the squared correlation between the true distances and the reconstructed distances." Young criticizes Kruskel as basing his index, i.e., Kruskal's Stress, upon how well the reconstructed distances reproduce the probably error-ridden dissimilarities. Young describes d_{ij} as nonmetrically reconstructed distances and d_{ij} as numbers which are monotonically related to the dissimilarities. He then chides Kruskal's formula as reflecting "the relationship between the reconstructed distances and the dissimilarities" whereas his own index of metric determinacy is based upon "the relationship between the true and reconstructed distances."[32] While Young's point is valid, it is not as useful as one might wish for as he himself admits, the true distances are usually not known for pyschological data. The investigation Young performed is based upon different combinations of given random coordinates.

Forest Young investigated the question of whether metric information can be recovered from nonmetric Scaling. He took a configuration of known structure, (i.e. Monte Carlo - random numbers) and added error to the points and performed a monotonic transformation on the points in the configuration. He is examining to what degree the original configuration can be reproduced. Young also compares his metric determining statistics to Kruskal's stress, a major statistic which has been used to analyze the degree of confidence the experimenter should have in the configuration he has produced.

Young found that metric determinancy is positively related to the number of points used but is inversely related to the number of dimensions and the amount of error in the configuration. Kruskal's measure of stress increases with the number of points used and the error but is inversely related to the number of dimensions. For low error, supposedly even a small number will give 99% metric determinacy. Note for 8 points with no error, the metric determinacy is practically perfect.[33]

As far as choosing dimensionality, Kruskal emphasizes the fact that scientific judgment is intimately involved. Two important components are (1) small stress and (2) interpretability, i.e., the theoretical significance of the factors. As error increases, Kruskal points out, that with error, it becomes increasingly likely that added dimensions are merely error.[34]

TORSCA

Since Shepard's work on nonmetric multidimensional scaling in 1962, a number of computer programs for nonmetric multidimensional scaling have been developed. While many are similar, among the ways they vary is how metrical they are. For example, Lingoes has written a multidimensional scaling program for nominal data.[35] One of the best available programs is TORSCA - 9.[36] This Fortran IV program is semi-metric and retains relatively more of the information provided by the original distance matrix than most other "non-metric" programs and perhaps thus can only be considered "quasi-non-metric."

The TORSCA program is divided into two parts. The first procedure is semi-metric. It requires that the number of dimensions - r - be given by the user on the program control cards. The distance matrix (i.e., the data) is converted to scalar products and then factor analyzed. Using only the first r factors, the distances are then reconstructed by using a formula generalized from the distance formula differing only in that any Minkowski metric number can replace the square and square root required for a Euclidean space:

$$d_{ij} = \left[\sum_{a}^{r}(v_{ia} - v_{ja})^m\right]^{1/m}$$

a = factor 1

v_{ia} = point i's "loading" on factor a

r = requested number of factors.

These derived distances are then monotonically transformed to best fit the original similarities -- i.e., original distance matrix. An index of fit is computed. The monotonically transformed distances, i.e., the disparities, are used through the next factor analysis cycle. After each cycle the index of fit is computed. When the index ceases improving, iterations stop and the initial configuration prior to the nonmetric algorithm is prepared. Young says concerning the semimetric first algoritm that, "when one realizes that this algoritm is being used only to prepare a starting configuration for the nonmetric algoritm, one also realizes that the basic assumptions of nonmetric scaling are not being violated."[37]

The derived distances are monotonically transformed to the disparities. Each block of disparities is compared with the previous one; if larger, go on to the next but if it is smaller, find the mean of both blocks and enter them in both blocks. The index of fit compares the derived configuration with the monotonically transformed disparities. The index of fit procedure is basically a regression problem. The degree to which "a plot of d_{ij} versus e_{ij} can be fitted by an equation of the form $y = ax$ is examined.

Factor Analysis, Nonmetric and Metric Multidimensional Scaling: Some Comments

This dissertation deals with the construction of a manifold for social objects wherein social change might be observed. The manifold to be constructed is to offer a continuous, homogenous, isotropic and metric framework from which to examine the discrete social objects.

Multidimensional scaling, metric as well as nonmetric, along with factor analysis have been characterized by applications which indicate theoretical inconsistency with the construction of the social manifold specified in this work. The source of the problem lies to a large degree with the conception of factor analysis which is associated with the first psychological uses of the technique. Factor analysis was used to determine the components underlying psychological tests; psychologists desired a means of reducing their batteries of tests to a smaller number of theoretical variables. Thus factor analysis came to be associated with discovering underlying "traits."

> Factor analysis provides a mathematical model which can be used to describe certain areas of nature. A series of test scores or other measures are intercorrelated to determine the number of dimensions the space occupies, and to identify these dimensions in terms of traits and other general concepts. The interpretations are done by observing which tests fall on a given dimension and inferring what these tests have in common that is absent from tests not falling on the dimension. Tests correlate to the extent that they measure common traits. By observing and analyzing the pattern of intercorrelations, the operation of one or more underlying traits or other sources of common variance is inferred.[38]

Measurement is often described as relating to properties or attributes (e.g., color, weight, or length) of objects. Torgerson, and Pfanzagl. This approach is consistent with the typical factor analysis wherein underlying properties, -- or in multidimensional scaling, dimensions of properties such as the brightness and chroma of color, are searched for to correspond to the dimensions (axes).

In physics, coordinate systems and points of coordinates (not unlike the imposing of axes and variables' loadings in factor analysis) are used to locate the position of objects. The motion of objects can be analyzed in the coordinate system. The axes may have measurement units such as distance, time, or even velocity but these are not properties of the objects being located.

Another way of conceiving of measurement is that it "presupposes no preexisting property", but instead involves "measuring operations with a more or less coherent results."[39] While one could argue against the atheoreticalness of this description, at least, it does not assume measurement of an object's property. Russel's definition of measurement also does not include the idea of property as a necessity. "Measurement of magnitudes is, in its most general sense, any method by which a unique and reciprocal correspondence is established between all or some of the magnitudes of a kind and all or some of the numbers, integral, rational, or real as the case may be."[40]

The latter definitions of measurement correspond more closely to the use of coordinate systems for the analysis of the motion of objects. This latter conception is also more consistent with the development of a social manifold which might be useful for the analysis of social motion. Distance, or velocity is not possessed by that dimension. Objects falling near the distance axis do not possess more distance while those near the other axis possess time. It is instead that correspondences are established between certain magnitudes -- e.g., distance, time, and velocity. These can then serve as an "explanation" of the motion of objects.

The too typical use of factor analysis has been in ways which do not satisfy the characteristics set forth as desirable for the construction of a social manifold; the spaces constructed through factor analysis have been more typically heterogeneous, anisotropic, and discrete. As factor analysis has been used, the development of a unbounded, metric space for the observation of social change has been inconceivable.

The interpretability and thus usefulness of factor analysis as it has been used can be questioned. The analysis of isolated factors and the relationship of variables from their loadings on isolated factors can be misleading. For example, a color's brightness or chroma alone would poorly serve as a basis for predicting how a subject perceives the similarity of colors of the same hue. Multidimensional scaling has improved on factor analysis by its simultaneous attention to the dimensions considered significant. Multidimensional scaling approaches are more likely to treat the scaled points -- or stimuli -- as phenomena to be plotted in a manifold characteristically analogous to a manifold. The metric system treat the input data as metric and give metric output as results. Metric multidimensional scaling has typically dealt with sensory stimuli. Objects are scaled in multidimensional scaling analysis. However, there is a tendency to assume that the axes must represent some attribute or property of the objects being scaled. This is because multidimensional scaling has typically been used for static analysis -- often for a descriptive presentation of the perceived similarity of the objects.

From the plots of the perceived similarity of the objects, there is an attempt to discover the underlying dimensions or properties of the objects which were used to determine similarity. The axes are then labeled for the properties, (i.e., dimensions) -- e.g., sportiness or luxuriousness.

The manifold to be constructed herein -- The Galileo System -- is distinguished by its social character. Social forces or processes at work, not immediately obvious when dealing at the level of individuals and properties of objects, might be made apparent.

Semantic Differential[41]

The conception of space in the semantic differential is contrary to the characteristics considered desirable for the construction of a social manifold. The space of the semantic differential does not provide a homogeneous continua from which to analyze discrete social phenomena. The semantic space instead is in itself discrete and heterogeneous and anisotropic.

Axes are treated as if they embody the psychological dimensions of evaluativeness, potency, and activity. Concepts are observed as to their loading on these axes with lesser regard to their interrelationships despite the fact that the polar adjective scales are correlated and also despite the fact that attention to loadings on isolated dimensions can not tell you how different concepts will be located in the semantic space.

The semantic differential technique involves the use of distance to construct a multidimensional semantic space. Polar adjectives are "assumed to represent straight lines passing through the origin of a space."[42] Osgood, et. al., says that a sample of polar coordinate adjective scales represent a multidimensional space. The more scales, the better the sample of adjective scales, and the better defined is the space. The goal is to find the orthogonal dimensions -- axes -- which define the space. A problem which is clear from the start is that the boundaries of the space are defined from the outset. The space is assumed to be polar and the subject is given the option of classifying concepts in terms of these polar adjective scales on a scale which runs from +3 to 0 to -3. The procedures used to limit the possibilities of the spread of the space is contradictory to the present desire to construct a manifold which reflects the conceptual space of the aggregate sampled. The degree to which this scale of measurement (=3 to -3) is equal interval, as assumed,[43] is dubious considering that is wholly externally imposed and thus should probably not be considered metric.

Osgood also states that "the sampling of concepts presented a less critical problem, since our purpose was a factor analysis of scales of judgement rather than of concepts."[44] This statement is matter for concern since it is orthogonal concepts which are truly spanning the space. Summing of cross products is done across adjective scales to determine the distance between concepts.[45] A concept or a pair of concepts which are highly loaded on adjective scales associated with evaluation, potency, or activity, are labeled as the dimension. The polar adjective scales which are supposedly measuring these dimensions are however, <u>not uncorrelated</u>. The extent to which the dimensionality of the space has been imposed becomes an issue. Questions of the validity of the procedure becomes more prominent with the recognition that though there does not seem to be a change in the factor structure due to subjects but there was one when concepts were changed.[46]

Osgood talks of the space as being defined by K dimensions of polar adjective scales. However, on examination it is apparent that the dimensions I, II, and III are passed through concepts with concepts being chosen according to which has the highest sum of squares of scores across scales.[47]

Summary and Conclusions

In this chapter, a number of technical procedures relevant to the construction of a social manifold were presented and evaluated, both in terms of their own validity and in terms of the criteria (homogeneous, continuous, isotropic, linear, unbounded and metric) set up for constructing a social manifold. These criteria for the spatial model do not require the data conform to these specifications.

Factor analysis of the cross products of a raw data matrix was found to be inadequate based both on a mathematical explanation of the technique and on empirical evidence. The traditional treatment of dimensions in factor analysis was shown to be inconsistent with the construction of a continuous, homogeneous, and isotropic space.

The Semantic Differential was found to be misleading in its designation of the three dimensions -- evaluative, potency, and activity -- as defining a semantic space. It was also explained to be discrete, anisotropic, and heterogeneous as opposed to continuous, isotropic, and homogeneous. The Semantic Differential's imposition of a unit of measurement was also noted to be inappropriate for constructing the desired social manifold.

Nonmetric multidimensional scaling techniques vary in the degree to which they pay heed to more than just ordinal information provided by the original data. Those that utilize only ordinal information would tend to lose some of the information provided by data collection techniques which collected subject's direct magnitude estimations with ratio properties (e.g., the galileo questionnaire -- to be discussed in the next chapter).

In general, metric methods of multidimensional scaling are procedurally most applicable to the construction of a social manifold according to the criteria specified. However, in practice, the use and conceptualization of this technique, as with nonmetric multidimensional scaling and factor analysis, has been greatly influenced by the posing of problems as conceived by psychologists and psychometricians. In chapters three and four, a basically metric multidimensional scaling technique will be used to analyze a sociological problem: the statics and dynamics of social objects in conceptual and behavioral space.

FOOTNOTES

[1]E. T. Bell, "Newton", in Robert Marks (ed.), The Growth of Mathematics (New York: Bantom Books, 1964), gives the mathematical meaning - which is the way they are used in this work - of the terms "variable" and "function." On pp. 177-78, he writes "a symbol which can take on many different values during an investigation is called a variable. Y is a function of x if every time x is assigned a numerical value, the value of Y is also determined. ...Let y be a function of x, say $y = f(x)$. The rate of change of y with respect to x, or as it is called, the derivative of y with respect to x is determined."

[2]A cardinal number "describes an abstract property shared by all sets which can be put into one-to-one correspondence with a given set." See Israel Rose, A Modern Introduction to College Mathematics (New York: Wiley, 1959), p. 8.

[3]See Webster's Seventh New Collegiate Dictionary (Springfield, Mass.: Merriam, 1965), p. 181.

[4]The resultant vector can not be resolved into its component parts in the way that a regression line can (i.e., the beta weights) since the orthogonal axes are merely arbitrary coordinate axes. Even if a behavioral vector laid right on an axis, you could not be sure that this particular combination of possible vector combinations resulted in the resultant vector, R.

[5]Rose, op. cit., p. 151.

[6]Note that it is possible for space to be unbounded and yet finite. Einstein seemed inclined, at least at some points, towards this view of the physical universe. See Max Jammer: Concepts of Space (Cambridge, Mass.: Harvard University Press, 1969).

[7]Rose, op. cit., on p. 31 writes: "The fundamental purpose of any coordinate system on a plane is to identify points of that plane by means of ordered pairs of real numbers." (Italics added.) This statement can be extended to an n-dimensional space without either changing the purpose or losing significance.

The possession, in physics, of a system of fundamental measurement meant that the point could be located as a function directly or indirectly of fundamental units of measure.

[8]Of course, functional relationships do not ensure theoretical importance. However, the additional emphasis on developing a system of fundamental measurement based on a few basic concepts is meant to support the development of concepts - jointly theoretical and empirical.

[9]The use of the term "technical" is not meant to suggest that the procedures to follow do not have theoretical implications.

[10]Some social scientists who have used physics, more or less, as a model are Lundberg, Dodd, followers of psychological "field Theory," and Catton.

[11]Seymour Schuster, Elementary Vector Geometry (New York: Wiley, 1962), pp. 46-48, 24, 34.

[12]Jum Nunnally, Psychometric Theory (New York: McGraw-Hill, 1967), pp. 385-386.

[13]P. M. Cohn, Solid Geometry (London: Routledge, and Kegan Paul, 1961), pp. 9-11.

[14]Cyrus MacDuffee, Vectors and Matrices (Menasha, Wisconsin: The Mathematical Association of America, 1943), pp. 18-19.

[15]Cohn, op. cit., pp. 3-7.

[16]Ibid.

[17]Factor analyzing a configuration without error will reproduce the object perfectly in the correct number of dimensions; see Chapter Four for a detailed discussion of this matter.

[18]See F. W. Young, "Nonmetric Multidimensional Scaling: Recovery of Metric Information," Psychometrika, 35 (December 1970), 455-74, W. S. Torgerson, Theory and Methods of Scaling (New York: Wiley, 1958), and R. W. Beals, D. H. Krantz, and A. Tversky, "The Foundations of Multidimensional Scaling," Psychological Review, 75 (1968), 127-42.

[19]See Gale Young and A. S. Householder, "Discussion of a Set of Prints in Terms of Their Mutual Distances," Psychometrika, 3 (March 1938), 19-22, and Torgerson, op. cit., pp. 254-56.

[20]Torgerson, op. cit., pp. 257-59.

[21]In a similar formula 2n is used where n^2 is used in Torgerson's formula.

[22]See Nunnally, op. cit., for an example of how the term "cross products" is used, p. 380.

[23]See Chapter Four for an extended discussion and graphs (figures 14-16) relating to dimensionality.

[24]See Nunnally, *op. cit.*, for distance formula, pp. 384, 407.

[25]*Ibid.*, p. 408.

[26]See figures 12 and 13 in Chapter Four.

[27]P. E. Green and D. S. Tull, *Research for Marketing Decisions* (Englewood Cliffs: Prentice-Hall, 1966).

[28]R. P. Abelson, "A Technique and a Model for Multidimensional Attitude Scaling," *Public Opinion Quarterly*, 18 (1954), 405-18.

[29]See J. B. Kruskal, "Multidimensional Scaling by Optimizing Goodness of Fit to a Non-metric Hypothesis," *Psychometrika*, 29 (March 1964), 2-3, and R. N. Shepard, "The Analysis of Proximities: Multidimensional Scaling with an Unknown Distance Function," *Psychometrika*, 27 (June 1962), 125-39.

[30]Young, *op. cit.*, p. 455.

[31]Kruskal, *op. cit.*, pp. 2-3.

[32]Young, *op. cit.*, pp. 457-58.

[33]*Ibid.*

[34]Kruskal, *op. cit.*, pp. 15-17.

[35]See J. C. Lingoes, "An IBM-7090 Program for Guttman-Lingoes Multidimensional Scalogram Analysis - I," *Behavioral Science*, 11 (1966), 76-78.

[36]See F. W. Young, *TORSCA - 9: A Fortran IV Program for Nonmetric Multidimensional Scaling* (Chapel Hill: Thurstone Psychometric Laboratory, 1968).

[37]*Ibid.*, p. 6.

[38]Benjamin Fruchter, *Introduction to Factor Analysis* (Princeton: Van Nostrand, 1954), p. 2.

[39]Brian Ellis, *Basic Concepts of Measurement* (Cambridge: Cambridge University Press, 1966), pp. 24-25.

[40]Torgerson, *op. cit.*, p. 43.

[41]Charles E. Osgood, George J. Suci, and Percy H. Tannenbaum, *The Measurement of Meaning* (Urbana: University of Illinois Press, 1957).

[42]*Ibid.*, p. 25.

[43]*Ibid.*, p. 93.

[44]*Ibid.*, p. 34.

[45]*Ibid.*, pp. 94, 108.

[46]*Ibid.*, pp. 37-39.

[47]*Ibid.*, pp. 332-34.

CHAPTER 3

THE CONSTRUCTION OF THE DESIRED MANIFOLD: THE GALILEO SYSTEM

Introduction

Chapter one expresses the desirability of constructing a theoretical paradigm for sociology wherein theory and research could be joined through the use of a parsimonious set of fundamental quantitative constructs--both theoretical and operational at the same time. Specifically the construction of a manifold for social objects was proposed to provide a continuous, quantitative backdrop through which relationships between discrete social and psychological objects could be expressed. Chapter Two examined the available techniques for constructing a manifold in terms of the theoretical characteristics desired.

This chapter examines the "Logic of Inquiry," and also specifies the techniques for constructing the desired social manifold--continuous, homogeneous, unbounded, linear, and metric--which has been termed the Galileo System. The jointly theoretical-methodological significance of data collection techniques such as the Galileo questionnaire is specified. The Galileo questionnaire is distinct in that it collects direct estimations of distances between social objects and in that it has a scale with a natural origin (i.e. zero distances between pairs of objects).

Prior Sociological Research Underlying Galileo System

There is a Logic of Inquiry--based on both a theoretical and a methodological analysis--underlying this dissertation which concerns the construction of a manifold for social objects.

Theoretically, the desire to construct a picture of social space underlies the origins of the Galileo System. Methodologically, factor analysis was focused on as the tool to handle the problem.

Factor analysis was conceived of as a descriptive technique for spatial analysis. It was proposed that if a matrix was formed whose elements contained the distances between pairs of objects e.g., objects in a room, factor analytic technique could be used to reproduce the dimensionality (i.e., length, width, and height--all arbitrary) and determine the projections of the objects on the dimensions with the end result of reproducing the spatial relation between the objects in the room.

This methodological question began to achieve theoretical significance with the following research; it was proposed that if people are to behave in physical space--i.e., walk through doors, not fall over chairs--they must have some conception of it. Moreover, if people act jointly in the same physical space, they must share a picture of that space. The following represents a test of the hypothesis:

A picture of Alice in Wonderland was presented to a social science class at the University of Illinois. (See Figure 3)

Figure 3. Alice in Wonderland Picture

The class members were asked to estimate the distances between all pairs of the objects in the picture, $\frac{N(n-1)}{2}$. (See Figure 3.) It was expected that through factor analysis the space could be reconstructed. Since the cross products of the distance matrix was computed (as opposed to the scalar products)[1] there were some distortions in the reconstructed picture, though the likeness was good enough for the results to be encouraging. (See Table 1 for example of what is meant by a distance matrix.)

The next logical step involved extending the "Alice" procedure to social behavior. Underlying this step is the theoretical position that for people to act jointly while engaging in social behavior, they must share a "picture of social space."

The first administration of the primary Galileo questionnaire involved only ten social objects. It was conjectured that if men must share a conception of physical space (Alice questionnaire) to act jointly in it and coordinate their actions in space, e.g., not bump into one another, they must also share a conception of social space to act conjointly. The Galileo system was attempting to reconstruct a picture of that social space[2] in the way the 'Alice' procedure attempted to reconstruct a picture of physical space from an aggregate's mean conception (i.e., conceptual space) of it.

