Quality and Quantity, 13 (1979) 215–232

ON THE DIMENSIONALITY OF PSYCHOLOGICAL PROCESSES

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Implicit to the method of multidimensional scaling is the notion that any set of stimuli which make up a single domain may be described with little or no loss of information upon some limited subset of underlying dimensions. While it is axiomatic that N stimuli may be described with no information loss by $N-1$ dimensions [1] MDS attempts to "... determine the minimum dimensionality of the set ..." (Torgerson, 1958, p. 248).

Through the orthogonal decomposition of the matrix (A) the number (m) of eigenvalues (roots) is determined. For each m roots there exists exactly m linearly independent eigenvectors, one associated with each eigenvalue (Noble, 1969, p. 281). The null space results because all the rows or columns [2] are not linearly independent. One of the assumptions of MDS is that the stimuli share attributes in varying magnitudes and are therefore linear combinations of each other. By factor analyzing the scalar products matrix the proportion of the variance of each m dimensions and the scale values of each stimuli on these dimensions is determined. Perhaps due to measurement error or the variety of attributes which subjects may use to differentiate the stimuli, some degree of variance is usually explained by all $N-1$ dimensions. This makes the actual number of underlying dimensions unclear. Thus, what is mathematically straightforward becomes a matter of confusion for psychometricians. How then does one determine the minimum number of dimensions upon which the domain of stimuli may be differentiated?

Shepard (1974) identifies determining the proper number of dimensions as one of the major problems facing the future of MDS. The authors concur in this opinion. They will review static methods curfently used to the number of dimensions: the scree test, the measure of stress, and the interpretation of the dimensions. They suggest that all three are inadequate, and that determination of the "true" dimensionality of an MDS space requires information from outside the domain of concepts themselves. As a theoretical question, the rank of an MDS space cannot be determined *solely* by methodological procedures. Next, they will discuss the determination of the dimensionality of psychological processes, through the use of time-series metric scaling. Finally, the utility of this conceptualization will be demonstrated.

1. Static measures

The scree test

One method of determining the proper dimensionality of a space is the scree test (Cattell, 1966; Tatsuoka, 1971). It operates as follows. Plot the absolute values of the eigenroots for each dimension of the space. Then connect these values. The proper number of underlying dimensions is determined where there is a drastic change in the slope of the curve. This quantity is the number of dimensions which lie off the line connecting the smallest root to this point and includes the largest root on the scree line. The remaining dimensions theoretically represent measurement error. In Fig. 1, there would be a three-dimensional solution.

The major fault with the scree test is its arbitrary nature. While it tells one which dimensions account for more variance than the others, it does not do this in an exact way. Only the fact that there is a clear break in the size of the eigenvalue is used to determine the correct number of dimensions. While each dimension in this subset is larger than each of the remaining dimensions, the solution may exclude as much variance as the retained dimensions take into account. The reason is that while the chosen dimensions are larger individually, collectively the remaining dimensions may sum to a total that

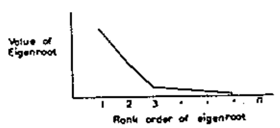

Fig. 1. The scree test.

approaches or even exceeds the sum of the chosen dimensions. Furthermore, the variance accounted for by a dimension may not indicate its relative importance for the theoretical issue under study; it only reflects the range of variation on the dimension for a set of stimuli.

Stress

Perhaps the most popular method for determining dimensionality is through Kruskal's (1964a,b) measure of stress. While the other methods discussed in this paper can be applied to both metric and nonmetric scaling, the utility of Kruskal's measure is limited to nonmetric. Stress is an indicator of goodness-of-fit between the obtained solution and the measured distances. A perfect solution (stress = 0.0%) indicates that there is a monotonic relationship between dissimilarities and the reconstructed distances. The criterion for selecting the proper number of dimensions is a solution where a minimum dimensionality is obtained in conjunction with a low stress value. This entails the production of a number of spaces of various dimensionality and then selecting the solution which is low in its number of dimensions and low in stress.

