A MULTI-DIMENSIONAL SCALING BASED PROCEDURE FOR DESIGNING PERSUASIVE MESSAGES AND MEASURING THEIR EFFECTS[1]

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Some psychometricians and market researchers have used multidimensional scaling solutions as a basis for designing message strategies. In a typical case, researchers will derive (by any of a variety of MDS routines), a configuration which includes the concepts to be manipulated (such as a product, service or political candidate), a set of attributes or descriptors of the concept, and some "ideal point" such as "the car I would buy," or "the ideal congressman," or a concept of self ("me"). Based on a careful examination of the configuration, a message strategy is designed which attaches attributes to the manipulated concept such that, in the researcher's judgment, recipients of the message will judge the concept maximally close to the ideal point.

A typical example is provided by Barnett, et al. (1976). Based on a telephone survey of a sample of voters in a congressional election, Barnett, et al. identified five key issues in the campaign. These issues, along with the names of the candidates, their political parities and the concept "me," were compiled into a complete pair-comparison questionnaire administered to 307 registered voters in the congressional district at three points in time prior to the election.

Based on a careful visual scrutiny of this plot (in several rotations), Barnett et al. suggested that the candidate attach the attributes "law and order" and Democrat" to his name as a means of moving closer to the concept "me." subsequent measures showed this strategy to be roughly successful; a reanalysis of these data by Serota, et al. (1977) showed the candidate concept in fact moved at an angle of about 30^{o} from the resultant of the component vectors "Democrat" and "law and order," increased his vote total by doing so, and won the election decisively against a popular incumbent in a state where no other incumbent was defeated, even though his opponent outspent him on the campaign by a factor of 3.

Depsite its apparent effectiveness, this procedure is flawed by its visual and intuitive character. In the experience of the present authors, visualizations of multidimensional configurations are frequently misleading, and in fact, Serota, et al.'s reanalysis shows the message chosen in this case yielded a resultant vector about 44^{o} off from the theoretically optimal strategy. The present article presents a simple mathematical procedure for designing messages of this sort based on falsifiable elementary assumptions, along with unambiguous mathematical procedures for measuring the effectiveness of these messages.

Theory

We begin by defining the vector space R_α^μ where each of the contravariant vectors R_α^μ represents the projections of the $\underline{\alpha}^{th}$ concept on a set of covariant (basis) unit vectors e_μ.[2] In practice we expect the R_α^μ to be the result of a multidimensional scaling analysis of a set of proximities data for $\underline{k}$ concepts where $\underline{r}$ is the number of dimensions retained. Therefore, we allow α to range from 1 to $\underline{k}$ and μ from 1 to $\underline{r}$.

We further designate the concept to be moved or manipulated (the "start" concept) as R_s^μ and the ideal point toward which it is to be moved as the "target" concept R_t^μ. The goal then becomes to move the start concept along the target vector R_t^μ - R_s^μ. For convenience, we first recenter the coordinate system with the start concept R_s^μ on the origin by the translation:

$$R_\alpha^\mu = \bar{R}_\alpha^\mu - \bar{R}_s^\mu \qquad (1)$$

where, R_α^μ = the position vector of the α^{th} concept after recentering,

$\bar{R}_\alpha^\mu$ = the original position vector of the α^{th} concept,

$\bar{R}_s^\mu$ = the original position vector of the concept to be manipulated (the "start" concept),

$\alpha = 1, 2, \ldots, k,$

$\mu = 1, 2, \ldots, r,$

Since R_s (the magnitude or length of R_s^μ) now is zero, the target vector is given by R_t^μ, which is represented in Figure 1 as the "target vector."

While our understanding of the dynamics of such spaces is very rudimentary, the original Barnett, et al. (1976) procedure is motivated by a sinple dynamic assumption: when two concepts in the space are associated (formally, when they are linked in an assertion of the form "x in y") they converge relative to one another along the line segment connecting them. In Figure 1, the sentence "the candidate is friendly" should therefore result in a motion of the candidate concept along the vector R_p^μ. This vector is labeled R_p^μ (predicted vector) in Figure 1. As yet, insufficient data are available to warrant predictions of the magnitude of this motion, but its direction is clearly given from our starting assumption.

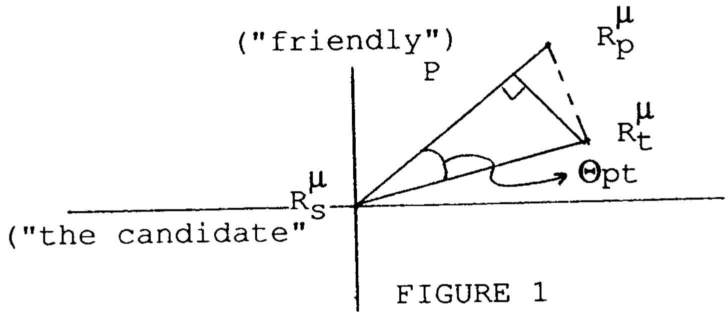

FIGURE 1

Hypothetical Representation of a Multidimensional Scaling Space

Based on this assumption, determination of a single optimal issued may be simply accomplished: first, the angle Θ_{pt} between any predicted vector $R_p{}^\mu$ and the target vector $R_t{}^\mu$ can be conveniently calculated as:

$$\Theta_{pt} = \cos^{-1}(g_{\mu\upsilon} R_p^\mu R_t^\upsilon R_p R_t) \qquad (2)$$

where,

$$R_p = /R_p^\mu/ = (g_{\mu\upsilon} R_p^\mu R_p^\upsilon)^{1/2} \qquad (3)$$

$$R_t = /R_t^\mu/ = (g_{\mu\upsilon} R_t^\mu R_t^\upsilon)^{1/2} \qquad (4)$$

and where the quantities $g_{\mu\upsilon}$ are given by the scalar products of the covariant basic vectors, i.e.

$$g_{\mu\upsilon} = e_\mu^j e_\upsilon^j \qquad (5)$$

The $g_{\mu\upsilon}$ can be shown to be a covariant tensor of the second rank which defines the metric properties of the space and is therefore referred to as the fundamental or "metric" tensor. If the covariant basis vectors e_μ are real and orthogonal, then the $g_{\mu\upsilon}$ take on the familiar form

$$g_{\mu\upsilon} = \delta_\upsilon^\mu = \begin{matrix} 1 \text{ if } \mu = \upsilon \\ 0 \text{ if } \mu \neq \upsilon \end{matrix}$$

That concept whose position vector forms the smallest angle with the target vector will represent the concept which lies most nearly in the direction of the "me" or ideal point. The amount of change advocated by this message strategy is given straightforwardly by the length of the predicted vector R_p, which is given by equation 3 above.

Although it is common practice for psychometricians to retain only real eigenvectors or dimensions, it is the prevailing practice of many communication researchers to perform metric analyses of ratio-scaled data averaged over very large samples, and most frequently all or nearly all eigenvectors (Woelfel, 1980; Woelfel and Danes, 1980).

Where the $\underline{p}^{th}$ through $\underline{r}^{th}$ roots are negative (corresponding to imaginary eigenvectors), the $g_{\mu\upsilon}$ are given by

$$g_{\mu\upsilon} = \begin{cases} 0 \text{ if } \mu \neq \upsilon \\ 1 \text{ if } \mu = \upsilon < p \\ -1 \text{ if } \mu = \upsilon \geq p \end{cases} \qquad (6)$$

Given these considerations, and equations 2 through 4, we can now solve any part of the triangle $R_s^\mu R_t^\mu R_p^\mu$ in Figure 1. If the message equating the start concept R_s^μ with the concept R_p^μ were completely successful, such that the concept represented by the end points of R_s^μ moved to the endpoint of R_p^μ, from the distance between the start concept and target concept (after the message) would be given by the distance $/R_p^\mu - R_t^\mu/$.

Such an outcome is very unlikely in most cases, since we could at best assume the point represented by R_s^μ would move only part of the distance toward R_p^μ. The point p in Figure 1 represents the orthogonal projection of R_t^μ on R_p^μ and gives the point of closest approach to R_t^μ.

This length of this line segment is given by

$$/PR_t^\mu/ = R_t \sin \Theta_{pt} \qquad (7)$$

where Θ_{pt} is as given in (2) above. Similarly, the distance along R_p^μ that the start concept must travel to reach P is given by

$$/PR_s^\mu/ = \frac{/PR_t^\mu/}{\tan \Theta_{pt}} \qquad (8)$$

The percentage of change advocated that must be achieved for this message to have its maximum effect is given simply by

$$\Delta \% \max = \frac{100}{R_p} \; \frac{/R_s^\mu/}{} \qquad (9)$$

These calculations, along with an empirically-measured estimate of the proportion of advocated change actually to be expected, provide ample data on the basis of which the optimal single issue may be chosen.

Multi-concept messages are very easily (and similarly) determined on the basis of an additional assumption: messages average like vectors in the space. This is equivalent to the assumption that order effects (like primacy-recency) are negligible over the life of the message campaign. Based on this assumption, the position vectors or any two or more issues may simply be averaged to yield a resultant vector given (for two vectors) by

$$R_p^\mu = (R_\alpha^\mu + R_\beta^\mu)/2 \qquad (10)$$

This resultant vector is then taken as the predicted vector and the procedures just described are repeated.

Equation (10) can easily be generalized for n vector sums, although the number of such combinations of possible messages grows very rapidly as n becomes large. (In practice, the Galileo_{TM} computer program with which we work computes all possible messages with up to four concepts to determine an "optimal message.")

Evaluation of the degree of success of the message strategy is also simply a matter of determining the angles included between the predicted, target and observed vectors over the time interval Δt. In practice, however, it is difficult to hold the origin of the space at the t + Δt precisely where it was at time t, and so it is convenient to choose yet a different origin. In our own work, we establish an origin at the centroid of the set of concepts not included or implicated in any message, and rotate the time t and time t + Δt spaces to least-squares best fit among only those unmanipulated concepts. This procedure may be seen as an effort to use the unmanipulated concepts to determine a stable frame of reference against which the relative motions of the manipulated concepts may be gauged. Time one variables transformed into these stable coordinates will be represented by double barred tensors (e.g., $R_{st1}{}^{\mu} = \bar{\bar{R}}_s{}^{\mu}$) and time two variables will be represented by hats (e.g., $R_{st2}{}^{\mu} = \hat{R}_s{}^{\mu}$) as shown in Figure 2.