The Galileo System represents an attempt at developing a coordinate system for social objects wherein social objects

TABLE 1

EXAMPLE OF A DISTANCE MATRIX
BASED ON A SEGMENT OF THE 'ALICE' DATA

	Alice	Door in tree	Cheshire Cat	March Hare	Chimney	Mad Hatter
Alice	0	2	22	16	40	15
Door in tree	2	0	21	17	42	16
Cheshire Cat	22	21	0	40	25	38
March Hare	16	17	40	0	35	1
Chimney	40	42	25	35	0	36
Mad Hatter	15	16	38	1	36	0

can be located in quantitative relations with one other. It also attempts to provide a frame of reference from which to view the rate of change of the relationship between social objects. The purpose of developing a fundamental system of measurement (e.g., a unit of measure from which other measures can be derived) argues at this time against using non-metric techniques which can lead to a loss of scale. Factor analysis of cross products is also not adequate because of its increasing error as the data varies from an axis like perpendicular representation.[3]

The Method of Constructing the Galileo System: A Spatial Manifold for Social Objects

Overview

Data collection is significant and an integral part of the Galileo System. The scale is continuous with ratio properties. The data does not necessarily conform to these attributes but the scale offers the opportunity for the most powerful data possible.[4] Moreover, it set up the basis for a metric system of measurement. The Galileo manifold based on the unit of galileo distance is a homogeneous and isotropic model. Technically, isotropy designates the "rotational symmetry" of space and homogeneity designates the "translational symmetry" of space.[5]

The Galileo Space

The Galileo Manifold is unbounded in the sense that it is administered at various points in time with the data being compared via a rotation process.

The Galileo Space should have the same metric unit of measurement throughout the manifold. The rate of social motion could be expressable in terms of how long it takes to cover a galileo unit of distance per unit of time. The linear model is expressed through the use of factor analysis - with its linear equations - and metric multidimensional scaling in a Euclidean space. This model does not enforce conformity of the data. It represents a mathematically powerful model whose correspondence with the data is an empirical question. Coombs, et. al., distinguishes between measurement theory and scaling because as he points out, rarely are the axiomatic assumptions of a model completely fulfilled.[6]

Scaling and the Galileo Questionnaire[7]

The participants were given the following written directions:

> Recent research has shown that people see different actions as more or less "far apart" or distant from each other. For example, "sitting" is closer to "lying down" than it is to "running." Unlike physical distance, which is measured in feet or miles or meters, etc., <u>social</u> distance is measured in "galileos." You are supposed to estimate how many galileos apart the following actions are. Please answer every question, even if you must guess, since any blank question ruins the whole questionnaire.

They were also supplied with a given unit of measurement: that sitting and lying down were one galileo apart. See Figure 4 for example of thirty-three object Galileo Questionnaire.

The Galileo Questionnaire was designed to collect data in a manner consistent with the underlying theoretical paradigm. For example, the estimations of the magnitudes of distance between objects were not numerically bounded. The scale is treated as continuous and isomorphic with the real number system. The natural zero point of no (i.e., zero) distance apart is consistent with the attempt to construct a ratio scale.

Some distinctions which can be made between methods of scaling are as follows:

1. What is being scaled; does the scale examine the relation between the subject and stimulus or does it concentrate on the stimuli? Coombs makes a distinction between "joint space" for the former and "stimulus space" for the latter.[8] The Galileo Manifold results in a stimulus space

Figure 4. The Galileo Questionnaire
(33 object)

ASSUME THAT "SITTING" AND "LYING DOWN" ARE ONE GALILEO APART: How far apart are....

(21,02) Sitting and fighting pollution............( )

(22,02) Sitting and ice skating...................( )

(23,02) Sitting and women's liberation............( )

(24,02) Sitting and skiing........................( )

(25,02) Sitting and going home....................( )

(26,02) Sitting and quitting school...............( )

(27,02) Sitting and smiling.......................( )

(28,02) Sitting and going to college..............( )

(29,02) Sitting and talking.......................( )

(30,02) Sitting and studying......................( )

(31,02) Sitting and war...........................( )

(32,02) Sitting and living in a commune...........( )

(33,02) Sitting and practicing medicine...........( )

(04,03) Strolling and running.....................( )

(05,03) Strolling and sleeping....................( )

(06,03) Strolling and fighting....................( )

(07,03) Strolling and revolution..................( )

(08,03) Strolling and marrying....................( )

(09,03) Strolling and singing.....................( )

(10,03) Strolling and smoking marijuana...........( )

(11,03) Strolling and making love.................( )

(12,03) Strolling and killing.....................( )

(13,03) Strolling and eating......................( )

(14,03) Strolling and stealing....................( )

(15,03) Strolling and laughing....................( )

(16,03) Strolling and getting ahead...............( )

(17,03) Strolling and going on vacation...........( )

(18,03) Strolling and sunbathing..................( )

(19,03) Strolling and swimming....................( )

(20,03) Strolling and me..........................( )

(21,03) Strolling and fighting pollution..........( )

(22,03) Strolling and ice skating.................( )

(23,03) Strolling and women's liberation..........( )

(24,03) Strolling and skiing......................( )

(25,03) Strolling and going home..................( )

(26,03) Strolling and quitting school.............( )

(27,03) Strolling and smiling.....................( )

(28,03) Strolling and going to college............( )

(29,03) Strolling and talking.....................( )

(30,03) Strolling and studying....................( )

(31,03) Strolling and war.........................( )

(32,03) Strolling and living in a commune.........( )

(33,03) Strolling and practicing medicine.........( )

(05,04) Running and sleeping......................( )

(06,04) Running and fighting......................( )

(07,04) Running and revolution....................( )

(08,04) Running and marrying......................( )

(09,04) Running and singing.......................( )

(10,04) Running and smoking marijuana.............( )

(11,04) Running and making love...................( )

based on the mean of the aggregate data of the sample.

2. level of measurement; ordinal, ordinal with natural origin, interval, or ratio scales represent, in inverse order, more tightly constraining (i.e., numerically) scales. The Galileo Questionnaire aims for ratio measures. Utilizing magnitude estimations of distances entailed using a scaling model which was isomorphic with the set of real numbers.

3. data collection techniques; Rating methods, methods of rank order, sorting methods, methods of paired comparison represent different scaling techniques. An important distinction is made between responses which are basically categorical and those which are comparative. While psychologists have made extensive use of the latter, sociologists have tended to use the former more. The advantages of the latter will be further discussed later.[9] The distinction between absolute judgments and relative-comparative judgments is also a central issue. The Galileo Questionnaire combines the advantages of paired comparisons and direct magnitude judgments (i.e., where the subject assigns a specific quantity to the difference).

4. cognitive versus affective responses; The soliciting of judgments as opposed to sentiments characterizes the Galileo Questionnaire. This distinction is described as scales of response vs. judgment by Torgerson and is associated by him with subject centered vs. stimulus centered data.[10] The Galileo System is stimulus centered as it asks for cognitive as opposed to affective judgments.

Clyde Coombs has developed a theory of data wherein his basic point is that:

> A measurement or scaling model is actually a theory about behavior, admittedly on a miniature level, but nevertheless a theory; so while building theory about more complex behavior it behooves us not to neglect the foundations on which more complex theory rests.[11]

Coombs is pointing out that the scales we construct will be based on our conception of the problem or the questions we ask. He says that mapping of behavior into a particular type of data and analyzing the category of data we gather by a type of model for analysis is closely related to the questions asked.[12]

5. order and/or proximity relations; According to Coombs, "an observation of a relation between two objects falls into one of only two classes: an order (dominance) relation or a proximity (consonance) relation.[13] Dominance includes the idea of more, heavier, louder, brighter. Correlation matrices represent a common form of proximity matrix. Distance measurements, including the Galileo system, represent proximity relations between pairs of objects. However, the distances between pairs of objects can be ordered. The 'metric data' from the Galileo Questionnaire is utilized in metric multidimensional scaling to try to construct a metric multidimensional spatial manifold for social objects.

6. subject/stimulus or stimulus/stimulus data; According to Coombs, besides distinguishing between modes of data analysis, data collection techniques can also be distinguished

on the basis of whether they compared stimulus and stimulus, or stimulus and subject.[14] Both the scaling of subjects with their results on a number of aptitude tests (by psychometricians) and the scaling of subjects and their thresholds to stimuli (by psychophysicists) represent typical uses of scaling of subjects and stimuli together. Psychologists have tended to use joint spaces of stimulus and subject more than sociologists because, for one reason, their concern has been more with the understanding of specific individual cases and secondly, they often do not use a large enough sample to justify aggregating their results and generalizing to any population, however limited (e.g., social science college students).

In the Galileo System, the elements (i.e., data) to be scaled are social objects, both tangible behaviors such as walking and eating, for example, and more abstract social phenomena such as fighting pollution and women's liberation. More intermediate abstraction's such as 'going to college' and 'living in a commune' are also social objects which are parts of the questionnaire.

The behavioral objects were chosen to be as diverse as possible. It was reasoned that if stability and small dimensionality could be shown with behaviors as unrelated as these, the support for a conceptual "behavioral space" of few dimensions would be more decisive.

Sampling

The Galileo data was collected differently than most psychological data is collected. For example, most psychological multidimensional scaling has been characterized by small numbers. The Galileo administrations were characterized by relatively large numbers. For each aggregate, the number of administrations varied from one to three per "group." In each case the sample was at least 80 and as many as 250. Some of the positive aspects of the sample size were diminished by the fact that the full eleven page questionnaire required splicing to three pages. All thirty-three social objects were present on every page however, with the one social object serving as the standard of comparison varying according to which segment was received by the subject. (The problems of splicing will be discussed further in the next chapter.)

Another distinction the Galileo System has from psychological scaling is that the emphasis is on the mean aggregate data of the sample and not on predicting individual differences.[15] It was reasoned that the large sample would lead to a cancelling out of extreme and random individuals estimations.

Specifically, the Galileo Questionnaires were administered to social science classes at the University of Illinois. An introductory class was administered once in January of 1971 (i.e., administration number seven). A social stratification class was administered twice (i.e., administrations five and six) in December and January of 1970-71. In the

Spring of 1971, a large research methods class was administered three times (i.e., administrations eight, nine and eleven), a very large interdisciplinary social science class was administered the questionnaires once in the spring of 1971 (i.e., administration ten).

Neither the sample or the subsamples are a 'group' in its technical sense -- although they may contain groups. While their similarity as social science students at the University of Illinois offers reason to believe they are more homogeneous than a random sample of respondents would be, they are still basically an aggregate as opposed to a social group.[16] This is problematic to some extent in that the idea of collective representation would best be expressed and tested via a true social group. However, access facilitated the choice of classes and their greater homogeneity than a random aggregate was hoped to permit the preliminary examination of the stability and dimension of a social manifold based on the mean aggregate data. Theoretical sampling[17] for true social groups would recommend itself as a comparison sample. The presence of stability in the conceptual space of this aggregate would, however, represent even more powerful evidence of the validity of the underlying theoretical framework. Another point of discussion is that behavioral distances may vary according to the context. Sitting and lying down may be closer or further according to the circumstances of the group at a particular <u>time</u> and within the group according to the context. Sitting and lying down may be further apart in school but closer together at home. At present, it is being proposed that the mean distances for the aggregate sample will reflect the general distances between objects. It is implicitly being suggested that overall, for the total population the sample represents, there will be a relative stability of distance between behavioral objects. In other words, it is implicitly being hypothesized that random individual variations in contextual factors will not be a factor when dealing with the mean responses of a large aggregate sample. Contextual factors such as where the questionnaire is answered is not considered a factor when dealing with the mean of the aggregate sample. These variations are taken as random error to be cancelled out by using large samples in the reconstruction of the picture of social space from the mean conceptions of the aggregate population.

Collective Representations

The theoretical view's relation to traditional sociological theory can be briefly reviewed. Some traditional sociological views from the discrete qualitative sociological tradition are convergent with the Galileo System, a continuous model. For example, Durkheim has attributed the source of coordinated action among people to collective representations; these representations can be conceived of as the shared cognitive categories that are created and learned by individuals in the context of a social group.

Paul Bohannan has interpreted collective representations in a useful way; as both the thing being perceived and the perceivor--i.e., as "perceivings." He sees perceivings as both the cause and result of social organization.[19]

The pattern of social psychological objects found in the Galileo System could be conceptualized similarly to "perceivings"--both a cause and reflection of social organization.

The Choice of Reference Objects
in the Social Manifold

The distance between sitting and lying down was given as one galileo apart. This was intended to set up a relative unit of measurement for the "social group" involved. Lying down and sitting were chosen because they seemed to be prosaic behaviors which would be characterized by stability for a group. They were assigned one galileo distance to provide a common frame of reference. There are a number of conceivable problems with this choice though.

The two behaviors which were chosen as the basic unit of reference are close together. The difference between sitting and lying down may thus vary just a little, e.g., be twice as psychologically distant as it was before, and make other behaviors which were twenty galileos apart according to the former frame of reference seem as if they were ten galileos according to the second frame of reference. Thus it might seem as if movement had occurred between behaviors whereas there had only been movement in the reference unit.[20]

The whole picture of the social world may not have expanded or contracted, just the behaviors given as the frame of reference may have.

The choice of behaviors further apart has been suggested as a means of lessening the effect of small changes in the psychological distances between the reference behaviors. Perhaps, even those farthest out might be best chosen for this reason. While there is merit to this possibility, there are other problems which may be increased by this schema. For example, behaviors which are very close together would be easier to utilize in relating the reference unit to distances between other behaviors (e.g., one galileo distance would be easier to use as a psychological standard from which to derive other distances in ratio relationship to the standard.) Also it might be expected that behaviors which were both close together and prosaic (e.g., sitting and lying down, the standard used for 1 galileo apart) would have a greater stability as unit of reference than concepts which were more esoteric (e.g., revolution which was far out) or not as apparently connected to the social space of the "group." Choosing behaviors not easily associated at this preliminary stage might have yielded completely incoherent results merely because the chosen unit of reference was difficult to utilize as a standard of reference.

Another possibility for the reference unit might be colors. There are problems with this choice also. For example, colors like behaviors also vary in their significance to groups and the distance between colors is open to cultural variations. The choice of black and white may vary culturally in terms of the distance separating them. It also has the added problem of imposing a bipolar frame of reference which might contaminate the picture of the social world reconstructed through the galileo questionnaire.[21]

The problem of standard of measurement might be reconceptualized in terms of the physical sciences. As a philosopher of science once asked, how would you know if you woke up one morning and everything (including yourself and all the objects on the earth) had expanded or contracted. How could science deal with the situation where the system of measurement had expanded or contracted along with everything else? For the Galileo system, the answer might not be so different as that for the physical earth. It is important to remember that "social worlds" exist within groups. Thus if a whole social world expanded or contracted, it might not be apparent within itself but there may be behaviors that have been far away which are behaviorally closer to other social groups. What may appear at first sight to be an indication of a behavior moving closer while examining only the one social group alone -- and plotted alone might be the whole social group moving closer to another social group in space which the behavior is associated with. This would indicate that eventually both social groups and behaviors may have to be plotted. In Coombs' terms, there would be a "joint space" - of behaviors and groups - rather than just a behavioral space for a particular group. The joint space might be even more fruitful for prediction and explanation and description.

It may not be necessary for a unit of reference to have the same psychological distance in different groups. In the long run, all that is necessary is that the unit be stable within its own frame of reference and that transformations be developed between frames of reference. To reiterate, what is needed is to discover some psychological reference that has stability within a given social frame of reference though it is not necessary that the behavioral objects chosen have the same distance for different social groups.

Treatment of Data

All thirty-three behaviors were assigned a number. They were written directly on the questionnaires next to their pair of objects. Larger numbers were always written first to permit ease of programming.; i.e., there were only thirty-three pairs of objects to identify since order of presentation of objects was not considered a factor. Five spaces were left free for the response -- estimation of Galileo distance. Thus a coding limit of 99,999 galileos apart was placed on the responses. Questionnaires with responses such as infinity were not coded.

The scalar products matrix was then submitted to a SSUPAC Principal Axis factor analysis program. The variance associated with the factors was used to determine the number of significant dimensions. Dimensionality was determined in accordance with where the variance explained dropped sharply.[23]

The "loadings" of each social object on the significant factor is utilized as that object's coordinate on that dimension. Thus a behavioral space is constructed by plotting the loadings of the social objects on the significant dimensions.

A Fortran rotation program was utilized to aid in the evaluation of the stability of the location of the behavioral objects in space across time. The social manifolds were taken two at a time (one criterion manifold which each of the other behavioral manifolds was compared with). The origins of the pairs of manifolds were given as walking in one case (i.e., object number one) and me in the other (i.e., object number twenty). Thus the two behavioral manifolds were placed on a common axis. With the angle of rotation being 1.00 degree, the coordinate systems were rotated to a least squares best fit. Specifically, each behavioral object's location is compared to itself in the two behavioral spaces. (The criterion manifold was taken as number seven for the thirty-three behavioral object manifold and number four for the ten behavioral object manifold.) The mean squared distance between manifold (i) and the criterion manifold was intended as a indicator of the stability of the objects across administrations. The problems encountered with using this to evaluate stability will be discussed in chapter four.

<u>Summary</u>:

This chapter has described the underlying logic and technical procedures followed in an attempt to construct a social manifold that is characterized by being continuous, linear, unbounded, metric, isotropic, and homogeneous.

These desired characteristics were central in the full design of the Galileo System's research procedure--e.g., in the questionnaire construction, spatial model, conception of multidimensional scaling, and time series sample.

The Galileo System is suggestive of a means of reconceptualizing qualitative and discrete cognitive categories in terms of continuous, quantitative variables like distance, time, and mass, since it provides a continuous homogeneous, linear, unbounded, isotropic manifold through which the discrete concepts may be seen to move. The motion is continuous. Be it that people do organize their behavior on the basis of discrete categories, this does not mean that the social scientist must perceive behavior in that and only that manner; i.e., scientific constructs must be evaluated on the basis of their explanatory power and their utility for social research.

Footnotes

[1]See Chapter Two for an explanation of the significance, of this statement. See the Appendix for the "Alice" questionnaire.

[2]See the Appendix for the Galileo questionnaire based on ten social objects and the 'Alice' questionnaire.

[3]See Chapter Two and Chapter Four for a discussion of factor analysis of cross products.

[4]By powerful is meant giving the most information (e.g., interval vs. ordinal data) and correspondingly open to high-powered mathematics.

[5]Max Jammer, Concepts of Space (Cambridge, Mass.: Harvard University Press, 1969), pp. 201-203. The physics of the microscopic have raised the question of the complete accuracy of isotropy. The physics of what has been termed "anti-matter" has dealt with the possible anisotropy or lack of parity between right and left in weak interactions.

[6]C. H. Coombs, et. al., Mathematical Psychology: An Elementary Introduction (Englewood Cliffs: Prentice-Hall, 1970), p. 31.

[7]See Appendix for copy of Galileo questionnaire (33 social objects).

[8]See C. H. Coombs, A Theory of Data (New York: Wiley, 1964), p. 431.

[9]See W. S. Torgerson, Theory and Methods of Scaling (New York: Wiley, 1958), Chapter 3, and Coombs, op. cit., Chapters 1 and 2.

[10]See Jum Nunnally, Psychometric Theory (New York: McGraw-Hill, 1967), Chapter 3; see also the references in the preceeding footnote.

[11]Coombs, op. cit., p. 50.

[12]Ibid., p. 29 and Chapter 1.

[13]Coombs, et al., op. cit., p. 32 and Chapter 3.

[14]Ibid., pp. 7-8.

[15]See W. S. Robinson, "Ecological Correlations and the Behavior of Individuals," American Sociological Review, 15 (June 1950), 351-57.

[16]"Basically it is the emergence of some pattern of social organization that distinguishes a group from a social category." Melvin De Fleur, W. D'Antonio, and L. De Fleur. Sociology: Man in Society (Glenview, Illinois: Scott, Foresman and Co., 1971), p. 38. Characteristics such as "we-they" feelings have also been used to distinguish groups from aggregates.

The sample used in this study was of social science college students at the University of Illinois. While this "aggregate" may not entail the degree of organization (i.e., norms, roles, social control, ranking systems, we-group and they-group feeling, shared perceptions, shared "goals") that a long established group of friends might have, these social classes certainly do have quite a degree of social organization. This factor is especially true in comparison to any random sample of disparate individuals.

[17]Barney Glaser and A. Strauss, The Discovery of Grounded Theory (Chicago: Aldine Publishing Co., 1968).

[18]See Emile Durkheim, The Elementary Forms of Religious Life (New York: Free Press, 1954).

[19]See Paul Bohannon, "Conscience Collective and Culture," in Kurt Wolff (ed.), Essays on Sociology and Philosophy (New York: Harper, 1964).

[20]I am indebted to Professor Kenneth Southwood for pointing out this problem and suggesting the use of very distant objects as the standard.

[21]Osgood's semantic differential has this same problem; see chapter four of the present work.

[22]Manifolds (or coordinate systems) can be very useful, as can be seen from the following example. A moving rod that is longer than a barn yet seems to be capable of being closed in the barn according to the frame of reference of the barn. However, the paradox is more apparent than real because of the problem of the "implicit assumption of simultaneity."

The great virtue of Minkowski diagrams is that they enable us, by drawing a picture of our colloquial verbal description of a situation, to get at this underlying reality. Once we have the picture in any frame in which it is simple to draw, we can immediately see how things must be described in any other frame simply by tilting the space and time axes according to the Minkowski rules. For this example and a related discussion see N. David Mermin, Space and Time in Special Relativity (New York: McGraw-Hill, 1968), pp. 194-99.