Generally, the stress measure decreases as the dimensionality increases. This suggests that one should apply the scree test to this measure. Ideally, the function relating stress to number of dimensions should drop abruptly to the proper number of dimensions and then decline only very slightly thereafter.

Kruskal (1964a) suggests the following evaluation of quality of solution, based on the stress measure.

Stress (%)	Goodness-of-fit
20	poor
10	fair
5	good
2.5	excellent
0.0	perfect

This method has been severely criticized by a number of researchers (Klahr, 1969; Stenson and Knoll, 1969; Young, 1970; Isaac and Poor, 1974; Cohen and Jones, 1974; Green, 1975). Young (1975) found that stress increased as the number of stimuli increased. He writes (1970, p. 471):

> If one relies heavily on the stress index the unfortunate situation exists that as he diligently gathers more and more data about an increasingly larger number

of stimuli, he will become less and less confident in the nonmetrically reconstructed configuration, even though it is more accurately describing the structure underlying the data. This situation suggests that one should not rely solely on the stress of a scaling solution when the solution is being evaluated. It has probably been suspected by most of those using nonmetric scaling that Kruskal's statement (1964a) that stress less than .05 is acceptable is too stringent. The results presented here support this suspicion. The results also suggest that one should not take stress too seriously as a measure indicating confidence in the results of the analysis. Stress should not really be taken as a confidence measure, but as a purely descriptive statistic, with μ (metric determinancy) being interpreted as the confidence statistic.

Based upon the above research, Green (1975, p. 75) makes the following comments concerning the use of stress to determine dimensionality:

1. The ratio of number of degrees of freedom for stimuli, $n(n-1)/2$, to number of degrees of freedom for solution, $4(n-1) - r(r-1)/2$ (where n is the number of stimuli and r is the number of dimensions) should be greater than 2.5, if at all possible. That is, with moderate degrees of error a ratio of 2.5 will generally overdetermine the solution well enough to provide good recovery.
2. Kruskal's stress measure overestimates goodness of recovery when too few points are used, when there is substantial error in the data, and when recovered dimensionality overestimates true dimensionality.
3. It is generally better to overestimate rather than underestimate the number of dimensions is one is interested primarily in configuration recovery with the least distortion.
4. Stress curves of the type proposed by Spence and Ogilvie (Stress vs. dimensionality for selected numbers of points) can provide a supplementary (and mechanical) measure for helping the researcher estimate dimensionality.
5. Kruskal's stress measure alone is not sufficient for determining the number of dimensions to be retained. At the very least, the dimensions that are retained should be interpretable substantively and reliable across subgroup or interoccasion scalings.

Interpretation

A third method which is used to determine the dimensionality of a multidimensional space is through the interpretation of the loadings on the dimensions. Can a meaningful substantive interpretation of the way the stimuli are arranged in a space of a given dimensionality be found? Shepard (1972) suggests that one should take the following factors into account when attempting to interpret a spatial representation. One should look for rotated or oblique *axes* that may be readily interpretable. Are there a set of *clusters* that can be interpreted? What other kinds of *orderly patterns* (such as the arrangement of the stimuli to form a circle, in the case of color names or chips) are there in the space?

Typically, the procedure is used in conjunction with the other methods. One attempts to strike the balance between an interpretable configuration and a good fit. It is considered undesirable to analyze the data in so many dimensions that they cannot be interpreted, even though the fit may be good. However, the important aspects of the configuration are not always in the first few dimensions. In some cases the higher dimensions may help clarify the interpretation. Thus, the rule of thumb as stated by Wish (1972, p. 2) is, "If an attribute or property can be fit well in n dimensions, but not in $n-1$ dimensions, then there is reason to keep the last dimension."