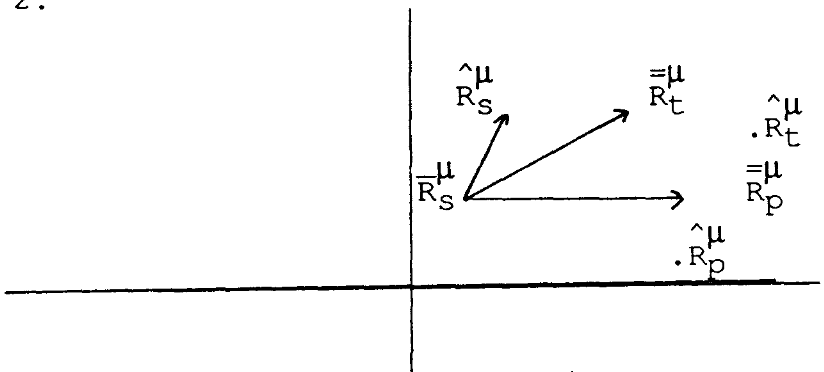

FIGURE 2
Multidimensional Scaling Space at time t and t + Dτ
Represented on Stabale Coordinates

Given these definitions we may define the predicted vector across the interval t as

$$R_p^{\mu} = \bar{R}_s^{\mu} - \bar{R}_p^{\mu} \qquad (11)$$

The target vector across Δt is defined as

$$R_t^{\mu} = \bar{R}_t^{\mu} - \bar{R}_s^{\mu} \qquad (12)$$

Similarly the observed motion vector is given by

$$R_o^{\mu} = \hat{R}_s^{\mu} - \bar{R}_s^{\mu} \qquad (13)$$

Evaluation of the extent to which the start concept has moved as predicted is given simply by the angle between the predicted and observed vector, which is given by

$$\Theta_{po} = \cos^{-1} (g_{\mu\upsilon} R_p^{\mu} R_o^{\upsilon} / R_p R_o) \qquad (14)$$

Also of interest is the extent to which the start concept has moved in the direction of the target, which is given by

$$\Theta_{to} = \cos^{-1} (g_{\mu\upsilon} R_t^{\mu} R_o^{\upsilon} / R_t R_o) \qquad (15)$$

Further Consideration

While these equations are sufficient to indicate the basic structure of the procedures, many valuable modifications can be derived easily by the interested reader. One such example is the unweighted summation of vectors in multiconcept messages given by equation (10) above, which assumes each concept to be equally effective. This assumption may be relaxed by providing weights β_{α} such that (10) is replaced by

$$R_p^{\mu} = \sum_{\alpha} \beta_{\alpha} R_{\alpha}^{\mu} / \sum_{\alpha} \beta_{\alpha} \quad , \qquad (16)$$

where the β_{α} are estimated empirically by the regression equation

$$R_o^{\mu} = \sum_{\alpha} \beta_{\alpha} R_{\alpha}^{\mu} + e_o \qquad (17)$$

where e_o is a least-squares error term.

The equations presented here, it may be noted, are all differences equations, reflecting the "before-after" or "treatment-control" designs typical of current practice. Clearly the emphasis on process implicit in this chapter suggests a much heavier emphasis on longitudinal designs. When such data sets become available, the transformation of these equations into differential form is strightforward, particularly when orthogonal MDS routines are chosen. Thus the infinitesimal displacement of the start vector ds_s is given by

$$ds_s = (g_{\mu\upsilon} dR_s^{\mu} dR_s^{\upsilon})^{1/2} \qquad (18)$$

where the dR_s^{μ} represent coordinate differentials. Similarly, the instantaneous velocity of the start vector at time t is given by

$$V_t = ds_s /dt \qquad (19)$$

and the instantaneous acceleration of the start concept at t is

$$a_t = d^2 s_s /dt^2 \qquad (20)$$

Summary

This chapter has presented a theory of cognitive change based on multidimensional scaling representations of concepts. Equations for devising messages to bring about specific effectes, and for evaluating these effects, were presented.

NOTES

1. An earlier version of this chapter was presented to the International Communication Association, Portland, OR, April 1976.

2. The tensor notation adopted here may seem initially cumbersome to those unfamiliar with it, but it is greatly simplifying for generalized coordinate systems where the reference axes may be of unique, imaginary or curved. It is also convenient to adopt the Einstein convention that repeated tensor indices are to be summed over so that sigma signs may be suppressed, i.e.,

$$g_{\mu\upsilon} \overset{\mu}{R}_p \overset{\upsilon}{R}_t = \sum_{\mu=1}^{r} \sum_{\upsilon=1}^{r} g_{m\upsilon} \overset{\mu}{R}_p \overset{\upsilon}{R}_t$$

under this convention.

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