[23]See Chapter Four for a discussion of the problems of determining dimensionality.

CHAPTER IV

RESULTS AND DISCUSSION

Introduction

Chapter one provided a discussion of the problems of the separation of theory and research in sociology. The lack of a fundamental system of measurement and of theoretical concepts defined quantitatively (i.e., concepts jointly theoretical and empirical at the same time) was proposed as a possible source of the problems of contemporary sociology. The significance of the concepts distance and time in physics along with the analytic utility of coordinate systems was explained. The development of a coordinate system or social manifold for social objects was suggested as a procedure which might provide a step toward developing concepts which are jointly theoretical and empirical as well as a system of fundamental measurement for sociology. An aim of this work has been to present and evaluate possible procedures for the construction of the manifold. This was done basically in chapters two and three. Chapter three described the procedures followed to construct the social manifold in accordance with the desired model, i.e., the Galileo System. Specific hypotheses concerning stability, lawfulness, and dimensionality were stated. This chapter reports some preliminary findings on the hypotheses and also evaluates the adequacy of the presently available procedures for the assessment of the concerns of the hypotheses.

This chapter also contains the results of some tests done to clarify the nature of factor analysis. This is important for an understanding of the methodological approach to the construction of a social manifold.

Overview of Hypotheses and Analysis Procedures

The aim of this dissertation has been to construct a spatial manifold for social objects and to present some preliminary findings relating to the reliability and face validity of the relations viewed in the social manifold. One hypothesis (Hypothesis 4) states that the dimensionality of the space will be small; probably three dimensions should be adequate. (See Chapter Three for a theoretical justification of this hypothesis.)

A second hypothesis (Hypothesis 1) proposes that the spatial relations between objects in space will be stable over the relatively short time intervals, i.e., across administrations of the Galileo questionnaires to a given sample. The position of an object in space is compared with itself across time. Hypothesis 2 proposes that the amount of movement between the objects in different manifolds (i.e., administrations) will not be great. (The mean of the sum of the least squares distances between social objects and themselves in different administrations will not be great. The configurations of points - i.e., the manifolds - of the different administrations are given a common origin and rotated to a least squares fit.)

Hypothesis Three proposes that the movement of objects in the manifold will be orderly.

The questions this dissertation have focused on relate to the stability of the configuration of social objects in the manifold, the orderliness or "lawfulness" of the movement of the social objects in the manifold, and the dimensionality of the social space. This chapter will report the findings from the data and assess both the utility and problems of the present data and analytic techniques for answering these questions.

First of all, the ascertainment of the stability of the mean conceptual configuration of the aggregate of similar individuals can be approached in a number of ways. Firstly, the raw mean distances between social objects can be examined for stability across administrations. This entails inspecting the cells of the mean distance matrices across time. Specific objects may be examined across time (i.e., reviewing a number of administrations) with all the other thirty-two social objects. Clusters of social objects may be examined across time. The magnitude of the mean distances can be surveyed. All of these things have been done.

Secondly, stability of the objects might be assessed by plotting -- i.e., the points representing the conceptual objects can be located on a coordinate system of the appropriate dimensionality. The "loadings" of the variables on the appropriate number of factors could be used as coordinates. Moreover, the rotation program delineates the coordinates of each conceptual object on three dimensions. These also could be plotted and compared across time since the rotation program has the advantage of imposing the same origin on the conceptual manifolds resulting from the different administrations. (The four samples with the ten social objects consist of one set of comparisons. The seven samples utilizing thirty-three social objects represent another set which can be compared through the procedure of imposition of the same origin and rotation to the least squares best fit.) This comparison is made for each manifold in a set and the criterion manifold; the seventh of the eleven samples was utilized as the criterion manifold for samples five through eleven; the first administration of the small sample was used as the criterion manifold for the other three ten social object samples. Difficulties were encountered in utilizing plotting as a means of assessing the degree of stability. The distortions found in the coordinates and factor "loadings" and the cause of these distortions will be discussed in this chapter. Since the coordinates of the variables had much distortion in them, they no longer were a useful technique for assessing the stability of the conceptual configuration. Therefore, only a few plots are graphed and they consist of configurations based on the ten concept sets which did not encounter the same problems as did the configurations based on the thirty-three object set.

A third way which was planned to assess stability is dependent on the coordinates of the social objects delineated in the rotation program. Hypothesis Two proposed (in its more technical form) that the mean of the least squares distances, (i.e., sum of the squared differences between the same object's coordinates on the three axes summed over all the objects) between a pair of manifolds (i.e., each of the manifolds in the set and the criterion manifold) will not be great. This technique for the estimation of stability becomes, of course, very dubious when the validity of the numerical values of the coordinates become questionable. Thus while the figures are recorded, their significance is doubtful because of the distortions in the coordinates on which the mean distance between the manifolds is based. Again, only in the samples using the ten object questionnaires -- where there are not the distortions in the coordinates -- are the resulting figures of mean distance between manifolds considered undistorted and meaningful.

Another hypothesis of this work involved the expected order or "lawfulness" of the motion of the social objects in the conceptual manifold. A large number of erratic responses as to the mean distances between pairs of social objects would weigh as evidence against the existence of a lawful conceptual manifold. Since the time periods are not that great between administrations, i.e., across time, to a class, it is not expected that extreme violations of seemingly existent conceptual patterns should occur. If these violations do occur and if a lawful conceptual manifold does exist for the mean of an aggregate of individuals, then either, one, there should be some possible explanation for the shift, or two, there should be reason to propose that the violation is due to error. The total number of administrations (eleven for some conceptual objects and seven for the rest) as well as the administration to the same classes more than once over a time period contributed to the possibility of discovering uniformities. The time factor and number of administrations provided a greater likelihood that uniformities would be recognizable despite the negative effects of error, and problems with the respondent and social object sample; i.e., a sample of respondents based on a true social group would more likely yield a stable and orderly conceptual manifold than an aggregate of "similar" individuals; likewise, social objects chosen within a more limited frame of reference as opposed to the very diverse sample of objects included in the present Galileo Questionnaires would also have been more likely to yield stable and lawful conceptual spaces. Returning to the point, the number of administrations to the same and different aggregates of similar individuals (as well as the sample size) aided the exploration for uniformities.

The appraisal that the motion of social objects is lawful does not require that the motion of all the objects be orderly in the hypothesized shared conceptual space. If the overall configuration is relatively stable and orderly, the "chaotic" and erratic movements of some conceptual objects might be an indication of a characteristic of that social object in relation to the sample of respondents. For example, the erratic movements of particular social objects might be indicating that the aggregate as a whole has no established pattern of behavior toward that social object and thus no stable place for it or even orderly pattern of movement for it; the disorderly motion of the social object might be an indication that the object was unimportant and/or quite removed behaviorally and conceptually from the aggregate of individuals. Thus, some disorder in the manifold - over time and samples - would not only be possible but also probable given the purposely desultory choice of objects for the social manifold.

The third hypothesis which this work began with, was that the dimensionality of the space would be small. While determination of the exact dimensionality entails some difficulty, the utilization of the number of factors accounting for the grand portion of the variance -- or the disavowal of all factors after there is a sharp break in the variance accounted for and the size of the eigenvalues - is most helpful and has been instrumental in determining the dimensionality of the social manifold. The procedure followed here to assess the number of dimensions is viewed when practiced on both the scalar products and cross products of the means matrix of a known solution (face and oblong solid). The difficulty of determining the correct dimensionality in the TORSCA non-metric multidimensional scaling program will also be discussed in this context. The general problem of ensuring the selection of the exact number of true dimensions will also be explained.

Stability of Mean Distances

The stability of the configuration of behavioral objects could indicate a) reliability and b) face validity. One procedure followed was to determine behavioral objects which were either stable or highly erratic and unstable, i.e., without any orderly pattern; a great deal of disorder and erratic movement would indicate that the Galileo system had failed at this preliminary step to prove its reliability.

Stability of Mean Distances Between Me and Other Social Objects

The most stable objects - in terms of range of galileos between me and the social objects - were eating, laughing, getting ahead, fighting pollution, going on vacation, sunbathing, and going to college. See Table 2. They vary from one to no more than seven galileos distance from me in seven administrations involving four different classes. Laughing is the most stable behavioral object, always being either two or three galileos away from me. (One could hypothesize from the pattern within a sample that "getting ahead" is slightly further away near exams but the difference is so slight one could not propose the difference to be significant at this time.)

Eating varies over a range of from three to ten galileos away from me, and in six of the seven administrations, eating

TABLE 2

MEAN DISTANCES OF ME FROM THIRTY-TWO SOCIAL OBJECTS[a] IN ADMINISTRATIONS 5 THROUGH 11

	5	6	7	8	9	10	11
S 1 Walk (3-12)[b]	12	9	4	6	4	3	4
S 2 Sit (4-12)	12	10	5	4	6	10	5
M 3 Stroll (3-29) (3-13)[c]	13	29	6	8	5	11	3
M 4 Run (6-54)(6-15)	15	8	6	54	6	13	5
M 5 Sleep (4-12)	12	5	7	8	4	11	5
M 6 Fight (8-4178) (8-26)	20	21	16	4178	8	14	8
M 7 Revol. (6-4173) (6-16)	16	16	8	4173	7	11	6
U 8 Marry (5-666) (5-82)	666	82	67	10	5	14	5
S 9 Sing (4-37) (4-7)	6	37	4	7	7	5	7
M 10 Pot (4-22)	17	5	20	5	12	22	4
S 11 Make Love (2-10)	10	3	3	2	8	9	2
U 12 Kill (13-3722) (13-129)	36	129	38	13	3722	18	15
S 13 Eat (3-10) (3-1)	10	4	5	5	6	3	4
M 14 Steal (8-259) (8-66)	259	66	20	13	8	26	10
S 15 Laugh (2-3)	2	3	2	3	3	2	3
S 16 Get Ahead (5-10)	5	10	8	6	6	9	7

[a] S = Stable object
M = Moderately stable object
U = Unstable object

[b] (3-12) = Range of mean distance estimations

[c] When two ranges are given in one box, the second range represents the mean distances with the largest distance removed.

TABLE 2 - CONTINUED

	5	6	7	8	9	10	11
S 17 Vacation (3-10) (3-5)	10	5	3	3	3	4	3
S 18 Sunbathing (3-10)	5	5	10	4	6	8	3
M 19 Swim (3-25) (3-15)	25	7	4	14	9	15	3
20 Me							
S 21 Fight Poll. (3-7)	6	7	4	5	3	5	3
M 22 Ice Skate (5-13)	10	12	6	5	7	13	5
M 23 Womens Lib. (4-31) (4-18)	18	5	8	6	10	31	4
M 24 Ski (4-35) (4-17)	8	11	6	7	17	35	4
M 25 Home (4-79) (4-12)	79	4	6	4	8	12	4
M 26 Quit School (6-20)	17	9	9	11	19	20	6
M 27 Smile (2-14) (2-7)	2	3	4	7	4	14	2
S 28 Go to Coll. (2-7)	3	4	3	7	2	6	2
M 29 Talk (2-38) (2-14)	14	3	3	38	4	7	2
M 30 Study (4-629)	12	6	4	629	5	20	4
U 31 War (6-6706) (6-146)	146	21	10	40	6706	109	6
U 32 Commune (3-53) (3-39)	15	9	9	39	6	53	3
U 33 Pract. Med. (6-286) (6-102)	286	16	13	10	25	102	6

varies only over a range of three to six galileos away from me. This indicates that eating is quite a stable behavioral object.

Fighting pollution is also a stable behavioral object, varying over only a range of three to seven galileos apart from me. Going to college ranges from two to seven galileos away from me. Reliability may be even greater than is indicated by this short range of two to seven galileos. When the results are looked at in terms of samples, administrations five and six were to the same class and they reported three and four galileos distance between themselves and going to college. The next administration, number seven, given at approximately the same time as administration six, involves a different class (Introductory Sociology) and the members of the class report three galileos between themselves and going to college. Administrations eight, nine and eleven were for the same introductory methods class and were seven, two and two galileos respectively between me and going to college. Administration eight also had a disproportionately large number of galileos between me and studying (six hundred twenty nine galileos), which suggests that there may have been participants present at that administration that were not present at later administrations for the same class. (Where studying was only five and four galileos away from me.) Another possible factor affecting the results might be the time of the semester, with studying becoming closer as finals approach. This is consistent with the movement in administrations five and six (same class) from twelve to six galileos for the distance between me and studying. The second administration was closer to the end of the semester and finals. Home also seems to be closer right before and after vacations: seventy-nine to four galileos and four, eight, four for the class administered to three times; the two fours were at the beginning of the semester and at the end. Most of the behavioral objects seemed to fall into this kind of category where the distance did not remain quite as stable as those designated stable but yet the movement was not so great as to be termed chaotic and where the movement seemed to be consistent with an orderly pattern.

Fighting and revolution have both moved in closer during the Spring semester as compared generally to the Fall semester (with the exception of the huge number reported for administration eight which could have been the result of just a few gigantic estimations, e.g., 99999. In addition, this is the Spring administration which is closest to the fall semester.

Administration number ten yielded a much greater galileo distance between war and walking, sitting and strolling than the other samples. Since this class was distinguished by being particularly associated with leftist politics, the socio-political context of the class may have affected the responses of the participants so as to emphasize the distance of war from everything and anything. It is possible that the interactional context may have affected the tone of all the responses. Yet perhaps inconsistently, it is administration number ten which has a mean distance of fifty-three galileos between me and living in a commune, the furthest distance of any of the administrations.

The overall mean distance in galileos between me and the thirty-two other behavioral objects is comparatively stable. The range in galileos for most behaviors is not very large over seven administration to three different samples. Thus, the reliability of this test utilizing divergent behaviors to aggregates of individuals (as opposed to social groups) is encouraging. One might guess that stability would increase given a more specified or limited behavioral space with a true social group.

Even those objects which you might wish to label unstable or unreliable vary in their "erraticness" and might not be as chaotic as they first appeared. Marrying, for example, ranges from five galileos to six hundred sixty-six galileos distance from me. However, there may be some meaning to the fact that the large distances all were in the Fall semester and the closer distances (five to fourteen galileos) all in the Spring (perhaps in conformity to statistics about Spring marriages). No theory is being proposed here, it must be remembered, except for explanatory purposes.

War, while having a very large range of distances associated with it--6 to 6706 galileos--and thus earning an "unstable" label, also suggests a pattern of being closer to me at the end of the semester. While this is very hypothetical and ad hoc (and not meant to be anything more at this stage), war being closer at the end of the semester made sense in terms of the relationship of flunking out and graduation to eligibility for the draft. The instability of practicing medicine from me suggests that the concept is not very significant to most of the sample in relationship to the concept me and thus there is little consistency in defining its relationship to me.

These hypothetical attempts to explain motion are problematic especially since movement of objects in the Galileo space quite possibly is not explainable in terms of "individual psychology."

Sensitivity to Error and Effect on Indications of Stability

Both the factor analysis and the error estimation program--mean squared distance between manifolds--have difficulties, for they are highly sensitive to atypical large numbers (i.e., distances). Yet, these atypical large numbers might only be an indication of an atypical large response (which greatly affected the mean distance) or even just a keypunch error.

The mean distance matrix for administration number ten (Sociology 199 X) is inconsistent with the spatial relations indicated from plots based on coordinates in the rotation program and factor analysis. The plotted distance between walking, sitting, strolling and running is greater than is actually the case according to the mean distance matrix. The mean distance between walking and sitting is only three galileos and only one and two galileos respectively between walking and strolling, and walking and running. Yet they appear far apart on the rotation coordinates and factor analysis "loadings" though not as far apart as they appear according to the rotations coordinates for administration number six. Administration number ten is distinguished, like number six, by having a behavioral object which is extremely distant from walking, while not being as distant from sitting, strolling, and running. War is nine hundred nineteen galileos from walking, while only one hundred fifty galileos from sitting, two hundred twenty-three galileos from strolling and only eighteen galileos from running. See the below figures as well as Table (walking).

Class 223, T1. Number 5. T_1 = Time 1.

Walking and war is 28 galileos.

Sitting and war is 27 galileos.

Strolling and war is 27 galileos.

Running and war is 19 galileos.

Class 223, T2. Number 6.

Walking and war is 18 galileos.

Sitting and war is 18 galileos.

Strolling and war is 29 galileos.

Running and war is 4 galileos.

TABLE 3

MEAN DISTANCES BETWEEN WALKING (AND OTHER BEHAVIORS IN WALKING CLUSTER)[a] AND FIVE SOCIAL OBJECTS

Administrations	1-20 Walking-Me	1-12 W. - Kill	1-31 W. - War	1-11 W.-Making Love	1-15 W.-Laugh
5 sit/stroll/run[a]	12	16	29	20	3.7
	12, 13, 15	41, 31, 18	27, 27, 20	22, 15, 11	4,3.7,3.8
6	9	23	18	14	11
	11, 30, 8	7, 18, 8	18, 30, 5	8, 30, 27	18, 27, 6
7	4	12	9	12	5
	5, 6, 6	11, 61, 9	52, 53, 7	4, 10, 16	4, 4, 5
8	6	23	9	11	6
	4, 8, 54	27, 16, 24	17, 18, 25	5, 9, 12	5, 4, 11
9	4	13	11	11	6
	6, 5, 6	12, 14, 6	13, 13, 7	6, 6, 8	5, 4, 5
10	3	11	920	7	3
	10, 11, 13	19, 43, 27	151,224,19	9, 17, 27	3, 11, 8
11	4	8	8	5	4
	5, 3, 5	9, 10, 5	11, 10, 4	4, 5, 5	4, 2, 5

[a] Sit, stroll, and run are the other behaviors in the Walking Cluster. Each of their distances from each of the five objects is reported in the bottom of each box.

In administration six, the 2622 galileos distance of women's liberation from walking has caused the factor analysis "loadings" and the coordinates in the rotation program to be greatly distorted. Note that the factor analysis shows the theoretically surprising results of only one factor explaining most of the variance. The problem operating becomes clear when the loadings of the thirty-three variables on this dominating factor are examined. Walking and women's liberation (number one and number twenty-three) override all other variables; e.g., walking and women's liberation are four digit whole numbers as compared to the largest other whole number being only one digit and most loadings being only decimals. In addition, they together define the positive and negative poles for the space.

The problem seems to be caused by 1) the tremendous relative distance between walking and women's liberation, and especially, 2) that the distance between walking and women's liberation did not coincide with their distances from other behaviors. For example, women's liberation is relatively close to sitting, strolling and running which are also close to walking; yet, walking and women's liberation are a tremendous distance apart. See Table 4 (Women's Liberation).

In this case, both the factor analysis and rotation program proved to be very sensitive to just one distance "error." One check for this type of error might be to have the median as well as the mean reported. This would <u>not</u> be

TABLE 4

DISTANCE OF WOMEN'S LIBERATION
FROM WALKING CLUSTER

	Sit	Stroll	Run	Walk
#5	10	11	11	8.6
#6	31	19	9	2621
#7	9	8	9	10.7
#8	13	12	272	7.4
#9	9	9	8	7.4
#10	40	39	18	18.7
#11	7.4	7.4	6	5.5

for theoretical reasons. However, the median distance would provide a check on the data distribution. For example, if the median distance between walking and women's liberation were ten galileos and the mean was two thousand, then a mis-punch error, or some kind of error would be indicated.

The distortion that is occurring becomes apparent when examining the rotation program for administration six which uses me (object twenty) as the origin. If a comparison is made with the mean distance matrix, inconsistencies can be sighted. For example, the mean distance between me and women's liberation is only five galileos apart. Yet the coordinates of the rotation program shows women's liberation (as well as walking) to be farther from me than any other behavioral object. This distortion in the rotation program's coordinates were fed in from the factor loadings and scalar products. It is the atypical and highly inconsistent (with the other distances) response in a cell of the mean distance matrix which affects the scalar products which confuses the factor analysis.

Administration number seven (an introductory sociology class) also demonstrates how inconsistencies (i.e. violations of the model of Euclidean Geometric space) lead to distortions in the behavioral objects "loadings" or coordinates. By examining the rotation which utilizes walking as the origin for administration seven (the criterion sample in the rotation program), it can be seen that making love (object eleven) is far from both walking and me (object twenty).

Yet on examining the mean distances, neither walking nor me are much of a distance from making love (twelve galileos and two galileos respectively). See Figures 5 and 6.

Making love seems very distant from other social objects because the mean distance associated with it and two other behaviors (killing--number twelve, and war--number thirty-one) are gigantically large relative to the scale of distance used between most other behaviors, including (and this is crucial) social objects which are supposed to be close to making love as indicated by their mean distances apart in galileos. Making love is approximately four hundred fifty galileos away from both killing and war. These large irregular mean distances have the end effect of hiding an overall stability of the galileo distances when just the resulting distorted coordinates are examined.

Walking is only twelve galileos away from killing and nine galileos away from war. Me, while distant relative to the other mean distances in the behavioral space, is only thirty-eight galileos away from killing; (most distances are fifteen galileos or under in administration seven).