The orientation of the resultant dimensions is entirely arbitrary. As a result, additional techniques may be used to help with the interpretation of the spatial manifold. Rotation of the points to simple structure may facilitate the analysis of the configuration. This has become standard procedure in certain MDS programs (cf. KYST). Cluster analyses, such as Johnson's (1967) hierarchical cluster analysis, may also facilitate interpretation. Additionally, linear regression may be used to identify attributes in the space (Barnett, 1976; Jones and Young, 1972; Gillham and Woelfel, 1977). This technique is particularly useful in spaces of large dimensionality.

While the interpretation of the dimensions may facilitate the determination of the proper dimensionality of the space, it is not without drawbacks. Linn's (1968, p. 38) comments concerning the same problem in factor analysis are applicable here.

Although meaningfulness or scientific interpretability are important considerations in the development of scientific theory, it is not a very sound basis for the inclusion of a construct or factor in the scientific domain. It appears that the post hoc meaningfulness given to a factor is limited only by the analyst's ability to rationalize the obtained factor loadings. The second, and more respectable criterion of replicability, while basically sound, is limited by the lack of a good method of comparing factors. The usual "eye ball" method of comparing factors is subject to the same criticism as is the meaningfullness criterion.

The use of graphic representation may aid in interpreting dimensionality. This may also lead to erroneous conclusions. Only three dimensions at a time can be visualized. The immediate result is that solutions which require more than three dimensions cannot be examined in this manner. Thus, there is a tendency to limit solutions to two or three dimensions so that they can be visualized and readily interpreted. However, often solutions with greater than three dimensions are common, especially when large numbers of stimuli are scaled. Clearly, the use of visual aids in interpreting the spatial manifold and its use in determining the proper dimensionality has severe limitations.

Perhaps the most severe criticism of labelling dimensions is that the practice may be running against the spirit of the method. The use of MDS requires that the dimensions be taken into account simultaneously. By attaching an attribute label to one dimension of the space, it is implied that *all* the variance on the dimension could be explained by that attribute. This need not be the case. Often, there is not an isomorphic relation between attributes and dimensions. There may be several linearly related attributes which can fit in and serve in interpretation of a space. Also, some attributes such as color or culture may be multidimensional. A dimension refers *only* to one of a set of orthonormal reference vectors, which is the result of mathematical operations, and not to an attribute. Thus, the removal of a dimension from a solution because it cannot be labelled with an attribute should not be a criterion for determining dimensionality.

Metric determinacy

The ultimate criterion for determining the "true" dimensionality of a spatial manifold *as measured* is the ability of the dimensions of the space to reconstruct the original dissimilarity matrix. Under ideal conditions, a space of m dimensions should be equivalent to the original $n \times n$ distance matrix. This idenity can be shown

$$\underset{n\times m}{A} \cdot A^{T} = \underset{n\times n}{A} \tag{1}$$

The discrepancy between the two matrices may be considered the error which a spatial manifold of a particular dimensionality enters into the analysis. Thus eqn. (2) becomes,

$$\underset{n\times m}{A} \cdot A^{T} = \underset{n\times n}{A} + E \tag{2}$$

The degree of error may be determined by the variance unaccounted for by the measure of metric determinacy (μ) (Shepard, 1966; Young, (1970).

The index of metric determinacy (μ) is defined as the squared correlation between the true distances

$$(\underset{n\times m}{A})$$

and the reconstructed distances

$$(\underset{n\times m}{A} \cdot A^{T}).$$

It follows that,

$$\epsilon = 1 - \mu \tag{3}$$

where ϵ is the variance unaccounted for by a space of a particular dimensionality. This value equals zero when all $n-1$ dimensions are used, and it increases as the dimensionality decreases. The metric determinacy statistic, however, assumes that the measured distances (A) are true, reliable measures. If a random component is included in (A), then a value of 0 for μ only indicates that the MDS configuration perfectly reproduces the (partially) erroneous raw scores, error and all. The use of metric determinacy as a sole criterion, therefore, would always result in a complete decomposition of the distance matrix, whether such was warranted or not.