In administration seven, laughing is five galileos from walking and two galileos from me. Yet in the rotation program the coordinates of laughing, using either walking or me as the origin, indicate that laughing is one of the most distant behaviors in the social manifold. The problem is the same again. Laughing and killing are four hundred and sixty-six galileos apart. It is the largest distance--and enormous

THE COORDINATES OF THE CRITERION MANIFOLD

1	-0.00000	0.00000	-0.00000
2	1.24731	0.15967	-3.17631
3	3.99533	-0.02765	-0.66314
4	-0.12410	-0.06690	0.06295
5	0.08345	0.05133	-0.18566
6	0.31659	0.03666	-1.64957
7	-0.50832	0.09507	0.24298
8	18.01973	1.49504	-45.46368
9	0.88446	0.54656	-0.59437
10	1.22370	195.80803	7.29618
11	240.20575	3.05037	-94.69337
12	-248.39165	3.19120	-91.05420
13	1.37953	-0.05414	0.50466
14	-0.00830	0.36530	0.38265
15	162.52254	-4.76270	150.32683
16	1.03069	-0.20732	0.87721
17	1.33534	-0.13070	0.90386
18	1.24308	-0.03541	1.02421
19	1.36507	-0.24156	1.04942
20	0.86931	-0.53797	2.38358
21	1.10101	-0.06596	1.06125
22	1.39511	-0.39932	0.63356
23	1.13474	-0.17795	0.90628
24	1.52803	-195.97937	-4.40975
25	1.16847	-0.49121	0.86087
26	-0.00400	-0.71289	0.01154
27	1.04550	-0.07171	0.56542
28	1.18734	-0.03395	0.63577
29	1.20469	-0.05973	0.96401
30	2.21499	-0.33652	1.97636
31	-146.21103	-5.88607	159.65552
32	1.59602	0.20836	-0.93003
33	1.07796	-0.51532	0.88618

Figure 5. Coordinates of Administration Seven

with Walk as Origin

	1	2	3	4	5	
1	0.0					
2	9.704	0.0				
3	2.074	8.214	0.0			
4	3.393	12.833	3.571	0.0		
5	11.111	6.483	6.241	8.296	0.0	
6	8.036	9.655	8.643	4.357	9.375	0.0
7	9.893	13.933	13.385	6.571	9.625	3.16
8	9.286	10.571	10.926	9.519	8.939	7.65
9	6.370	5.036	4.630	5.556	8.091	12.03
10	8.241	4.207	8.571	8.111	7.273	11.36
11	11.966	4.241	9.464	15.607	4.303	13.44
12	11.750	10.724	61.679	8.968	10.543	2.93
13	8.893	3.862	6.107	6.967	5.813	10.65
14	9.333	8.571	14.444	4.645	11.212	5.42
15	5.444	4.464	3.893	4.774	5.875	14.67
16	5.571	7.433	9.000	4.242	10.485	4.82
17	5.966	6.207	4.448	7.394	6.394	14.79
18	7.926	3.107	3.929	8.781	5.625	15.27
19	9.214	10.769	4.630	4.969	8.688	11.96
20	4.296	4.643	5.692	5.938	7.000	26.48
21	7.607	14.846	16.462	6.152	10.656	5.16
22	4.929	7.423	4.731	3.906	9.875	11.22
23	10.600	9.111	8.440	8.576	9.588	5.78
24	8.500	8.444	6.500	4.774	9.824	11.12
25	13.759	5.889	4.692	6.030	5.176	11.87
26	7.429	7.077	7.280	4.406	6.727	11.83
27	5.321	3.536	3.556	6.000	5.598	11.59
28	3.857	6.207	6.037	5.394	5.559	6.52
29	3.519	3.500	3.630	6.156	6.667	10.967
30	12.250	4.379	9.321	6.774	6.061	6.513
31	8.500	52.222	53.143	7.094	11.212	35.60
32	7.179	6.036	9.679	9.031	5.167	14.567
33	10.241	12.036	10.607	7.469	13.484	7.367

Figure 6. Mean Distance Matrix of Administration Seven

5	6	7	8	9	10	11
.0						
.375	0.0					
.625	3.167	0.0				
.939	7.655	27.867	0.0			
.091	12.034	28.733	13.680	0.0		
.273	11.367	6.581	7.370	4.926	0.0	
.303	13.448	21.774	3.444	6.250	8.769	0.0
.543	2.933	3.710	29.741	26.357	30.407	449.750
.813	12.655	17.276	6.577	7.308	9.370	8.125
.212	5.429	7.241	10.692	9.885	13.214	21.417
.875	14.679	17.862	8.423	2.556	6.857	2.959
.485	4.828	23.067	12.925	10.036	12.714	5.360
.394	14.703	35.167	14.444	4.571	8.778	4.290
.625	15.276	19.172	16.320	6.500	7.407	4.800
.688	11.966	12.483	7.000	6.667	13.407	4.290
[illegible]	26.483	9.214	67.160	4.107	10.741	2.292
.656	5.161	5.067	22.042	9.000	16.000	8.500
.875	11.226	12.500	8.680	7.500	10.253	6.440
.588	5.788	4.710	6.615	9.536	9.462	7.280
.824	11.125	11.410	9.444	21.111	301.962	6.400
.176	11.875	10.548	6.333	5.815	20.654	5.760
.727	11.830	6.517	7.160	7.846	25.731	12.020
.598	11.594	27.414	3.577	7.077	7.333	1.830
.559	6.500	22.067	8.962	25.231	6.704	4.458
.667	10.967	33.555	3.741	6.462	6.925	3.360
.061	6.517	23.966	6.593	21.222	17.083	13.156
.212	35.600	6.308	198.519	30.714	35.375	449.458
.167	14.567	9.333	25.760	5.192	3.292	3.609
.484	7.367	39.077	10.500	10.640	22.480	7.208

	12	13	14	15	16	17
12	0.0					
13	41.708	0.0				
14	26.435	10.176	0.0			
15	456.243	11.000	20.000	0.0		
16	37.529	8.471	19.625	3.625	0.0	
17	41.188	8.059	20.813	2.870	5.500	0.0
18	40.882	9.059	23.125	5.261	4.458	1.958
19	42.500	9.200	23.563	2.696	4.625	2.000
20	37.706	4.824	20.500	1.917	8.042	3.043
21	39.500	14.563	23.933	7.043	7.833	6.348
22	41.000	10.313	23.625	2.870	6.958	3.478
23	39.471	10.200	21.438	6.043	5.739	5.875
24	40.824	8.118	21.813	3.652	5.440	2.542
25	39.059	5.235	21.438	4.174	6.040	3.773
26	14.941	11.176	20.375	10.739	13.960	5.174
27	36.056	5.118	10.348	1.636	8.160	3.130
28	38.471	7.529	8.913	4.609	4.917	9.524
29	40.118	5.647	8.870	2.435	5.174	5.136
30	56.200	6.882	10.609	12.000	7.913	13.682
31	3.525	21.375	3.000	29.435	7.609	15.273
32	40.189	4.941	11.565	5.087	8.136	12.150
33	39.176	8.125	8.818	10.913	4.261	13.455

	23	24	25	26	27	28
23	0.0					
24	16.100	0.0				
25	15.381	8.700	0.0			
26	8.714	10.571	5.400	0.0		
27	9.286	4.400	8.476	7.667	0.0	
28	10.286	6.588	9.045	13.727	7.364	0.0
29	7.295	5.813	5.000	7.091	3.667	3.583
30	11.429	9.222	34.714	7.455	22.833	14.077
31	9.789	16.833	14.286	6.455	14.417	16.692
32	6.286	12.105	63.850	5.455	6.500	7.333
33	9.250	11.524	12.619	22.273	10.583	7.000

≠ 7 MEANS

Figure 6. Continued

6	17	18	19	20	21	22
0	0.0					
8	1.958	6.000				
5	2.000	1.696	0.0			
2	3.043	10.087	3.696	0.0		
3	6.348	11.136	6.364	4.136	7.000	
8	3.478	11.087	5.810	6.217	21.609	0.0
9	6.875	12.087	9.048	7.793	12.125	18.048
0	2.542	7.826	5.048	5.826	15.440	6.667
0	3.773	9.040	7.636	5.696	14.917	10.714
0	5.174	8.720	8.400	9.825	14.565	15.952
0	3.130	4.000	4.810	4.174	13.217	5.364
7	9.524	7.520	6.762	2.652	6.520	8.818
4	5.136	8.667	6.389	2.682	6.875	8.333
3	13.682	9.250	8.091	4.182	6.091	18.350
0	15.273	10.565	10.455	10.136	10.571	20.632
6	12.150	7.870	8.864	9.304	9.478	14.211
1	13.455	9.391	9.045	13.043	8.130	14.750

7	28	29	30	31	32	33
4	0.0					
7	3.583	6.500				
3	14.077	3.667	0.0			
7	16.692	7.417	11.692	0.0		
0	7.333	3.692	26.846	43.938	0.0	
3	7.000	8.308	3.286	8.923	35.357	0.0

EANS

compared to most of the distances--in administration seven. The fact that laughing's reported distance from killing is so great in comparison to their clusters of other behavioral objects which laughing is close to in terms of their mean distance apart leads to laughing being relatively distant from these other behavioral objects in terms of their respective coordinates--i.e., factor loadings.

Since both the coordinates of the factor analyses and the coordinates based on the rotation program are very misleading as to the amount of movement and instability, i.e., it seems to be highly exaggerated, only a few social manifolds will be plotted, since they have many distortions caused by the phenomena I have already suggested. The plots for the thirty-three objects would be worse than useless--they would misrepresent the actual mean aggregate Galileo distance data. See the Appendix for copies of all the rotations.

The two behavioral objects which are furthest out in administration seven are killing (number twelve), and war (number thirty-one). Both in the factor analysis and rotation program, making love (number eleven), smoking pot (number ten), laughing (number fifteen), and skiing (number twenty-four) are all quite distant from the other behavioral objects. This seems to be a distortion caused by the large number of galileo distance they are from one or two <u>far out</u> objects like war or fighting.

TORSCA[1] and Distortions

TORSCA has both some similar and some different problems than the metric multidimensional scaling program. The principal axis factor analysis program for administration seven has laughing (15), smoking pot (10), making love (11), killing (12) and war (31) as most distance from the axis with marrying (8) a moderate distance.

The TORSCA program has similar distortion problems but they are not so obvious because all the differences are squeezed together. The overall scale in TORSCA is compressed, so that much of the information relating to scale of difference is lost. Many behaviors are relatively far out in TORSCA's administration seven (Revolution, marrying, fighting, smoking pot, stealing, fighting pollution, living in a commune, practicing medicine) and only killing has somewhat of a real distance break. However, using absolute distance in galileos, something like stealing (14) is perhaps, at most, twice as distant (.3 to .4 more on a 1.0 scale) as strolling (3) and killing is only the same amount again more distant on one axis only (.3 to .4 more on a 1.0 scale) as stealing. Thus the scale is lost.

However TORSCA does have the virtue of not paying disproportionate attention to just one or two large distances out of the 32 possible pairs. Thus behavioral objects like making love and laughing, which have gigantic distances from war, but are close to many other behaviors including me, have coordinates that place them as less distant from the other behavioral objects in the space than does the metric multidimensional scaling. See TORSCA configuration matrix for administration seven. See Figures 7 through 10.

One possible approach that would improve metric methods, (e.g., the Galileo System) which utilize information about scale is to remove the few large numbers before analysis. This would permit the use of the metric method (which can utilize more information) while removing the highly distorting large numbers. At least part of the reason that the factor analysis and rotation programs work out on the sample using only ten behaviors is that the behaviors which are distant from, for example, walking are also distant from other behaviors close to it in a behavioral cluster-like sitting, strolling and running. Administration one (time one, small sample) shows revolution, marrying, and practicing medicine being relatively distant from walking, sitting, strolling and running, but they are all within a smaller range. For example, marrying varies only from 16 to 20 galileos distance from the walking behavioral cluster. Practicing medicine ranges over 13 to 18 galileos distance from that cluster. Revolution ranges from 16 to 23 galileos distance for the walking behavioral cluster.

<u>Assessing Dimensionality with Galileo, Cross Products and TORSCA</u>

The resulting number of factors required to explain the great bulk of the variance of the thirty-three social objects (i.e., variables as conceived in factor analysis) is small.

	CONFIGURATION 1	2	3		TORSCA SYMBOL
1	0.124	0.047	-0.478	WALK	A
2	-0.334	0.360	0.237	SIT	B
3	-0.338	0.225	-0.394	STROLL	C
4	0.254	-0.158	-0.245	RUN	D
5	-0.027	0.434	0.474	SLEEP	E
6	0.655	-0.215	0.119	FIGHT	F
7	0.780	0.284	-0.214	REVOLUTION	G
8	0.043	0.044	0.779	MARRY	H
9	-0.247	0.495	-0.330	SING	I
10	0.062	0.811	0.123	SMOKE POT	J
11	-0.554	-0.018	0.322	MAKE LOVE	K
12	1.138	0.201	-0.303	KILL	L
13	-0.307	0.118	0.589	EAT	M
14	0.797	-0.077	0.408	STEAL	N
15	-0.471	-0.048	-0.134	LAUGH	O
16	-0.079	-0.540	0.013	GET AHEAD	P
17	-0.461	-0.034	-0.374	GO ON VACATION	Q
18	-0.688	0.224	-0.004	SUNBATHING	R
19	-0.432	-0.322	-0.099	SWIM	S
20	-0.199	-0.153	-0.327	ME	T
21	0.326	-0.682	-0.104	FIGHT POLLUTION	U
22	-0.176	-0.152	-0.683	ICE SKATE	V
23	0.434	0.448	0.291	WOMEN'S LIB.	W
24	-0.552	-0.537	-0.037	SKIING	X
25	-0.371	-0.454	-0.488	GOING HOME	Y
26	0.271	0.444	-0.479	QUIT SCHOOL	Z
27	-0.417	0.267	-0.045	SMILE	1
28	-0.223	-0.344	0.341	GO TO COLLEGE	2
29	-0.111	-0.023	0.172	TALK	3
30	0.247	-0.422	0.593	STUDY	4
31	0.836	-0.249	-0.405	WAR	5
32	-0.167	0.706	0.179	LIVE IN COMMUNE	6
33	0.187	-0.680	0.509	PRACTICE MEDICINE	7

Figure 7. TORSCA Configuration for Administration 7

Figure 8. TORSCA--Plot of
Dimensions 1 Versus 2--Administration 7

1

```
DIMENSION 1 (X AXIS) VS. DIMENSION 2 (Y AXIS)
          .   -0.9000.  -0.7000.  -0.5000.  -0.3000.  -0.1000.
        -1.0000   -0.8000   -0.6000   -0.4000   -0.2000    -0.
        *.****.****.****.****.****.****.****.****.****.****.
   0.90 .                                                  I
   0.87 *                                                  I
   0.83 *                                                  I
   0.80 ..                                                 I
   0.77 *                                                  I
   0.73 *                                                  I
   0.70 .                                                  I
   0.67 *                                                  I
   0.63 *                                                  I
   0.60 ..                                        I        I
   0.57 *                                                  I
   0.53 *                                                  I
   0.50 .                                                  I
   0.47 *                                                  I
   0.43 *                                                  I
   0.40 ..                            R C                  I
   0.37 *                                      1           I
   0.33 *                                                  I
   0.30 .                                                  I
   0.27 *                                                  I
   0.23 *                                                  I
   0.20 ..                     Q                           I
   0.17 *                                                  I
   0.13 *                            O              A      I
   0.10 .                      V                           I
   0.07 *                                      K           I
   0.03 *                                                  I
  -0.00 ..-------------------------------T------------------
  -0.03 *                                                  3
  -0.07 *                                                  I
  -0.10 .                                                  I
  -0.13 *                          S                       I
  -0.17 *                                                  D
  -0.20 ..              Y                                  I
  -0.23 *                                                  I
  -0.27 *                                                  I
  -0.30 .                                                  I
  -0.33 *                   X                          2   I
  -0.37 *                                                  I
  -0.40 ..                                                 I
  -0.43 *                                                  I
  -0.47 *                                      P           I
  -0.50 .                                                  I
  -0.53 *                                                  I
  -0.57 *                                                  I
  -0.60 ..                                                 I
  -0.63 *                                                  I
  -0.67 *                                                  I
  -0.70 .                                               U  I
  -0.73 *
  -0.77 *
  -0.80 ..
  -0.83 *
  -0.87 *
  -0.90 .
        *.****.****.****.****.****.****.****.****.****.****
          .  -0.9000.  -0.7000.  -0.5000.  -0.3000.  -0.1000
```

2

```
.   0.1000.   0.3000.   0.5000.   0.7000.   0.9000.
.0000     0.2000    0.4000    0.6000    0.8000    1.0000
.****.****.****.****.****.****.****.****.****.****.*
I                                                  .      0.90
I                                                  *      0.87
I                                                  *      0.83
I                                                  ..     0.80
I                                                  *      0.77
I                                                  *      0.73
I                    J                             .      0.70
I           6                                      *      0.67
I                                                  *      0.63
I                                                  ..     0.60
I                                                  *      0.57
I                                                  *      0.53
I                                                  .      0.50
I    Z                                             *      0.47
I                                                  *      0.43
I                                                  ..     0.40
IB                                                 *      0.37
I                                                  *      0.33
I                   E                              .      0.30
I                                                  *      0.27
I                                                  *      0.23
I                                 W                ..     0.20
I                                                  *      0.17
I                                                  *      0.13
I                            G                     .      0.10
I                                                  *      0.07
I      M                                           *      0.03
---------------------------------------------------..    -0.00
I                                                  *     -0.03
I                                      L           *     -0.07
I                                                  .     -0.10
I                                                  *     -0.13
I                        H                         *     -0.17
I                                                  ..    -0.20
I                                                  *     -0.23
I                                                  *     -0.27
I                                                  .     -0.30
I                                                  *     -0.33
I               5                                  *     -0.37
I                                       N          ..    -0.40
I                       F                          *     -0.43
I                                                  *     -0.47
I                                                  .     -0.50
I                                                  *     -0.53
I                                                  *     -0.57
I                 4                                ..    -0.60
I                                                  *     -0.63
I                                                  *     -0.67
I                                                  .     -0.70
I                                                  *     -0.73
I                                                  *     -0.77
I       7                                          ..    -0.80
I                                                  *     -0.83
I                                                  *     -0.87
I                                                  .     -0.90
.****.****.****.****.****.****.****.****.****.****.*
.   0.1000.   0.3000.   0.5000.   0.7000.   0.9000.
```

Figure 9. TORSCA--Plot of
Dimensions 1 Versus 3--Administration 7

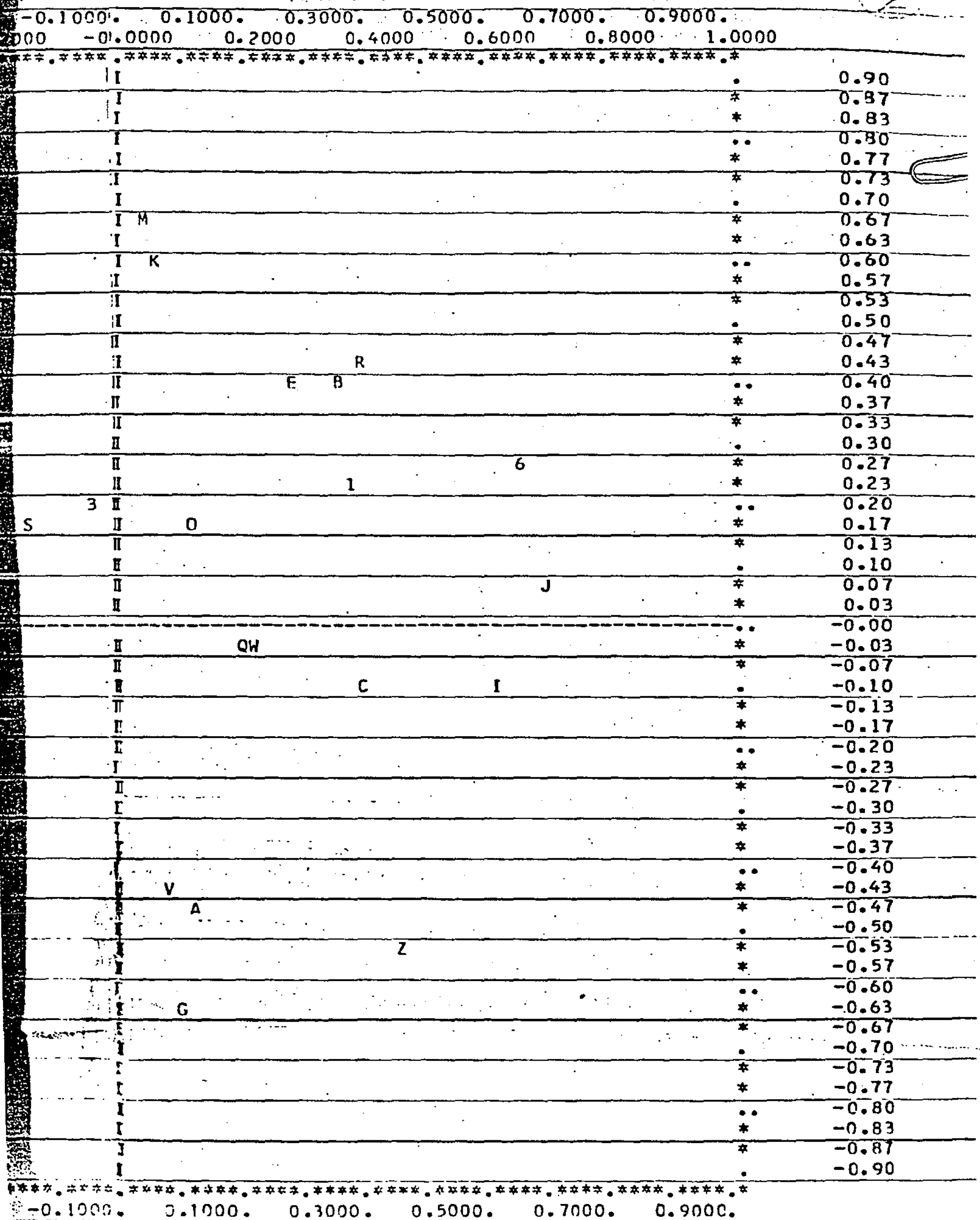
```
DIMENSION 1 (X AXIS) VS. DIMENSION 3 (Y AXIS)                     1
          .  -0.9000.  -0.7000.  -0.5000.  -0.3000.  -0.1000
         -1.0000   -0.8000   -0.6000   -0.4000   -0.2000   -0.
          *.****.****.****.****.****.****.****.****.****.****.
    0.90  .                                                  I
    0.87  *                                                  I
    0.83  *                                                  I
    0.80  ..                                                 I
    0.77  *                                                  I
    0.73  *                                                  I
    0.70  .                                                  I
    0.67  *                                                  I
    0.63  *                                                  I
    0.60  ..                                    K            I
    0.57  *                                                  I
    0.53  *                                                  I
    0.50  .                                                  I
    0.47  *                                                  I
    0.43  *                             R                    I
    0.40  ..                                           2     I
    0.37  *                                                  I
    0.33  *                                                  I
    0.30  .                  X                               I
    0.27  *                                                  I
    0.23  *                                      1           I
    0.20  ..                                                 3
    0.17  *                         S  O                     I
    0.13  *                                                  I
    0.10  .                                                  I
    0.07  *                                                  I
    0.03  *                                     P            I
   -0.00  ..--------------------------------------------------
   -0.03  *                      Q                           I
   -0.07  *                                                  I
   -0.10  .                               C         I        I
   -0.13  *                               T                  I
   -0.17  *              Y                                   I
   -0.20  ..                                                 I
   -0.23  *                                                  I
   -0.27  *                                                  I
   -0.30  .                                              U   I
   -0.33  *                                                  I
   -0.37  *                                                  D
   -0.40  ..                                                 I
   -0.43  *                      V                           I
   -0.47  *                                          A       I
   -0.50  .                                                  I
   -0.53  *                                                  I
   -0.57  *                                                  I
   -0.60  ..                                                 I
   -0.63  *                                                  I
   -0.67  *                                                  I
   -0.70  .                                                  I
   -0.73  *                                                  I
   -0.77  *                                                  I
   -0.80  ..                                                 I
   -0.83  *                                                  I
   -0.87  *                                                  I
   -0.90  .                                                  I
          *.****.****.****.****.****.****.****.****.****.****.
          .  -0.9000.  -0.7000.  -0.5000.  -0.3000.  -0.1000.
```

```
.  0.1000.   0.3000.   0.5000.   0.7000.   0.9000.
.0000     0.2000    0.4000    0.6000    0.8000    1.0000
.****.****.****.****.****.****.****.****.****.****.*
I                                                  .      0.90
I                                                  *      0.87
I                                                  *      0.83
I                                                  ..     0.80
I                                                  *      0.77
I                                                  *      0.73
I                                                  .      0.70
I      M                                           *      0.67
I                                                  *      0.63
I                      H                           ..     0.60
I                                                  *      0.57
I                                                  *      0.53
I                                                  .      0.50
I                                                  *      0.47
I                                                  *      0.43
IB                  E                              ..     0.40
I                                                  *      0.37
I                                                  *      0.33
I                4                                 .      0.30
I       7   6                                      *      0.27
I                                                  *      0.23
3                                                  ..     0.20
I                                                  *      0.17
I                                                  *      0.13
I                                                  .      0.10
I                    J                             *      0.07
I                                                  *      0.03
---------------------------------------------------..    -0.00
I                                W                 *     -0.03
I                                                  *     -0.07
I                                                  .     -0.10
I                                                  *     -0.13
I                                      N           *     -0.17
I                                                  ..    -0.20
I                                                  *     -0.23
I                                                  *     -0.27
I                     F                            .     -0.30
I                                                  *     -0.33
D                                                  *     -0.37
I                                                  ..    -0.40
I                                                  *     -0.43
I                                                  *     -0.47
I                                                  .     -0.50
I    Z                                             *     -0.53
I                                                  *     -0.57
I                                                  ..    -0.60
I                           G                      *     -0.63
I                                                  *     -0.67
I                                                  .     -0.70
I                                                  *     -0.73
I                                                  *     -0.77
I                                                  ..    -0.80
I              5                                   *     -0.83
I                                                  *     -0.87
I                                                  .     -0.90
.****.****.****.****.****.****.****.****.****.****.*
.  0.1000.   0.3000.   0.5000.   0.7000.   0.9000.
```

Figure 10. TORSCA--Plot of

Dimensions 2 Versus 3--Administration 7

```
DIMENSION 2 (X AXIS) VS. DIMENSION 3 (Y AXIS)
          .   -0.9000.  -0.7000.  -0.5000.  -0.3000.  -0.1000.   0.10
       -1.0000   -0.8000   -0.6000   -0.4000   -0.2000   -0.0000
      *.****.****.****.****.****.****.****.****.****.****.****.****.**
  0.90 .                                                 I
  0.87 *                                                 I
  0.83 *                                                 I
  0.80 ..                                                I
  0.77 *                                                 I
  0.73 *                                                 I
  0.70 .                                                 I
  0.67 *                                                 I M
  0.63 *                                                 I
  0.60 ..                                       H        I  K
  0.57 *                                                 I
  0.53 *                                                 I
  0.50 .                                                 I
  0.47 *                                                 I
  0.43 *                                                 I
  0.40 ..                               2                I
  0.37 *                                                 I
  0.33 *                                                 I
  0.30 .                  4              X               I
  0.27 *        7                                        I
  0.23 *                                                 I
  0.20 ..                                             3  I
  0.17 *                                          S      I     O
  0.13 *                                                 I
  0.10 .                                                 I
  0.07 *                                                 I
  0.03 *                         P                       I
 -0.00 ..------------------------------------------------------------
 -0.03 *                                                 I
 -0.07 *                                                 I
 -0.10 .                                                 I
 -0.13 *                                                 I
 -0.17 *                             N          Y        I
 -0.20 ..                                                I
 -0.23 *                                                 I
 -0.27 *                                                 I
 -0.30 .              U             F                    I
 -0.33 *                                                 I
 -0.37 *                                         D       I
 -0.40 ..                                                I
 -0.43 *                                                 I   V
 -0.47 *                                                 I     A
 -0.50 .                                                 I
 -0.53 *                                                 I
 -0.57 *                                                 I
 -0.60 ..                                                I
 -0.63 *                                                 I    G
 -0.67 *                                                 I
 -0.70 .                                                 I
 -0.73 *                                                 I
 -0.77 *                                                 I
 -0.80 ..                                                I
 -0.83 *                               5                 I
 -0.87 *                                                 I
 -0.90 .                                                 I
      *.****.****.****.****.****.****.****.****.****.****.****.****.**
          .   -0.9000.  -0.7000.  -0.5000.  -0.3000.  -0.1000.   0.100
```



-0.1000. 0.1000. 0.3000. 0.5000. 0.7000. 0.9000.
-0.0000 0.2000 0.4000 0.6000 0.8000 1.0000
0.90
0.87
0.83
0.80
0.77
0.73
0.70
0.67
0.63
0.60
0.57
0.53
0.50
0.47
0.43
0.40
0.37
0.33
0.30
0.27
0.23
0.20
0.17
0.13
0.10
0.07
0.03
-0.00
-0.03
-0.07
-0.10
-0.13
-0.17
-0.20
-0.23
-0.27
-0.30
-0.33
-0.37
-0.40
-0.43
-0.47
-0.50
-0.53
-0.57
-0.60
-0.63
-0.67
-0.70
-0.73
-0.77
-0.80
-0.83
-0.87
-0.90
M
K
R
E B
6
1
3
S O
J
QW
C I
V
A
Z
G
-0.1000. 0.1000. 0.3000. 0.5000. 0.7000. 0.9000.

Specifically, in administration one, there is a sharp break in the percent variance explained after the third factor: thus this manifold is interpreted as requiring three dimensions or coordinate axes. Likewise, the percent variance explained in sample two drops sharply after the third factor. Sample three and four also drops after the third factor. Administration number five is a more problematic case with more factors than usual being needed to explain all the variance. Four factors do explain the largest bulk of the variance though. Administration number six, according to this procedure, has one gigantic factor which explains the vast amount of variance with no other factor seeming to have much significance. In administration number seven, the variance drops sharply after the third factor. In administration number eight, three factors explain most of the variance, however, the dimensionality is less clear with the fourth and fifth factors having moderate significance. In administration number nine, we again confront the apparent complete domination of all the variance by one factor. In administration number ten, the first two factors explain most of the variance; however, factor three especially, and factors four and five less so, explain a moderate amount of the variance. In administration number eleven, the first two factors explain the greatest proportion of the variance; the third, and to a much lesser extent factors four and five, explain a small portion of the variance.

The procedure being used for the determination of dimensionality entails using percent of variance explained as the indicator. The assessment of the number of axes needed to represent the configuration of social psychological objects is derived from tests using factor analysis of the scalar products of distances between points for a configuration whose dimensions were known. An eight by eight matrix with cells consisting of the distances between pairs of the eight corner points of an oblong solid was calculated. After these distances were converted into the scalar products of the pairs of points, they were factor analyzed (Principal Axis). Three factors cumulatively accounted for 99.99 percent of the variance. See oblong in Figure 11. When a factor analysis of the oblong solid was done using a distance matrix which had error in it (i.e., incorrect distances between two pairs of points). See Figure 12. Factors four through eight were merely error. There is however, a sharp drop in the variance after the third dimension which we know to be the correct dimensionality. The forth dimension is indicated to explain a small amount of the variance although we know this is just a reflection of the error in the matrix. This, of course, becomes a source of difficulty when, as is the normal case, the true dimensionality of the configuration is unknown. Note also that the errors in the oblong solid matrix seem to result in the increase of negative eigenvalues, negative percent variances, and in the cumulative percent variance (i.e., more than 100 percent). Error results in the production of more factors. Likewise, factor analysis (Principal

PROGRAM NUMBER 2.
EXECUTING PRINCIPAL COMPONENTS/PRINCIPAL AXIS FACTOR ANALYSIS PROGRAM
ADDRESS OF MEMORY WORK AREA IS 0AF000. LENGTH OF MEMORY WORK AREA IS 015A00.

FACTOR	VARIANCE	PERCENT VARIANCE	CUMULATIVE PERCENT
1	17.9992	64.2845	64.2845
2	7.9992	28.5693	92.8538
3	1.9990	7.1395	99.9933
4	0.0011	0.0040	99.9973
5	0.0011	0.0040	100.0013
6	0.0009	0.0033	100.0046
7	0.0000	0.0000	100.0046
8	-0.0013	-0.0045	100.0000

THE TRACE IS 27.9993

THE SUM OF THE FIRST 3 ROOTS IS 27.9974

FACTOR MATRIX

	1	2	3
1	1.49997	-0.99995	-0.49988
2	1.49997	0.99995	-0.49988
3	1.49997	0.99995	0.49988
4	1.49997	-0.99995	0.49988
5	-1.49997	-0.99995	-0.49988
6	-1.49997	0.99995	-0.49988
7	-1.49997	0.99995	0.49988
8	-1.49997	-0.99995	0.49988

Figure 11. Factor Analysis of Scalar Products of Perfect Oblong Solid

```
PROGRAM NUMBER 2.
EXECUTING PRINCIPAL COMPONENTS/PRINCIPAL AXIS FACTOR ANALYSIS PROGRAM
ADDRESS OF MEMORY WORK AREA IS 048000.  LENGTH OF MEMORY WORK AREA IS 015A00.
```

FACTOR	VARIANCE	PERCENT VARIANCE	CUMULATIVE PERCENT
1	17.4160	60.8436	60.8436
2	8.8146	30.7942	91.6378
3	2.9024	10.1397	101.7776
4	0.7822	2.7326	104.5100
5	0.0010	0.0036	104.5136
6	-0.0000	-0.0000	104.5135
7	-0.0001	-0.0002	104.5133
8	-1.2919	-4.5134	99.9999

THE TRACE IS 28.6243

THE SUM OF THE FIRST 3 ROOTS IS 29.1331

FACTOR MATRIX

	1	2	3
1	1.47382	0.96597	0.10882
2	1.47381	-0.90991	0.64058
3	1.47382	-0.96597	-0.10882
4	1.47381	0.90991	-0.64058
5	-1.37512	1.34846	0.78655
6	-1.57251	-0.90990	0.64060
7	-1.37512	-1.34846	-0.78655
8	-1.57251	0.90990	-0.64060

Figure 12. Factor Analysis of Scalar Products of Oblong Solid with Error

Axis) of a distance matrix constructed from the known distances between eight points on a face (two eyes, two eyebrows, a nose, two corners of the mouth, and the chin) but with incorrect figures entered in two pairs of cells lead to all eight of the factors being reported by the factor analysis of the scalar products. See Figure 13. However, again it should be observed that there is a sharp drop in variance explained after the second factor. By examining the coordinates or factor "loadings" of the third factor as well as observing the cells with incorrect distances, it becomes apparent that the "loadings" on the third factor primarily entail an attempt to compensate for the inconsistencies caused by the incorrect distances between the eyebrows (points four and five) and the corners of the mouth (points six and seven); it is points five and seven which have the highest "loadings" on this dimension.

The use of a configuration of known dimensionality can also be used to examine the adequacy of a Principal Axis factor analysis of cross products (instead of scalar products) for determining dimensionality. Factor analysis of the cross products of an errorless distance matrix based on an oblong solid do not result in a perfect reproduction of the oblong solid. See Figure 14. Factor one accounts for 78 percent of the variance alone, while the second and third factors account for 15 and 4 percent of the variance respectively. The resulting configuration constructed from the coordinates (i.e., loadings) in the factor matrix is only two dimensional

EXECUTING PRINCIPAL COMPONENTS/PRINCIPAL AXIS FACTOR ANALYSIS PROGRAM
ADDRESS OF MEMORY WORK AREA IS 085000. LENGTH OF MEMORY WORK AREA IS 015A00.

FACTOR	VARIANCE	PERCENT VARIANCE	CUMULATIVE PERCENT
1	220.5308	73.0310	73.0310
2	64.8657	21.4809	94.5119
3	29.0791	9.6298	104.1417
4	10.0279	3.3208	107.4625
5	5.0519	1.6730	109.1354
6	0.0000	0.0000	109.1354
7	-4.3606	-1.4440	107.6914
8	-23.2261	-7.6916	99.9998

THE TRACE IS 301.9688

THE SUM OF THE FIRST 3 ROOTS IS 314.4753

FACTOR MATRIX

	1	2	3
1	-4.16214	-3.55557	-1.00074
2	-3.91992	4.77765	-0.77214
3	1.09082	0.24729	-0.62955
4	-5.58006	-2.97923	-1.73686
5	-5.55173	2.12403	3.65438
6	3.72552	-2.79070	1.80521
7	5.30545	2.79012	-2.47447
8	9.09207	-0.61359	1.15418

Figure 13. Factor Analysis of Scalar Products of Face with Error

PROGRAM NUMBER 3.

EXECUTING PRINCIPAL COMPONENTS/PRINCIPAL AXIS FACTOR ANALYSIS PROGRAM

ADDRESS OF MEMORY WORK AREA IS 048000. LENGTH OF MEMORY WORK AREA IS 013200.

FACTOR	VARIANCE	PERCENT VARIANCE	CUMULATIVE PERCENT
1	351.3900	78.4372	78.4372
2	68.4458	15.2785	93.7156
3	19.5435	4.3625	98.0781
4	4.2082	0.9394	99.0175
5	2.3538	0.5254	99.5429
6	0.8798	0.1964	99.7393
7	0.6247	0.1395	99.8787
8	0.5438	0.1214	100.0001

THE TRACE IS 447.9890

THE SUM OF THE FIRST 3 ROOTS IS 439.3792

FACTOR MATRIX

	1	2	3
1	6.62750	2.92502	1.56299
2	6.62750	2.92502	-1.56299
3	6.62750	2.92502	-1.56299
4	6.62750	2.92502	1.56299
5	6.62750	-2.92502	1.56299
6	6.62750	-2.92502	-1.56299
7	6.62750	-2.92502	-1.56299
8	6.62750	-2.92502	1.56299

Figure 14. Factor Analysis of Cross Products of Perfect Oblong Solid

as opposed to three despite the fact that the data treated was error free. The two dimensionality can easily be recognized by the fact that each pair of points (2 and 3, 1 and 4, 5 and 8, 6 and 7) are identical. These points depicted the oblong's depth in the original configuration. This test indicates the superiority of the factor analysis of scalar products as opposed to cross products for determining the correct dimensionality. When the cross products were used on data with known error in the distance matrix, i.e., the face matrix discussed above, the results were very poor. See Figure 15. Incorrect (i.e., inconsistent) distances were included between two pairs of points. The resulting configuration based on the loadings of the factor matrix was extremely distorted. The first dimension which accounts for 77.7 percent of the variance was quite distorted. See Figure 16. The third factor (note points 3 and 4, and 6 and 7)

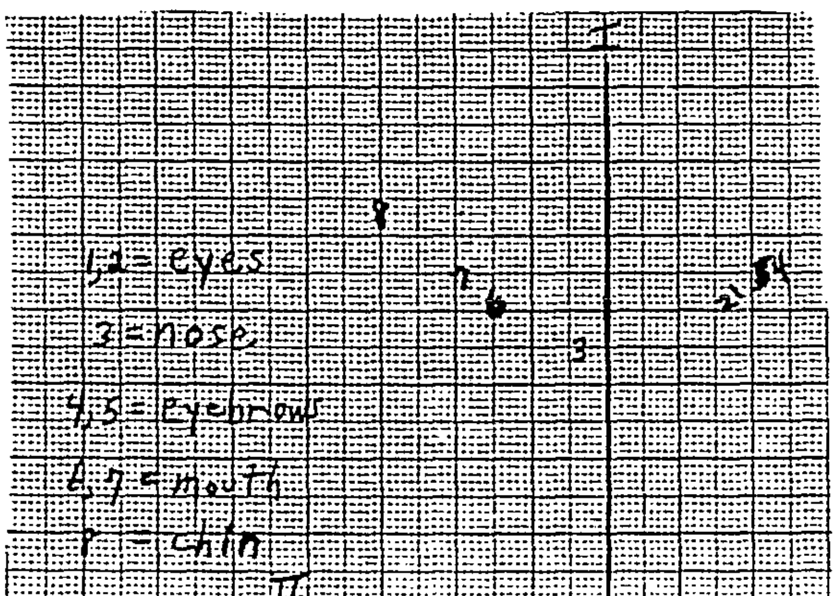


Figure 15. Loadings of Cross Products of Face with Error

EXECUTING PRINCIPAL COMPONENTS/PRINCIPAL AXIS FACTOR ANALYSIS PROGRAM
ADDRESS OF MEMORY WORK AREA IS 095000. LENGTH OF MEMORY WORK AREA IS 013200.

FACTOR	VARIANCE	PERCENT VARIANCE	CUMULATIVE PERCENT
1	3758.1202	77.7837	77.7837
2	771.3636	15.9653	93.7490
3	180.6731	3.7395	97.4885
4	62.8471	1.3008	98.7893
5	42.4838	0.8793	99.6686
6	12.0599	0.2496	99.9182
7	3.9157	0.0810	99.9992
8	0.0367	0.0008	100.0000

THE TRACE IS 4831.5000

THE SUM OF THE FIRST 3 ROOTS IS 4710.1523

FACTOR MATRIX

	1	2	3
1	20.64191	8.68865	-6.55706
2	20.56954	8.23547	7.29516
3	17.88541	-2.20821	0.16372
4	22.34197	10.65821	-5.99573
5	22.34723	10.55877	5.66150
6	19.88781	-7.51289	-2.92208
7	21.55102	-10.00456	2.77467
8	27.02782	-15.51533	-0.43963

Figure 16. Factor Analysis of Cross Products of Face with Error

which is shown explaining only 3.7 percent of the variance is, nonetheless, the dimension indicating the only horizontal distinctions (e.g., right and left eye).

The ascertainment of the correct dimensionality is also a problem when using TORSCA, the generally nonmetric method for multidimensional scaling. Two indicators are included with TORSCA scaling results. One is Kruskal's stress and the other is the Index of Fit which is another indicator of the degree of fit between the derived distances and the disparities - i.e., the original distances monotonically transformed.