An illustration of static tests for dimension:

In February 1975, 15 subjects bilingual in French and English completed 45 direct pair comparisons between 10 different lexical items dealing with the mass media. The concept's language, English or French, was randomly assigned. The assumption under investigation was that a dimension would be present in the space which would differentiate the English words from the French. The data were aggregated to produce the lower triangle of the mean matrix presented in Table I. This matrix was then entered into a metric scaling program (Galileo 2.4, Serota, 1974) and a non-metric program (KYST). The spatial coordinates for the entire $N-1$ dimensions from the Galileo program are presented in Table II.

Next, the authors attempted to determine the proper dimensionality of this configuration using the methods described above. The scree test of the eigenroots from the metric data produced a three-dimensional solution as shown below (see Fig. 2). The measure of stress suggested that a six dimensional solution provided the best fit. This was arrived at through a scree test of the stress values. This is shown in Fig. 3. According to Kruskal (1964a), this solution (stress = .039) provides a good to excellent fit. Green (1975) suggested that for a space generated from 10 concepts that only two dimensions should be retained if the solution is to exceed 2.5 for ratio of the number of degrees of freedom for the stimuli, to the number of degrees of freedom for the solution [4].

At this point, an attempt was made to interpret the spatial manifold. The first dimension in both the metric and non-metric solution separated the electronic media from the print media. The second dimension could be described as an entertainment-information dimension. Beyond

TABLE I

Mean distance matrix for bilinguals in a mixed language condition

	1	2	3	4	5	6	7	8	9	10
1. Books	.0									
2. Magazines	60.80	.0								
3. Des Journeaux	79.93	40.20	.0							
4. La Musique	96.73	91.46	94.00	.0						
5. Radio	105.86	88.71	67.50	25.71	.0					
6. La Télévision	108.43	92.36	68.64	54.57	48.27	.0				
7. Sports	161.57	143.00	89.93	111.29	72.93	54.43	.0			
8. Le Cinéma	95.79	118.71	94.43	49.42	72.64	42.07	99.50	.0		
9. L'Information	65.93	51.36	20.21	98.00	30.50	26.00	83.43	60.00	.0	
10. Entertainment	31.43	48.00	53.62	18.14	36.00	40.64	33.57	19.07	67.36	.0

$N = 15$

TABLE II

Spatial coordinates for bilinguals in a mixed language condition

1. Books	−73.85	17.12	19.40	15.00	9.66	−9.66	−.06	−12.33	2.19	−25.10
2. Magazines	−60.12	−24.18	−23.52	−1.56	−11.74	−3.25	−.05	14.40	−3.08	−19.33
3. Des Journaux	−16.97	−41.68	−2.81	2.70	−3.24	10.46	−.01	−6.59	−6.32	8.99
4. La Musique	6.59	50.47	−31.52	−7.61	−4.28	2.43	.01	−4.57	27.95	3.53
5. Radio	20.72	2.47	−23.15	−22.10	20.35	−.84	.02	.49	−23.54	−4.43
6. La Télévision	30.48	.06	9.02	−15.16	−18.01	−4.59	.02	−14.22	−16.53	4.13
7. Sports	80.19	−29.87	−.58	23.12	2.87	−1.47	.06	.60	13.76	−31.50
8. Le Cinéma	19.42	42.55	33.62	−4.92	−3.73	4.19	.02	14.15	−9.62	−13.07
9. L'Information	−8.56	−30.24	24.76	−23.38	6.48	−2.63	−.01	4.05	25.46	29.39
10. Entertainment	2.11	13.29	−5.22	33.92	1.67	−2.38	.00	4.01	−10.28	48.40
Eigenvalues (roots) of eigenvector matrix										
	17,624.34	8962.95	4320.12	3266.46	1066.40	183.64	.01	−859.44	2698.65	−5445.09
Percentage of distance accounted for by individual vector										
	49.78	25.29	12.19	9.22	3.01	.52	.00	−2.43	−7.61	−15.36
Cumulative percentages of real distance accounted for										
	49.78	75.07	87.26	96.47	99.48	100.00	100.00	97.58	89.96	74.59
Cumulative percentages of total (real and imaginary) distance accounted for										
	66.73	100.63	116.97	129.33	133.36	134.05	134.05	130.80	120.60	100.00

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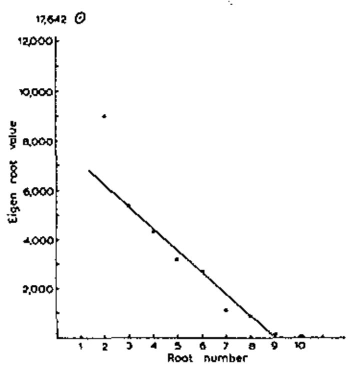

Fig. 2. Scree test of eigenroots.

these two vectors, the other dimension could not be readily interpreted. It is interesting to note that no language dimension could be readily identified. Clusters were next examined and the only ones present in the space were the ones discussed above dealing with the mass media. The graphic representation of the first two dimensions is presented in Fig. 4.

Since the language variable was not manifest in the configuration,

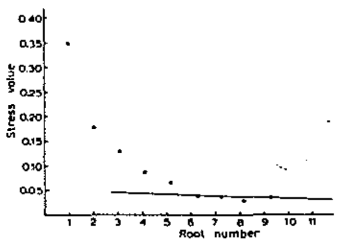

Fig. 3. Scree test of stress values.

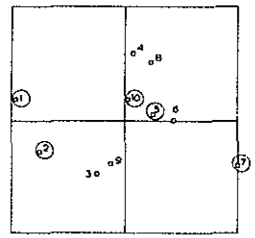

Fig. 4. Bilingual mixed-language space (English concepts are circled). $N = 15$. 1, Books; 2, Magazines; 3, Des journaux; 4, La musique; 5, Radio; 6, La télévision; 7, Sports; 8, Le cinéma, 9, L'information; 10, Entertainment.

it was decided to regress a vector of zeros (English) and ones (French) through the space. This operation was performed on all nine dimensions which resulted from the metric solution [5]. The correlation coefficients of the language vector with each dimension are presented in Table III. Because of the problem in regression analysis that as the number of columns approaches the number of rows, the multiple correlation approaches 1.0, only those dimensions with an $r^2 > .15$

TABLE III

Correlations of individual dimensions from bilingual mixed language space with language vector

Dimension	r	r^2
1	.147	.027
2	.141	.020
3	.318	.101
4	−.535	.287
5	−.441	.194
6	.460	.212
7*		
8	−.155	.024
9	.255	.024
10	.274	.075

* Dimension 7 was not included in the analysis because it was a vector of zeros.

were retained in the regression. These were dimensions 4, 5 and 6. Together they produced an $R^2 = .69$, significant at the .056 level. Clearly, the language vector was present in the space, although small compared to other attributes on which the words were arrayed. It is of interest to note that if only the dimensions suggested by the standard procedures were retained as the solution, the phenomenon under investigation might have been removed from the data.

But the question of concern is, which solution, 2, 3, 6 or all 9 dimensions is the best? While the two dimensional solution was readily interpretable, it accounted for only 68.2% of the variance, leaving an error term of .318 and a stress level of .175 (a poor to fair fit). The scree test suggested three dimensions. However, the third dimension could not be interpreted and over 20% of the variance was removed from the analysis ($\mu = .788$, $\epsilon = .212$). The inclusion of this dimension reduced stress to only .126; still a poor to fair fit. Six dimensions produced a good to excellent stress level (.039) and accounted for 90% of the variance. Additionally, the language vector could be explained quite well. All the dimensions with an r^2 of .15 or greater were included in the six dimensions. Clearly, the six dimensional solution is the best discussed so far, in that it retains an attribute known to span the space, even though that attribute does not correspond to any dimension.