The TORSCA multidimensional scaling program was tested with the error free oblong solid distance matrix. The results showed zero Stress, an excellent Stress and an index of 1.00000 for the three dimensional solution. See Figure 17. Deter-

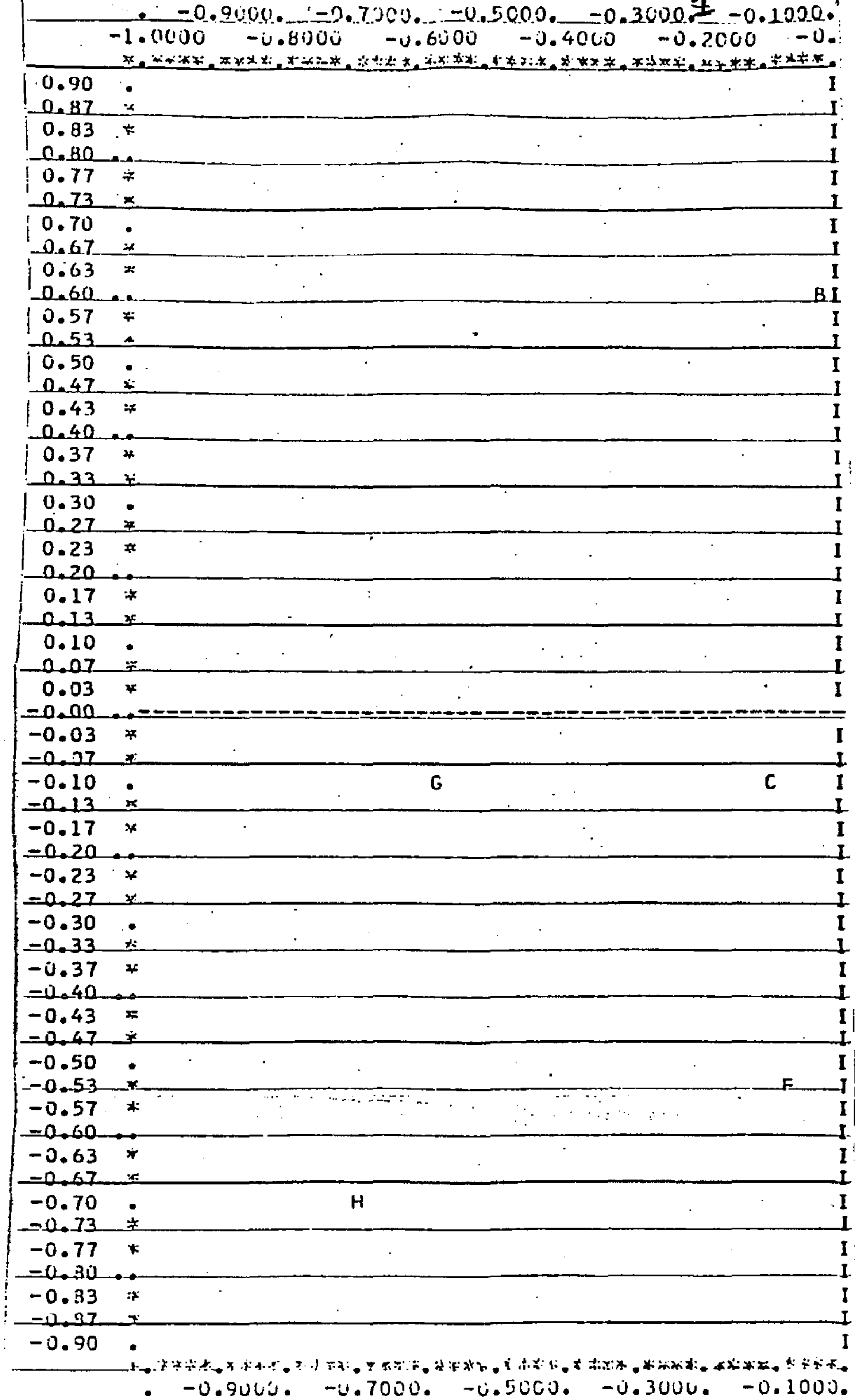
CONFIGURATION	1	2	3
1	0.530	-0.354	0.177
2	0.530	0.354	0.177
3	0.530	0.354	-0.177
4	0.530	-0.354	-0.177
5	-0.530	-0.354	0.177
6	-0.530	0.354	0.177
7	-0.530	0.354	-0.177
8	-0.530	-0.354	-0.177

Figure 17. TORSCA Coordinates of Perfect Oblong Solid

mination of dimensionality is only possible within the TORSCA program by the achievement of a satisfactory level of Stress and/or a sufficient Index of Fit. Difficulties arise in discovering the true dimensionality since error in the original distance matrix leads to satisfactory Stress being achieved only by using a larger number of dimensions. The problem of determining dimensionality with the TORSCA multidimensional scaling program can be demonstrated by examining the results of a test case using a given matrix of distances between pairs of points on a face wherein point five (the eyebrow) -- or point E as designated by TORSCA -- has incorrect distances (inconsistent with the other distances in a Euclidean space and resulting in violations of triangle inequality) given with both mouth corners -- points F and G -- in the original distance matrix. The program results show satisfactory stress for three dimensions but only minimum stress for two dimensions. The Index of Fit on the two dimensions barely meets the minimum of .999 suggested as appropriate with the Index of .99995 being much better in the three dimensional case. However, since the true configuration of points is actually known, i.e., the face (A and B are the eyes, C the nose, D and E the eyebrows, F and G the corners of the mouth, H the chin), the derived configuration can be evaluated in terms of its conformity to the true relationships between the points. Plotting the coordinates of dimensions one and two yields a relatively acceptable representation of the face. The third dimension appears mainly to be an attempt to respond to the given spatial inconsistencies or errors in the distance matrix. See Figures 18 - 20 for the plots of dimensions one and two, one and three, and two and three respectively. The program run asking for only two dimensions seems somewhat better than the plots of dimensions one and two on the three dimensional plot. See Figure 21. Of course, since the true configuration is not known, and since there are likely to be violations of triangle inequality among the distances in the original data matrices, determination of dimensionality via the level of Stress and/or Index of Fit become both difficult and questionable. Furthermore, as is evident from observing the plots with the third dimension of the face, the resulting configuration can be highly distorted while the stress may be reported as satisfactory and the index of fit quite sufficient. It would thus seem that the only indicators available in TORSCA for the assessment of accuracy may not only be insufficient (as in the case of the oblong solid whose data is distorted but yet receives a quite satisfactory Stress and Index of Fit) but even worse may be misleading (e.g., as with the third dimension of the face).

Figure 18. TORSCA Plot of Dimensions 1 Versus 2 of Face with Error

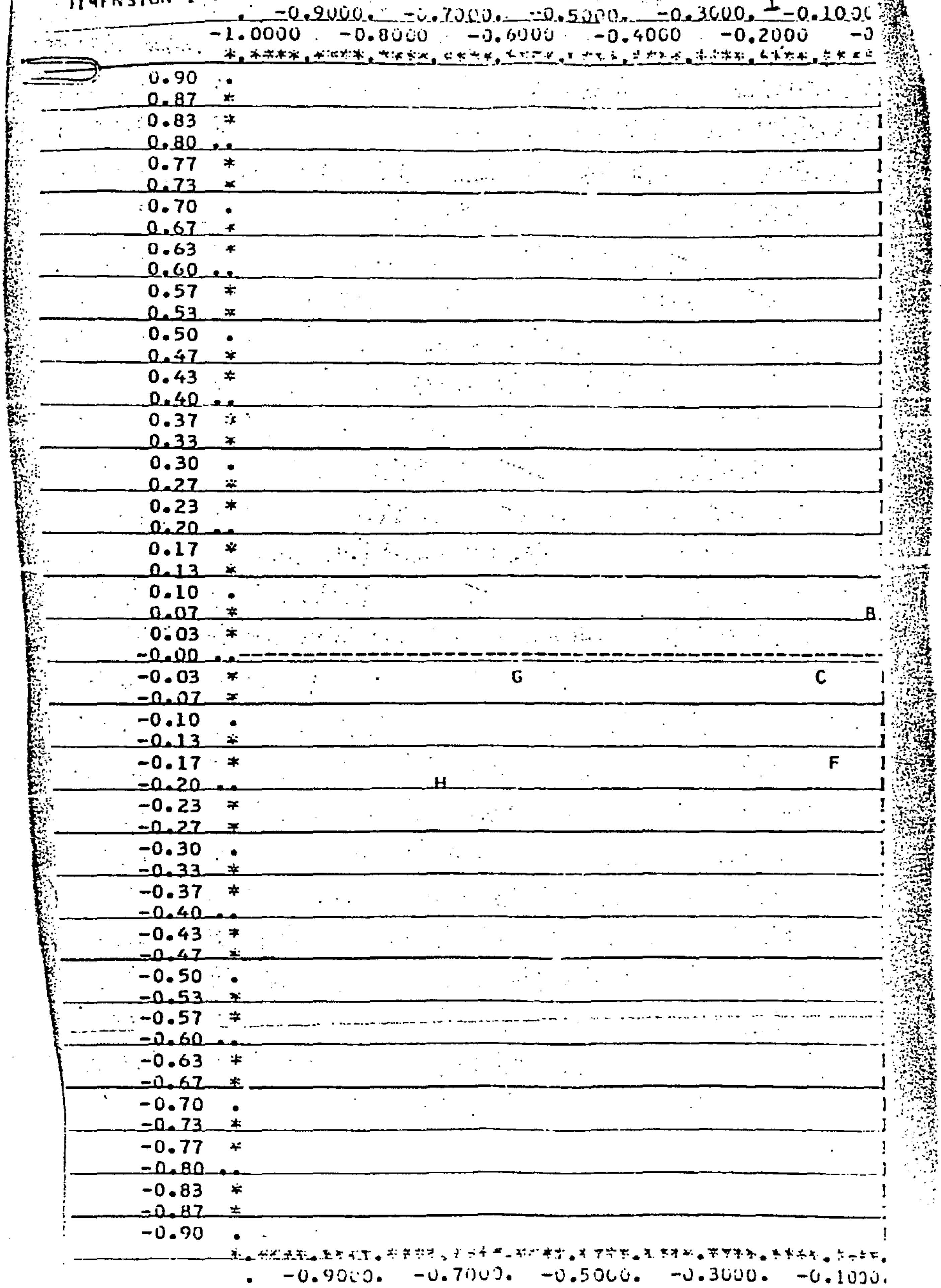
ION 1 (X AXIS) VS. DIMENSION 2 (Y AXIS)



```
.   0.1000.   0.3000.   0.5000.   0.7000.   0.9000.
.0000     0.2000     0.4000     0.6000     0.8000     1.0000      2
.****.****.****.****.****.****.****.****.****.****.*
I                                                   .        0.90
I                                                   *        0.87
I                                                   *        0.83
I                                                   ..       0.80
I                                                   *        0.77
I                                                   *        0.73
I                                                   .        0.70
I                                                   *        0.67
I                                                   *        0.63
I                                                   ..       0.60
I                     E                             *        0.57
I                                                   *        0.53
I                                                   .        0.50
I                                                   *        0.47
I                                                   *        0.43
I                                                   ..       0.40
I                                                   *        0.37
I                                                   *        0.33
I                      D                            .        0.30
I                                                   *        0.27
I                                                   *        0.23
I                                                   ..       0.20
I                                                   *        0.17
I                                                   *        0.13
I                                                   .        0.10
I                                                   *        0.07
I                                                   *        0.03
---------------------------------------------------..       -0.00
I                           A                       *       -0.03
I                                                   *       -0.07
I                                                   .       -0.10
I                                                   *       -0.13
I                                                   *       -0.17
I                                                   ..      -0.20
I                                                   *       -0.23
I                                                   *       -0.27
I                                                   .       -0.30
I                                                   *       -0.33
I                                                   *       -0.37
I                                                   ..      -0.40
I                                                   *       -0.43
I                                                   *       -0.47
I                                                   .       -0.50
I                                                   *       -0.53
I                                                   *       -0.57
I                                                   ..      -0.60
I                                                   *       -0.63
I                                                   *       -0.67
I                                                   .       -0.70
I                                                   *       -0.73
I                                                   *       -0.77
I                                                   ..      -0.80
I                                                   *       -0.83
I                                                   *       -0.87
I                                                   .       -0.90
.****.****.****.****.****.****.****.****.****.****.*
.   0.1000.   0.3000.   0.5000.   0.7000    0.9000
```

Figure 19. TORSCA Plot of Dimensions 1 Versus 3 of Face with Error

DIMENSION 1 (X AXIS) VS. DIMENSION 3 (Y AXIS) 1



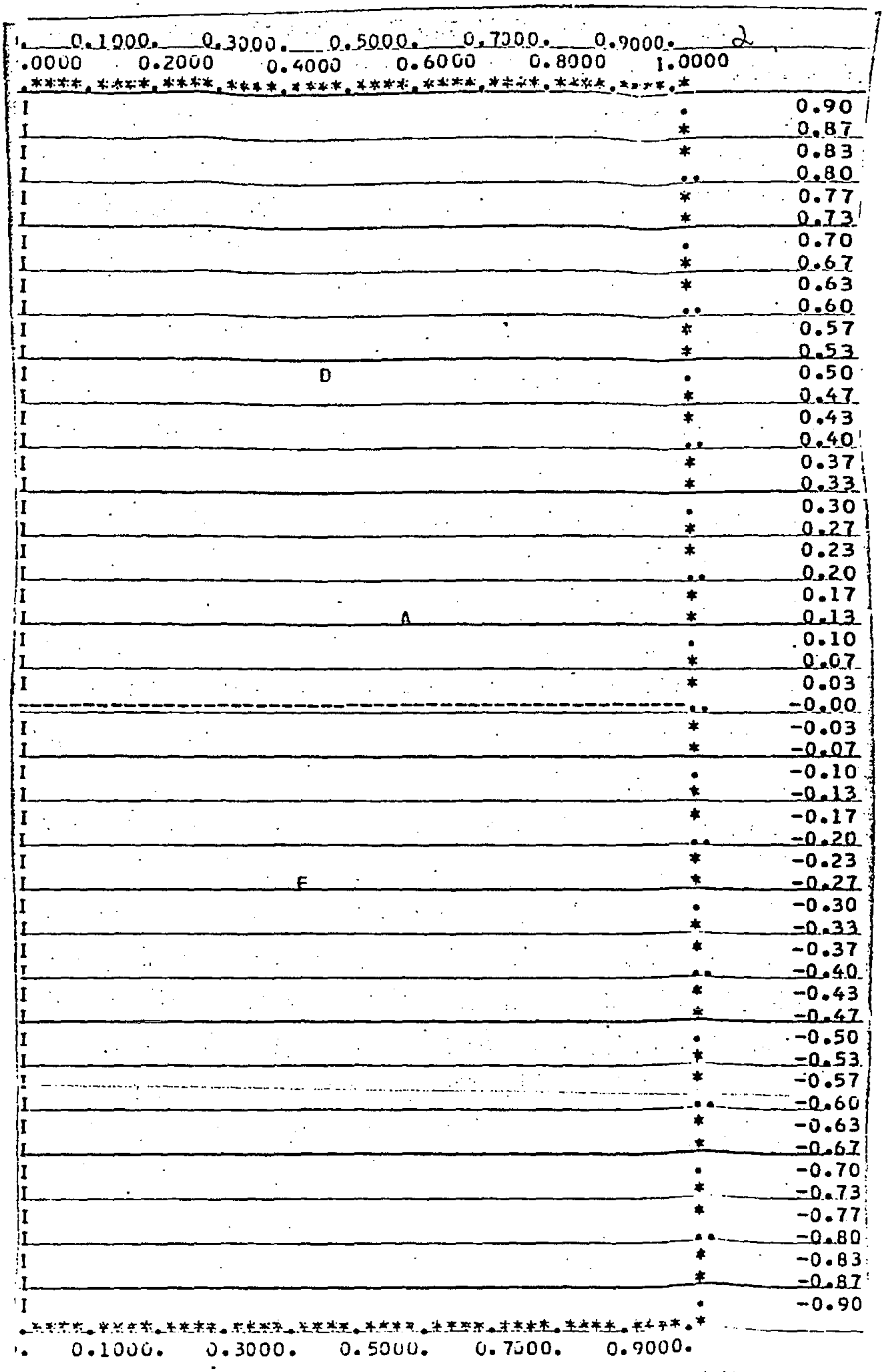
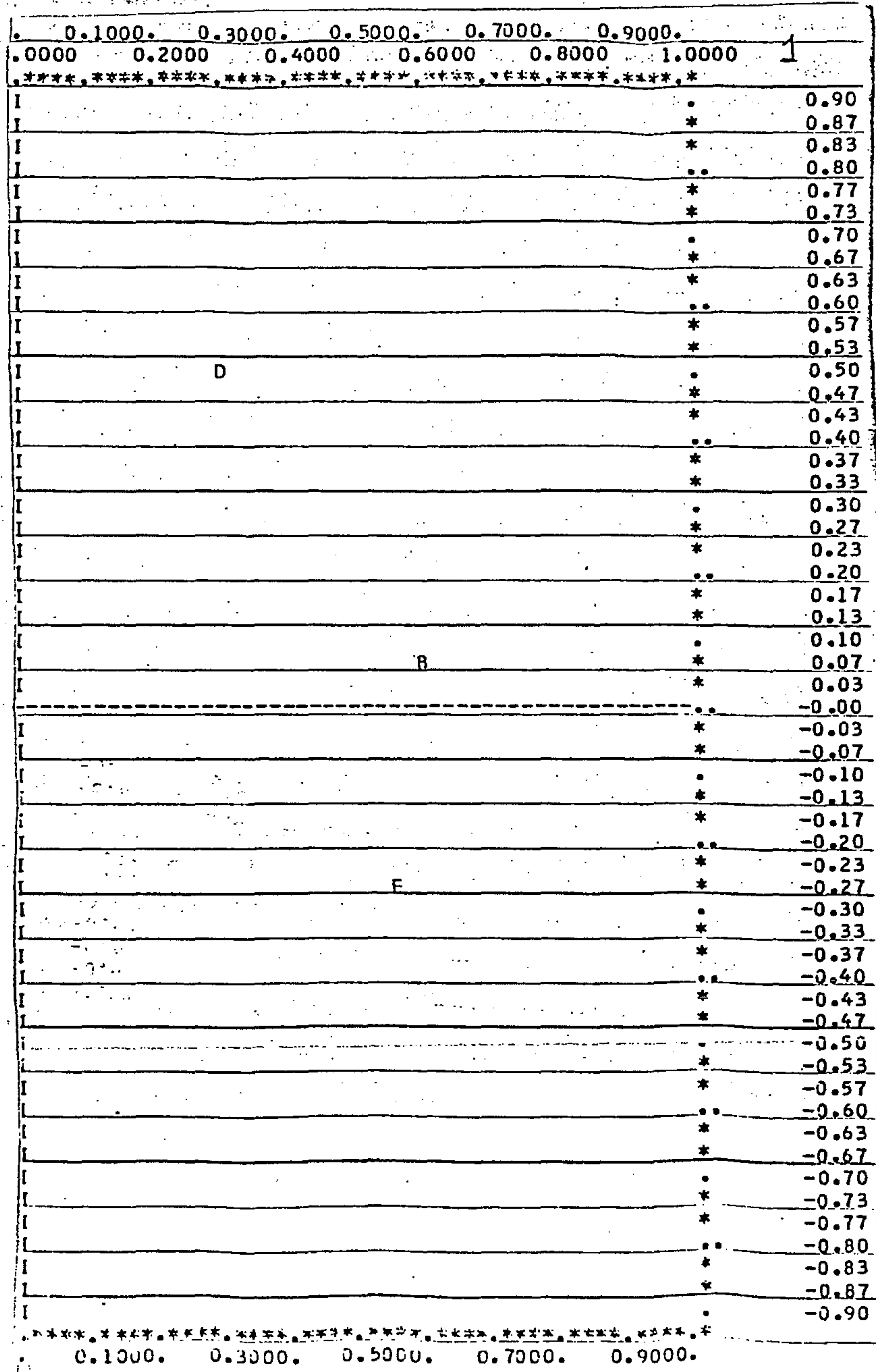
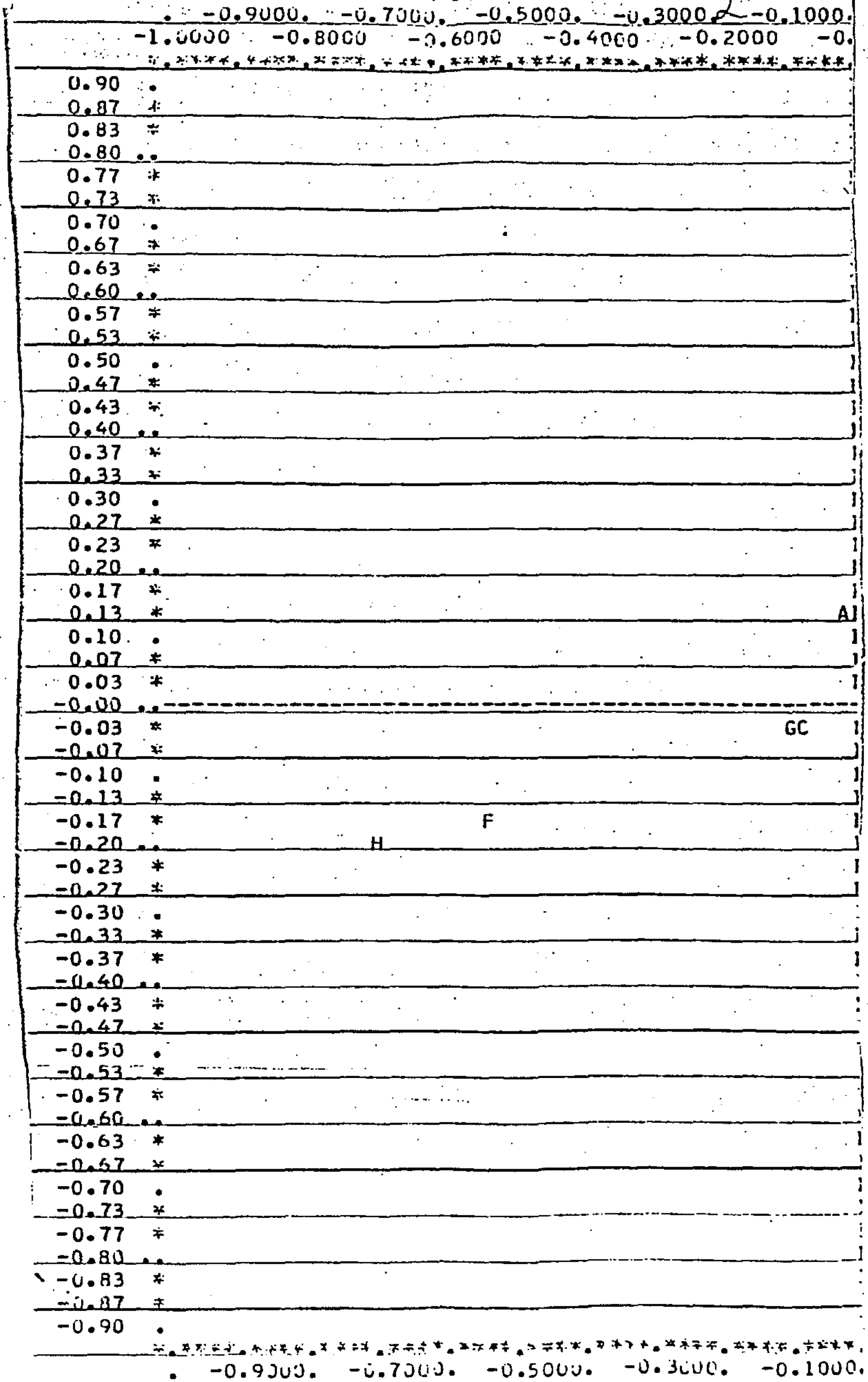