2. Dynamic measures

When time-ordered measurements are taken, additional measures of dimensionality are made available. Ultimately the reasoning for selecting a subset of the dimensions from the empirically identified set of dimensions, even though that subset less-than-perfectly reproduces the raw data, is that the raw data themselves are assumed to be in error. Usually this error component is assumed to be random measurement error or unreliability; it is usually assumed also that it is this random component which lies along the smallest eigenvectors. At least two procedures suggest themselves as a test of this hypothesis. First, if the latter dimensions are unreliable, their over-time auto correlations should be near zero. Danes and Woelfel (1975) report correlations of the 16 dimensions recovered from a 17 × 17 matrix of dissimilarities across a five-week interval (see Table IV). These data show the last five eigenvectors are randomly related across time, but note that the 11th eigenvector is substantially more stable than the 7th through 10th eigenvectors.

A second, more rigorous method follows from an extension of

TABLE IV

Stability coefficients of the coordinates obtained at two points in time *. Source: Danes and Woelfel (1975)

Coordinate	Over-time correlation	(Correlation) ^{2}
1.	.97	.94
2.	.81	.65
3.	.86	.74
4.	.70	.49
5.	.76	.57
6.	.64	.40
7.	.38	.14
8.	.32	.10
9.	.37	.14
10.	.44	.19
11.	.56	.31
12.	.04	.02
13.	−.014	.00
14.	.011	.00
15.	−.29	.08
16.	−.074	.00

* The coordinates are rank ordered in terms of their absolute eigenroot value.

the regression method suggested earlier. Given that an attribute is regressed onto the set of factors at two or more points in time, stability and "meaningfulness" of a dimension across time requires that the regression coefficient of the factors vis à vis the attribute vector remain the same, since those coefficients represent the projection of the attribute vector on the factors. Thus, Gillham and Woelfel (1977) present the regression coefficients for two attribute vectors across the factors

TABLE V

Unstandardized regression coefficients from multiple regression of qualitative position upon perceived position. Source: Gillham and Woelfel (1975)

	Axis	Time one	Time two	Time three
Political judgments	1	−.16	−.20	−.15
	2	−.17	−.07	−.02
	3	.07	−.11	−.03
	R =	.91	.93	.92
Quantitativeness judgments	1	.11	.19	.15
	2	.22	.16	.11
	3	−.27	−.28	−.06
	R =	.80	.79	.75

$p < .01$, one-tailed.

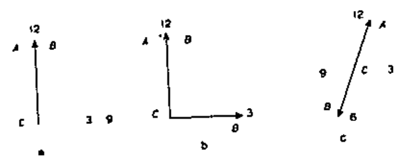

Fig. 5.

of a metric MDS space at three points in time in Table V. These data show the three factors in the illustration remaining quite stable over the intervals.

Neither of these two procedures can really be called "dynamic" in an important sense, since they really represent comparisons of static configurations over time. When genuinely processual data are at hand, however, more powerful methods are available. To illustrate these procedures we define three points, A, B, and C, as respectively the endpoints and axis of two equal-length clock hands in Fig. 5. Assuming a rotation scheme that holds C on the origin and rotates to analytic simple structure at each time interval (Woelfel *et al.*, 1975), the dimensionality of the set of points (A, B, C) will be one on the hour and (roughly) 38 minutes past the hour, and two dimensional at all other times. Even assuming perfect measures of the interpoint distances among A, B and C continuously across time, it is very unlikely that the most diligent psychometrician, using all the techniques named heretofore, would ever uncover the simple dynamic (and cyclical) pattern in this configuration which is given to an approximation in Fig. 6. Two procedures do suggest themselves, however, to detection

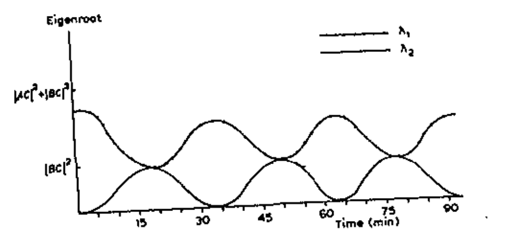

Fig. 6. Eigenvalues of the set of interpoint distances in Fig. 5 by time (min).

of this cyclical pattern, both based on auto regressive models.