0.1000. 0.3000. 0.5000. 0.7000. 0.9000.
.0000 0.2000 0.4000 0.6000 0.8000 1.0000
0.90
0.87
0.83
0.80
0.77
0.73
0.70
0.67
0.63
0.60
0.57
0.53
0.50
0.47
0.43
0.40
0.37
0.33
0.30
0.27
0.23
0.20
0.17
0.13
0.10
0.07
0.03
-0.00
-0.03
-0.07
-0.10
-0.13
-0.17
-0.20
-0.23
-0.27
-0.30
-0.33
-0.37
-0.40
-0.43
-0.47
-0.50
-0.53
-0.57
-0.60
-0.63
-0.67
-0.70
-0.73
-0.77
-0.80
-0.83
-0.87
-0.90
D
A
E
0.1000. 0.3000. 0.5000. 0.7000. 0.9000.

Figure 20. TORSCA Plot of Dimensions 2 Versus 3 of Face with Error



0.1000. 0.3000. 0.5000. 0.7000. 0.9000.
.0000 0.2000 0.4000 0.6000 0.8000 1.0000
D
B
E
0.90
0.87
0.83
0.80
0.77
0.73
0.70
0.67
0.63
0.60
0.57
0.53
0.50
0.47
0.43
0.40
0.37
0.33
0.30
0.27
0.23
0.20
0.17
0.13
0.10
0.07
0.03
-0.00
-0.03
-0.07
-0.10
-0.13
-0.17
-0.20
-0.23
-0.27
-0.30
-0.33
-0.37
-0.40
-0.43
-0.47
-0.50
-0.53
-0.57
-0.60
-0.63
-0.67
-0.70
-0.73
-0.77
-0.80
-0.83
-0.87
-0.90
0.1000. 0.3000. 0.5000. 0.7000. 0.9000.

DIMENSION 2 (X AXIS) VS. DIMENSION 3 (Y AXIS)



FOR THE 2 DIMENSIONAL SOLUTION:

MINIMUM STRESS ACHIEVED.

INDEX = 0.99962 STRESS = 0.039

CONFIGURATION	1	2
1	0.431	0.423
2	0.431	-0.426
3	-0.171	-0.005
4	0.641	0.329
5	0.642	-0.325
6	-0.460	0.300
7	-0.474	-0.303
8	-1.040	0.008

3 DIMENSIONAL SOLUTION:

SATISFACTORY STRESS ACHIEVED.

INDEX = 0.99995 STRESS = 0.015

CONFIGURATION	1	2	3
1	0.417	-0.430	-0.050
2	0.415	0.435	0.054
3	-0.134	0.006	0.007
4	0.622	-0.183	0.349
5	0.618	0.179	-0.356
6	-0.457	-0.310	-0.131
7	-0.474	0.311	0.135
8	-1.007	-0.009	-0.006

Figure 21. TORSCA 3 Dimensional Versus 2 Dimensional Solution of Face with Error

FOR THE 3 DIMENSIONAL SOLUTION:

SATISFACTORY STRESS ACHIEVED.

INDEX = 0.99994 STRESS = 0.015

D I S P A R I T I E S

	1	2	3	4	5	6	7	8
1	0.0	0.819	0.872	0.392	0.965	1.262	1.320	1.034
2	0.819	0.0	0.392	0.872	1.262	0.965	1.034	1.320
3	0.872	0.392	0.0	0.819	1.320	1.034	0.965	1.262
4	0.392	0.872	0.819	0.0	1.034	1.320	1.262	0.965
5	0.965	1.262	1.320	1.034	0.0	0.819	1.034	0.392
6	1.262	0.965	1.034	1.320	0.819	0.0	0.392	0.872
7	1.320	1.034	0.965	1.262	1.034	0.392	0.0	0.819
8	1.034	1.320	1.262	0.965	0.392	0.872	0.819	0.0

THERE ARE NO VIOLATIONS OF THE TRIANGULAR INEQUALITY IN THE MATRIX ABOV

D I S T A N C E S

	1	2	3	4	5	6	7	8
1	0.0	0.800	0.871	0.381	0.955	1.270	1.334	1.036
2	0.800	0.0	0.381	0.900	1.253	0.975	1.047	1.306
3	0.871	0.381	0.0	0.800	1.334	1.036	0.955	1.270
4	0.381	0.900	0.800	0.0	1.047	1.306	1.253	0.975
5	0.955	1.253	1.334	1.047	0.0	0.839	1.005	0.403
6	1.270	0.975	1.036	1.306	0.839	0.0	0.403	0.846
7	1.334	1.047	0.955	1.253	1.005	0.403	0.0	0.839
8	1.036	1.306	1.270	0.975	0.403	0.846	0.839	0.0

THERE ARE NO VIOLATIONS OF THE TRIANGULAR INEQUALITY IN THE MATRIX ABOV

C O N F I G U R A T I O N

	1	2	3
1	0.488	-0.412	0.140
2	0.474	0.381	0.239
3	0.488	0.412	-0.140
4	0.474	-0.381	-0.239
5	-0.463	-0.450	0.223
6	-0.499	0.386	0.173
7	-0.463	0.450	-0.223
8	-0.499	-0.386	-0.173

Figure 22. TORSCA 3 Dimensional Solution for Oblong with Error

Summary

When examining the mean distance matrices, the stability of the mean aggregate galileo distances seems relatively stable (considering the infinity of numbers possible) across administrations both within a particular class and between classes. However, there is difficulty in assessing how significant the degree of stability indicated is.

As to the orderliness of the motion of social psychological objects in the behavioral manifold, some preliminary interpretations were made as to the 'reasonableness' of the motion. However, they were not meant to be more than 'plausible' interpretations. Long term examination of motion in the behavioral manifold on data which is less open to the problems of this exercise (e.g., splicing, keypunching error) seems indicated. Moreover, the exact mapping relation of this behavioral manifold to overt behavior has not yet been clearly assessed and will require further work.

The procedure for estimating distance change was found faulty because of its sensitivity to large distances in the means matrix. It is suggested that prior to factor analysis, the distances which are exceedingly large on only one or two comparisons be removed so as to not distort their relationship to the other behaviors.

As to the dimensions of the manifold, they were found to be small.as indicated by the number of large positive eigenvalues.

FOOTNOTES

[1]See F. W. Young, TORSCA-9: A Fortran IV Program for Nonmetric Multidimensional Scaling (Chapel Hill: Thurstone Psychometric Laboratory, 1968).

CHAPTER V

SUMMARY AND CONCLUSIONS

Chapter one presented some problems of contemporary sociology which may be heeding its progress as a science. The gap between the theoretical and research concepts of sociology is cited as an element of the "problem." A perspective on the nature of the trouble is sought by examining the relationship between theory and research in physics. A number of factors seemed to characterize physics: the basic concepts were both theoretical and empirical at the same time; all other concepts could be derived from the fundamental ones. Thus, stated differently, theory in physics was built on a base of concepts characterized by fundamental measurement -- distance, time, and to a lesser degree, either mass or force. All other concepts could be derived from these concepts. Measurement in the social sciences is, on the other hand, characterized by fiat.

Physics can be characterized as studying motion, forces, and matter. The simple application of a measuring stick or scale is not a sufficient procedure for the analysis of motion. Motion is not a property of a physical object. Motion, or velocity, is analyzed relative to a fixed point. Coordinate systems in physics have provided a very useful tool of analysis in physics. The object in motion is located against the continuums of distance and time relative to a fixed origin.

Descartes' development of the coordinate system involved preliminary understanding of the concepts of 'variable' and 'function' which along with derivative played such an important role in the development of Newton's calculus which focused attention on rates of change (or rates of motion or flow).[1] Descartes' coordinate system brought the power of algebra to the geometry of space.[2] Any point could be located in terms of its coordinates. The Cartesian coordinate system contributed fundamentally to the development of calculus--which permitted the mathematical study of motion or, i.e., rates of change of moving objects. It is the coordinate system which permitted the methodical treatment of the functional relationships between the fundamental measures of distance and time and velocity and acceleration, i.e., motion.

This work has suggested the possible value of constructing a social manifold or coordinate system for social objects. This work has tried to demonstrate how the concepts of distance and time -- both theoretical and empirical -- have been used in physics to join fundamental measurement to a set of theoretical laws through derived measurement. There has been an attempt to demonstrate in a very preliminary way, how the concepts of distance and time might be useful in developing a fundamental system of measurement for sociology.

In both chapter one and more elaborately in chapter two the characteristics of the desired manifold were described. The manifold for social psychological objects was to be continuous, homogeneous, isotropic, linear, unbounded, and metric. These characteristics, with the exception of linarity, have contributed to the creation of a fundamentally common quantitative and continuous framework from which to locate and observe discrete social objects and their dynamics. While linearity may not eventually prove an appropriate model for social space, it was proposed at this stage because of its simplicity and utility for mathematical analysis. These two reasons while not ultimately adequate, given evidence for other models, do suffice at this preliminary stage. The possible contribution of a model which facilitates mathematical analysis can not be ignored. Throughout this work, in the body and in the footnotes, contributions to scientific advance associated with the conceptualization of the problem mathematically have been noted. Not the least of which has been the place mathematical analysis has played among Descartes, Keplar, and Galileo's three great achievements upon which Newton built his grand accomplishments of dynamics and celestial mechanics. Specifically, a factor supposedly essential to the scientific revolution in physics was Galileo's reconceptualization of motion in terms of an initial impetus rather than a constantly necessary force in contact with the moving object.[4] This reconceptualization of motion freed it of assumptions which were mathematically cumbersome. Thus, the choice of a spatial model for a manifold of social psychological objects was affected by the desire not to hinder the social manifold with assumptions which were mathematically burdensome. It is necessary to understand that this model neither ensures nor requires the data to conform to the spatial model and the data did not completely conform to the model. The degree to which the inconsistencies between the data and model were a matter of error, sample inadequacies, or the inappropriateness of the spatial model is not clear. However, the data results, as will be discussed further shortly, seemed to indicate some degree of stability and small dimensionality. It is also necessary to understand that while there are important mathematical reasons to support the use of the present spatial model, there is nothing to prevent the adoption of an alternative spatial model if the empirical evidence indicates the latter's possible superiority for reflecting simply behavioral relationships.

To reiterate the model underlying the development of a social manifold was of a space which was homogeneous, isotropic, continuous, metric and linear. Chapter two examined the available technical procedures which bore upon the construction of a manifold for social psychological objects. These procedures were examined both in terms of their own attributes (as isolated techniques) and own past uses as well as in terms of the criteria identified above as desirable for a social manifold. Factor analysis, the Semantic Differential, metric multidimensional scaling, and nonmetric multidimensional scaling were all described and evaluated. The advisability of factor analyzing the cross products as opposed to the scalar products of distances was examined at length. The evaluation was done both theoretically, in terms of the underlying mathematical assumptions of factor analyzing cross products -- i.e., orthogonality -- as well as empirically, in terms of evidence based upon the inadequacy of the results of a factor analysis of the cross products of a symmetric distance matrix of the eight corners of an oblong solid. This extended criticism was necessary in the light of a noted psychometrician's support for factor analysis of cross products as well as his misleading use of the term cross products as a substitution for scalar products.[5] They are the same however, only when calculating the cross products of two vectors coordinates or loadings on the axes (i.e., basis vectors or orthogonal dimensions accounting for all variance). It should be clear that factor analysis of cross products of a distance matrix was considered an inadequate way to construct the social manifold. It is important to understand that the typical factor analysis package if permitting more than correlation will include an option for computing the cross products but not the scalar products of raw data.

Factor analysis in general was described as suffering from its close association with its past applications; psychologists were first using factor analysis to try to determine if what they were measuring with intelligence tests was unidimensional or really composed of several traits. One of the principal applications of factor analysis has been by psychologists who were administering n number of tests and wanted to know if they could find less than n dimensions, or factors, which would help indicate the underlying structure. Axes were often treated as if they embodied a dimension or variable with loading on a factor being taken in isolation as a measure of the degree to which the factor explained a variable. The inadequacy of conceiving of an axis as embodying some psychological dimension is discussed in chapter two and in chapter four which reports and analyzes the findings of this preliminary attempt at constructing a social manifold. It is chapter four which includes primarily the factor analysis of the respective distance matrix's of an oblong solid and a face. These tests were meant as empirical evidence of the similarity of the resulting factor matrix and factor loadings to the axes and coordinates of a coordinate system.

The nature of the typical applications of factor analysis has been essentially such as to construct social spaces which are heterogeneous, anisotropic, and discrete, and thus inconsistent with the criteria specified as desirable in the construction of a social manifold. This problem of past applications is increased by the fundamental problem that factor analysis programs will include an option for cross products (since it must be computed in the correlation program anyway) but not for scalar products.

The Semantic Differential was devised in an attempt to "measure meaning" or, more specifically, determine the underlying structure or dimensions of the semantic space. The question asked was, what psychological dimensions are used in ordering the relations between concepts. Osgood, et. al., have indicated that the semantic space is dominated by three scales -- evaluative, potency, and activity. The space is described as being spanned by polar adjective scales. At the technical level, there is question as to the accuracy of this description since the dimensions are actually passed through the concepts. And since it is really the concepts which the dimensions are passed through which are orthogonal, the dimensions of evaluation, activity, and potency (abstracted out of the polar adjective scales) are actually correlated empirically.

Besides these technical problems with the method of constructing a social space, procedurally the space is bounded by an imposed polar unit of measurement. The space as conceived is discrete, heterogeneous, and anisotropic.

Multidimensional scaling, metric and nonmetric, was also examined. This technique's ability to construct a configuration from only the distances between pairs of objects represents an advance over factor analysis. Young and Householder's geometric derivation from any third point was converted by Torgerson into formula utilizing cross products to express the same essentially geometric formula which also used the average of all n points to replace any particular point i.

Completely nonmetric methods of multidimensional scaling were considered inappropriate for a construction of a social space according to the criteria specified basically because they ignore information - from the input data - of greater than an ordinal level. This was inconsistent with a data collection technique (i.e., the galileo questionnaire) which asked respondents for direct magnitude judgements on a ratio basis. Generally, metric methods of multidimensional scaling seemed most appropriate for the construction of a social manifold as specified by the criteria given. In the Galileo System, metric multidimensional scaling is directed at the analysis of sociological questions:

what is the configuration of social objects in the conceptual (and perhaps behavioral) space of an aggregate of similar individuals; what are the dynamics of these social objects? In this work, the major concern in chapters two through four has been with the necessary preliminary steps of evaluating possible methods for the construction of the social manifold, constructing the space according to the criteria specified, and evaluating the results.

Chapter Three is concerned specifically with how the desired social manifold -- the Galileo System -- was constructed. Both the logic of inquiry underlying the preliminary spacial studies and the specific technical procedures followed are described. The significance of the method of data collection was discussed. Direct estimations of the distances between pairs of objects was utilized which provided the advantages of both direct magnitude judgement by the subject and also the method of comparative judgement. The scale of galileo distances is continuous with a natural zero point existing in terms of no galileos apart or zero distance separation between a pair of objects. The subject is given the distance of one galileo apart between sitting and lying down in order to provide a frame of reference. Other distance estimations were then expected to be estimated proportionally. The Galileo Questionnaire was thus devised to facillitate the development of a ratio scale. The problems involved in the choice of objects to impose a frame of reference is discussed at length.

The sample the Galileo Questionnaires were given to was five social science classes, each of which received from one to three administrations of the questionnaire. The samples were more similar than a random aggregate of individuals would be, however, they were not social groups which may have affected the data results. The samples were large with the intention that error would cancel itself out. The technical procedure followed in the analysis of the data was: 1. constructing a matrix of mean distances between all pairs of objects; 2. converting the distance matrix into a matrix of scalar products; 3. performing a principal axis factor analysis on the matrix of scalar products; 4. rotation of pairs of manifolds (resulting from different test administrations) to a least squares best fit on a common origin.

Chapter four presents the data results from the galileo administrations. A substantial portion of this section is devoted to demonstrating how idiosyncracies and/or inconsistencies in the data (distances and mean distances between pairs of objects) interact with the technical procedures of scalar products, factor analysis, and the rotation program, to result in distorted configurations of points (i.e., social psychological objects). The configuration constructed often conceals the more typical, stable and patterned relationships which can be seen by scrutinizing the mean distance matrices. These problems bear directly, of course, on one of the principal intentions of this work -- the assessment of the reliability and face validity of the Galileo System (the procedure devised for the construction of a manifold for social psychological objects).

It was hypothesized (1) at the very beginnings of this work that overall, the multidimensional configurations of social objects would be relatively stable across time, i.e., administrations. It was reasoned that if the existence of a shared conceptual space underlies joint behavior, at least some moderate degree of stability should characterize the reflections of that shared conceptual space if the orderliness of behavior across time is to be explained and/or expressed through the resulting "pictures" of social space. Likewise, the hypothesis (3) that movement of objects in space will be orderly was meant as a preliminary indication as to the face validity of the spatial configuration over time. The preliminary assessment of the Galileo System's utility as a measure of lawfulness in human behavior was intended. The procedure for evaluation of possible "lawfulness" of object's motion (apart from the question of stability) was problematic. Analysis centered upon the "reasonableness" of the movement of social psychological objects. The potential appropriateness of this type of procedure could be questioned in terms of its often psychological emphasis. The influences or forces acting upon the changes in the configuration of social psychological objects may be reflecting social factors acting on the aggregate over time and not immediately apparent or interpretable in terms of social psychology. Some degree of disorderly movement was not considered as inconsistent with the basic lawfulness of the social manifold. Objects such as war, abstract for many of the respondents, and far removed from their everyday lives and behavior, was particularly irregular with regard to its distances from the other objects in the social manifold. Highly unstable cases (there were not a great many of them) such as war were interpreted in the context of the much greater stability of most of the objects in the social manifold. In many cases the erratic pattern was associated with the particular object rather than as characterizing the social manifold as a whole.

The stability of objects in the social manifold was interpretable in three ways: 1.)the mean distance matrix could be scrutinized with regard to both stability between pairs of social objects across time as well as to stability of object clusters; 2.) plots of coordinates (in the rotation program) or loadings (in the factor analysis) could be used as a basis for comparing the configurations of social objects resulting from the different administrations; 3.) the mean least squares distance between object configurations resulting from different administrations could be examined. This last procedure was formalized in the form of a hypothesis (2) that "the total amount of movement in the manifold -- measured in terms of the distance between the same object at different points in time (different administrations) -- should not be great."

The mean least squares distance between the thirty-three behavior administrations was quite large although the distance was not great for the administrations using the ten behavior Galileo Questionnaires. The large least squares distance reflect distortions caused by the technical procedures' sensitivity to error and/or to a small number of inconsistencies involving very large numbers (i.e., in the mean distance matrix). There is an extensive analysis of this problem in chapter four. However, stability as indicated by procedure number one -- i.e., via the mean distance matrix -- is greatly underestimated and distorted in the configurations and thus plots (procedure two) based on the coordinates and loadings of the rotation and factor analysis respectively are also distorted. This, of course, results in apparent instability according to procedure three (the estimation of least squares distance between the configurations of social objects) appearing much greater than instability according to procedure one (observation of the mean distance matrices).

Chapter four also examines the data results relevant to the determination of the dimensionality of the social manifold. In chapter one it was proposed (Hypothesis 4) that the dimensions or axes required to present the relations between the objects in the social manifold would be few, perhaps as few as three, permitting ease of graphic representation. This latter limitation to three dimensions was not necessary to the construction of a social manifold. Just as analytic geometry (geometry in a coordinate system in very simple terms) is not confined to three dimensions neither is the Galileo System necessarily confined to three dimensions.

In chapter four, the procedure for determining dimensionality in the Galileo System, TORSCA (i.e., the young program for semi-nonmetric multidimensional scaling), and factor analysis of the cross products of a distance matrix are compared. The distance matrix of known configurations (i.e., an oblong solid and face) and thus also known dimensionality is used to test the above procedures' adequacy for assessing dimensionality. The effect of imposed error in the given distance matrices upon the assessment of dimensionality was also observed. With regard to the perfect distance matrix of the oblong solid, both the Galileo System and Torsca reported the correct dimensionality and reconstructed perfectly the oblong solid. Factor analysis of the cross products failed to report the correct dimensionality even for the perfect matrix and also could not reproduce the oblong solid. The addition of error created difficulties for all three procedures, although the cross products was still by far the least adequate. Error affected the galileo procedures by reporting all eight points as factors, however the variance explained dropped sharply after the third factor. Analogously, in constructing the social manifold for the Galileo System -- wherein there is of course error -- dimensionality was taken to be the number of factors reported up until the point where the variance explained drops sharply. This procedure became somewhat problematic under the circumstances of the presence of distortions in the factor loadings themselves. (These are the same distortions which were discussed above.) With TORSCA, the reported level of the Index of Fit and Kruskal's Stress are the only indicators which aid in the assessment of the user's choice of dimensionality. Error in the data however, leads to a greater number of dimensions being needed to achieve satisfactory stress, making it difficult to assess correctly the dimensionality.

Finally, one would have to conclude by saying that it is too early and the results too preliminary to make a final evaluation of the goal of constructing a social manifold -- an aggregate or group's conceptual space of social objects which has implications for the aggregate or group's behavioral space. Difficulties were encountered with all procedures. However, the stability of the objects in the social manifold was sufficient, considering many of adversities associated with the procedures, to encourage continued study. Moreover, the jointly theoretical and methodological questions posed in this work are of sufficient magnitude as to warrant further work in this area despite technical difficulties.

FOOTNOTES

[1]E. T. Bell, "The Beginning of Modern Mathematics, 1637-1687," in Robert Marks (ed.), The Growth of Mathematics (New York: Bantam Books, 1964), p. 141.

[2]Ibid; p. 133.

[3]E. T. Bell, "Newton," in Marks (ed.), op. cit., p. 172.

[4]See Herbert Butterfield, The Origins of Modern Science (New York: Free Press, 1957).

[5]See Jum Nunnally, Psychometric Theory (New York: McGraw-Hill, 1967).

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```
FORTRAN IV G LEVEL  18                    MAIN                    DATE =

          C      MEANS MATRIX PROGRAM
          C      TIME 5
                 INTEGER SUM(33,33)/1089*0/,JJ(8),KK(8),X(8),N(
                 REAL*8 DAVE(33,33)/1089*0.D0/
          10     READ(5,1,END=25) (JJ(I),KK(I),X(I),I=1,8)
                 DO 14 I=1,8
                 J=JJ(I)
                 IF(J .EQ. 0) GO TO 12
                 IF(J.GT.33)GO TO 12
                 K=KK(I)
                 IF(K.GT.33)GO TO 12
                 SUM(J,K)=SUM(J,K)+X(I)
                 N(J,K)=N(J,K)+1
          14     CONTINUE
          12     CONTINUE
                 GO TO 10
          25     DO 20 J=1,33
                 DO 20 K=1,33
                 IF(N(J,K) .EQ. 0) GO TO 20
                 DAVE(J,K)=SUM(J,K)/(1.0D0*N(J,K))
          20     CONTINUE
                 DO 30 I=1,3
                 K1=11*(I-1)+1
                 K2=K1+10
                 WRITE(6,2) (K,K=K1,K2)
                 DO 30 J=K1,33
                 K3=J
                 IF(K3 .GE. K2) K3=K2
          30     WRITE(6,3) J,(DAVE(J,K),K=K1,K3)
          1      FORMAT(9X,8(2I2,I5))
          2      FORMAT('1',11I11)
          3      FORMAT(I3,11F11.3)
                 DO 100 I=1,33
                 WRITE(7,105) (DAVE(I,J), J=1,33)
          105    FORMAT (7F9.3/4F9.3)
          100    CONTINUE
                 STOP
                 END
```

```
EVEL 18                     MAIN                    DATE = 71183          22/49/25

      MEANS MATRX PROGRAM
      TIME 5
      INTEGER SUM(33,33)/1089*0/,JJ(8),KK(8),X(8),N(33,33)/1089*0/
      REAL*8 DAVE(33,33)/1089*0.D0/
0     READ(5,1,END=25) (JJ(I),KK(I),X(I),I=1,8)
      DO 14 I=1,8
      J=JJ(I)
      IF(J .EQ. 0) GO TO 12
      IF(J.GT.33)GO TO 12
      K=KK(I)
      IF(K.GT.33)GO TO 12
      SUM(J,K)=SUM(J,K)+X(I)
      N(J,K)=N(J,K)+1
4     CONTINUE
2     CONTINUE
      GO TO 10
5     DO 20 J=1,33
      DO 20 K=1,33
      IF(N(J,K) .EQ. 0) GO TO 20
      DAVE(J,K)=SUM(J,K)/(1.0D0*N(J,K))
0     CONTINUE
      DO 30 I=1,3
      K1=11*(I-1)+1
      K2=K1+10
      WRITE(6,2) (K,K=K1,K2)
      DO 30 J=K1,33
      K3=J
      IF(K3 .GE. K2) K3=K2
      WRITE(6,3) J,(DAVE(J,K),K=K1,K3)
      FORMAT(9X,8(2I2,I5))
      FORMAT('1 ',11I11)
      FORMAT(I3,11F11.3)
      DO 100 I=1,33
      WRITE(7,105) (DAVE(I,J), J=1,33)
05    FORMAT (7F9.3/4F9.3)
00    CONTINUE
      STOP
      END
```

```
C     SCALAR PRODUCTS
      REAL*8 RDSQ(33,33)/1089*0.0/, SSDSQ, SDSQJ(33)
     1SDSQK(33)/33*0.0/,RBSTAR(33,33) /1089*0.0/
      DIMENSION RD(33,33)
      N=8
      SSDSQ =0.0
      DO 10 J=1,N
      READ (5,20)(RD(J,K),K=1,N)
   20 FORMAT  (8F6.4)
   10 CONTINUE
      DO 1000 J=1,N
      DO 1000 K=1,N
 1000 RDSQ(J,K)=RD(J,K)**2
      DO 11 J=1,N
      DO 11 K=J,N
      RDSQ(J,K)=RDSQ(K,J)
   11 CONTINUE
      DO 12 J=1,N
      DO 12 K=1,N
       SSDSQ=SSDSQ  + RDSQ(J,K)
   12 SDSQJ(J)= SDSQJ(J) +RDSQ(J,K)
      DO 9 K=1,N
      DO 9 J=1,N
      SDSQK(K)= SDSQK(K) + RDSQ (J,K)
    9 CONTINUE
      DO 4 J=1,N
      DO 4 K= 1,N
      RBSTAR(J,K) = ((SDSQJ(J)/N) + (SDSQK(K)/N)- (SS
     1 (J,K))/2
    4 CONTINUE
      DO 100 I=1,N
      WRITE(7,105)(RBSTAR(I,J),J=1,N)
  105 FORMAT (4F12.5/4F12.5)
  100 CONTINUE
      DO 30 I=1,3
      K1=11*(I-1)+1
      K2=K1+10
      WRITE(6,2) (K,K=K1,K2)
      DO 30 J=K1,33
      K3=J
      IF(K3 .GE. K2) K3=K2
   30 WRITE(6,3) J,(RBSTAR(J,K),K=K1,K3)
    1 FORMAT(8X,8(2I2,I5))
    2 FORMAT ('1',7I11/' ',4I11)
    3 FORMAT(I3,4F17.3/4F17.3/3F17.3)
      STOP
      END
```

```
R PRODUCTS
B RDSQ(33,33)/1089*0.0/,  SSDSQ, SDSQJ(33)  /33*0.0/,
(33)/33*0.0/,RBSTAR(33,33) /1089*0.0/
SION RD(33,33)

 =0.0
 J=1,N
(5,20)(RD(J,K),K=1,N)
T  (8F6.4)
NUE
00 J=1,N
00 K=1,N
J,K)=RD(J,K)**2
 J=1,N
 K=J,N
J,K)=RDSQ(K,J)
NUE
 J=1,N
 K=1,N
D=SSDSQ   + RDSQ(J,K)
(J)= SDSQJ(J) +RDSQ(J,K)
K=1,N
J=1,N
K)= SDSQK(K) + RDSQ (J,K)
NUE
J=1,N
= 1,N
(J,K) = ((SDSQJ(J)/N) + (SDSQK(K)/N)- (SSDSQ/N**2) -RDSQ
)/2
UE
 I=1,N
7,105)(RBSTAR(I,J),J=1,N)
 (4F12.5/4F12.5)
UE
I=1,3
(I-1)+1
10
6,2) (K,K=K1,K2)
 J=K1,33

.GE. K2) K3=K2
(6,3) J,(RBSTAR(J,K),K=K1,K3)
T(8X,8(2I2,I5))
T ('1',7I11/' ',4I11)
T(I3,4F17.3/4F17.3/3F17.3)
```

```
SOUPAC OPTIONS=EXECUTE,LIST,PGM,NOLET,NODYNAM

          1        PRIN (C)(S2/P)(7)(100).
          2        MAT.
             1        PUN (S2)"(7F11.5)".
             2        OUT (S2)(I).
                      ENDP
                   ENDS

       6 PROGRAM CARDS HAVE BEEN READ BY SOUPAC

 EXECUTE TIME FOR SYNTAX INTERPRETER PROGRAM IS 0.41 SECO
 * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

PRINCIPAL AXIS FACTOR ANALYSIS

ROTATION PROGRAM I

FORTRAN IV G LEVEL 18 MAIN DATE = 71

```
      REAL*8 X(50),Y(50),Z(50),XP(50),YP(50),ZP(50),TH
      M=1
      READ(5,1) MIDDLE,THETA,N,NSETS
    1 FORMAT(I3,F5.2,2I3)
      WRITE(6,361)
  361 FORMAT('1',50X,'WELCOME TO THE GALILEO SYSTEM',
      WRITE(6,362)MIDDLE, THETA
  362 FORMAT (' THE ORIGIN IS BEHAVIOR NUMBER',I3,//
     1TATION IS',F5.2,' DEGREES')
      THETA=THETA*3.1416/180
      DO 100 I=1,N
      READ(5,4) X(I),Y(I),Z(I)
  100 CONTINUE
    4 FORMAT(3F11.5)
      CALL MATCH (X,Y,Z,MIDDLE,N)
  105 DO 102 I=1,N
  102 READ(5,4) XP(I),YP(I),ZP(I)
      CALL MATCH (XP,YP,ZP,MIDDLE,N)
      CALL ROTATE (XP,YP,X,Y,THETA,N)
      CALL ROTATE (XP,ZP,X,Z,THETA,N)
      CALL ROTATE (YP,ZP,Y,Z,THETA,N)
      SDSQ=0.0
      DO 2 I=1,N
      DSQ=(X(I)-XP(I))**2+(Y(I)-YP(I))**2+(Z(I)-ZP(I)
      SDSQ=SDSQ+DSQ
    2 CONTINUE
      SDSQ=SDSQ/N**2
      WRITE(6,201) M
  201 FORMAT('1',39X,'THE COORDINATES OF MANIFOLD NUM
      DO 3 I=1,N
    3 WRITE(6,200) I,XP(I),YP(I),ZP(I)
  200 FORMAT(' ',32X,I5,2X,3(F11.5,5X))
      WRITE(6,203)M,SDSQ
  203 FORMAT(//,4X,'THE MEAN SQUARED DISTANCE BETWEEN
     1I4,'AND THE CRITERION MANIFOLD IS',3X,F11.5)
      M=M+1
      IF(M.LE.NSETS) GO TO 105
      WRITE(6,202)
  202 FORMAT('1',37X,'THE COORDINATES OF THE CRITERIO
      DO 7 I=1,N
    7 WRITE(6,200) I,X(I),Y(I),Z(I)
      WRITE(6,363)
  363 FORMAT(' ',/////,44X,'END PROGRAM GALILEO',////
      STOP
      END
```

ROTATION PROGRAM I

MAIN DATE = 71202 14/37/12 PA

```
B X(50),Y(50),Z(50),XP(50),YP(50),ZP(50),THETA
5,1) MIDDLE,THETA,N,NSETS
T(I3,F5.2,2I3)
(6,361)
T('1',50X,'WELCOME TO THE GALILEO SYSTEM',//////////)
(6,362)MIDDLE, THETA
T (' THE ORIGIN IS BEHAVIOR NUMBER',I3,//,' THE ANGLE OF RO
N IS',F5.2,' DEGREES')
=THETA*3.1416/180
O I=1,N
5,4) X(I),Y(I),Z(I)
NUE
T(3F11.5)
MATCH (X,Y,Z,MIDDLE,N)
2 I=1,N
5,4) XP(I),YP(I),ZP(I)
MATCH (XP,YP,ZP,MIDCLE,N)
ROTATE (XP,YP,X,Y,THETA,N)
ROTATE (XP,ZP,X,Z,THETA,N)
ROTATE (YP,ZP,Y,Z,THETA,N)
0.0
I=1,N
X(I)-XP(I))**2+(Y(I)-YP(I))**2+(Z(I)-ZP(I))**2
SDSQ+DSQ
NUE
SDSQ/N**2
(6,201) M
T('1',39X,'THE COORCINATES OF MANIFOLD NUMBER', I6//)
I=1,N
(6,200) I,XP(I),YP(I),ZP(I)
T(' ',32X,I5,2X,3(F11.5,5X))
(6,203)M,SDSQ
T(//,4X,'THE MEAN SQUARED DISTANCE BETWEEN MANIFOLD NUMBER',
ND THE CRITERION MANIFOLD IS',3X,F11.5)
LE.NSETS) GO TO 105
(6,202)
T('1',37X,'THE COORDINATES OF THE CRITERION MANIFOLD',//)
I=1,N
(6,200) I,X(I),Y(I),Z(I)
(6,303)
T(' ',/////,44X,'END PROGRAM GALILEO',//////////////////////)
```

FORTRAN IV G LEVEL 18 ROTATE DATE = 71

```
      SUBROUTINE ROTATE (A,B,C,D,ALPHA,J)
      REAL*8 A(J),B(J),C(J),D(J),ALPHA,DSIN,DCOS,AP(5
      SDSQ=0.0
      SDSQ0=0.0
  701 DO 702 I=1,J
      DSQ=(C(I)-A(I))**2+(D(I)-B(I))**2
  702 SDSQ0=SDSQ0+DSQ
      GO TO 705
  704 SDSQ0=SDSQ
      SDSQ=0.0
  705 DO 703 I=1,J
      AP(I) = A(I)*DCOS (ALPHA)-B (I)*DSIN (ALPHA)
      BP(I) = A(I)*DSIN (ALPHA)+B (I)*DCOS (ALPHA)
      A(I)=AP(I)
      B(I)=BP(I)
      DSQ=(C(I)-A(I))**2+(D(I)-B(I))**2
      SDSQ =SDSQ +DSQ
  703 CONTINUE
      IF(SDSQ.LT.SDSQ0) GO TO 704
  706 SDSQ0=SDSQ
      SDSQ=0.0
  707 DO 708 I=1,J
      AP(I) = A(I)*DCOS (ALPHA)+B (I)*DSIN (ALPHA)
      BP(I) = B(I)*DCOS (ALPHA)-A (I)*DSIN (ALPHA)
      A(I)=AP(I)
      B(I)=BP(I)
      DSQ=(C(I)-A(I))**2+(D(I)-B(I))**2
      SDSQ =SDSQ +DSQ
  708 CONTINUE
      IF(SDSQ.LT.SDSQ0) GO TO 706
      RETURN
      END
```

II

ROTATE DATE = 71202 14/37/12 PA

```
UTINE ROTATE (A,B,C,C,ALPHA,J)
8 A(J),B(J),C(J),D(J),ALPHA,DSIN,DCOS,AP(50),BP(50)
D.0
=0.0
2 I=1,J
C(I)-A(I))**2+(D(I)-B(I))**2
=SDSQO+DSQ
 705
=SDSQ
G.0
3 I=1,J
 = A(I)*DCOS (ALPHA)-B (I)*DSIN (ALPHA)
 = A(I)*DSIN (ALPHA)+B (I)*DCOS (ALPHA)
AP(I)
BP(I)
C(I)-A(I))**2+(D(I)-B(I))**2
=SDSQ +DSQ
NUE
SQ.LT.SDSQO) GO TO 704
=SDSQ
D.0
8 I=1,J
 = A(I)*DCOS (ALPHA)+B (I)*DSIN (ALPHA)
 = B(I)*DCOS (ALPHA)-A (I)*DSIN (ALPHA)
AP(I)
BP(I)
(I)-A(I))**2+(D(I)-B(I))**2
SDSQ +DSQ
UE
Q.LT.SDSQO) GO TO 706
```

III

FORTRAN IV G LEVEL 18 MATCH

```
      SUBROUTINE MATCH(A,B,C,MID,K)
      REAL*8 A(K),B(K),C(K)
      AMID=A(MID)
      BMID=B(MID)
      CMID=C(MID)
      DO 601 I=1,K
      A(I)=A(I)-AMID
      B(I)=B(I)-BMID
      C(I)=C(I)-CMID
  601 CONTINUE
      RETURN
      END
```

WELCOME TO SOUP

CROSS PRODUCTS AND FACTOR ANALYSIS

```
SOUPAC OPTIONS=EXECUTE,LIST,PGM,NOLET,NODYNAM

              1        MAT.  OBLONG SOLID
                 1         MOVE(C)(S1).      STORE MEANS
                           END P
              2        COR(S1)( )(P)(S2/P).        CROSS PRODUCTS MATRIX
              3        PRI(S2)(S6/PRINT(F))(3)(100).
              4        VARIMAX (S6) (PRINT(F)).
                       END SOUPAC

        7 PROGRAM CARDS HAVE BEEN READ BY SOUPAC

EXECUTE TIME FOR  S Y N T A X   I N T E P R E T E R  PROGRAM IS 0.41 SEC
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

WELCOME TO SOUPAC

S PRODUCTS AND FACTOR ANALYSIS

```
,PGM,NOLET,NODYNAM

 OBLONG SOLID
MOVE(C)(S1).     STORE MEANS
END P
S1)( )(P)(S2/P).     CROSS PRODUCTS MATRIX
S2)(S6/PRINT(F))(3)(100).
MAX (S6) (PRINT(F)).
SOUPAC

VE BEEN READ BY SOUPAC

X  I N T E P R E T E R  PROGRAM IS 0.41 SECONDS.
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

TORSCA
YOUNG-TORGERSON NONMETRIC M
VERSION 9 (O
PSYCHOMETRIC LABORATORY, UNI

```
PROBLM CARD PROCESSED
   33 STIMULI
    3 MINIMUM DIMENSIONS
    4 MAXIMUM DIMENSIONS
  100 ITERATIONS (PART 1)
  100 ITERATIONS (PART 2)
      INPUT ON TAPE  5
      MINKOWSKI CONSTANT =   2.00000
      SYMMETRIC MATRIX INPUT MODE
      PLOTS REQUESTED
      PUNCHED OUTPUT REQUESTED
      TITLE:: MULTI-DIMENS SCALING--GALILEC-TORSCA    TIME 1
      FORMAT: (4(8F9.3/),1F9.3)
```

TORSCA 9

YOUNG-TORGERSON NONMETRIC MULTIDIMENSIONAL SCALING

VERSION 9 (03/16/68)

PSYCHOMETRIC LABORATORY, UNIVERSITY OF NORTH CAROLINA

)

)

= 2.00000

NPUT MODE

UESTED

NS SCALING--GALILEO-TORSCA TIME 1

,1F9.3)

VITA

Gail Rae Wisan was born on August 4, 1945 in New York City. She received a B. A. from Hunter College in February of 1966. During that month she began a period of three semesters of teaching in the New York City public school system. In September of 1967 she entered the Graduate College of the University of Illinois at Urbana-Champaign. She was awarded the M. A. degree in sociology in 1969. During 1967-68 she was a research assistant; from 1969 to 1971 she was a teaching assistant in the Department of Sociology at the University of Illinois. In the fall of 1971 she began teaching at Long Island University; there she is an assistant professor and a member of the Graduate Faculty.