First, we might calculate the correlation between the value of the *ith* eigenroot with itself at each earlier point in time, giving the series of autocorrelations

$$r_{\lambda t \cdot (t-1)}, r_{\lambda t \cdot \lambda(t-2)}, \ldots r_{\lambda t \cdot \lambda(t-n)} \tag{4}$$

for n periods of time. In the present example, for both λ_1 and λ_2, $r_{\lambda t \cdot \lambda(t-59)} = 1.0$, indicating a precise one-hour cycle. (Due to the complications of both motions, other autocorrelations of near 1.0 scores will be found as well over longer time intervals.)

A second, more comprehensive procedure would require regressing the value of each eigenroot at t on its entire history of values in the recursive multiple regression equation (see Glass *et al.*, 1976; Box and Jenkins, 1970).

$$\lambda_t = \alpha_1 \lambda_{(t-1)} + \alpha_2 \lambda_{(t-2)} \cdots \alpha_{(t-n)} \lambda_{(t-n-1)} + \epsilon \tag{5}$$

Not only does eqn. (5) present the information available from the set of zero-order autoregressions in (4), but it makes possible the empirical determination of external events on the process. If we assume that a stimulus is introduced into the sequence at a point in time t_P we may create the dummy variable X, where $X = -1$ for all points prior to t_P and +1 for all points afterward. Any number of such variables may be fit into eqn. (5) to yield

$$\lambda_t = \alpha_1 \lambda_{(t-1)} + \alpha_2 \lambda_{(t-2)} + \ldots + \alpha_{(t-n-1)} \lambda_{(t-n)} + \beta_i X_t + \epsilon \tag{6}$$

In eqn. (6), the α_i give the effects of each prior value of λ on its present value, and $\Sigma_i\, \alpha_i$ = the total effect of the *history* of λ in its value at t. The β_i give the effects of the *ith* outside event on the process, and Σ_i, β_i gives the total effect of all outside events measured.

If we expect that variations in the rank of the space are affected by *continuously variable* outside variables Z_i, we may incorporate their effects into the model similarly as

$$\lambda_t = \alpha_1 \lambda_{(t-1)} + \alpha_2 \lambda_{(2t-2)} + \ldots + \alpha_{(t-n)} \lambda_{(t-n-1)} + \beta_i X_t + \gamma_1 Z_{(t)} + \gamma_2 Z_{(t-1)} + \ldots \text{ etc.} \tag{7}$$

where the γ_i represent the effects of variable Z at each point in time, and $\Sigma_i\, \gamma_i$ represent the total effect of the history of Z on λ.

While eqn. (7) can be further generalized to include effects of interactions among outside events, variables and their histories, these generalizations are fairly obvious in the context of the regression equation and will not be dealt with here. Of greater salience, however, is the observation that the time-series of measures represented in eqns. (4) through (7) anticipate occasional zero values for some of the λ_i at some of the points in the series. This means that even a null eigenvector will frequently be retained in a time series. Put another way, for time-series analyses of MDS *processes*, the dimensionality of the process should not be assumed to be the small set of eigenvectors which persist across all time points, nor even a relatively stable subset, but rather the largest dimensionality found at any point. Thus for any two points in time i and j, if r (the rank of the space) at t_i is r_i, and at $t_j = r_j = r_i + k$, then k zero eigenvectors need to be added to the space at t_i before it is included in the time-series equations.

Summary

In general, most of the techniques now in common use for the determination of the dimensionality of MDS spaces tend to underestimate the number of dimensions needed to represent the underlying configuration accurately. This tendency is exaggerated when time-sequenced MDS observations are under study. Fortunately, econometric time-series (autocorrelation) procedures may prove helpful in resolving this problem [6].

Notes

1 Any two points may be connected by a line, yielding a single dimension differentiating the objects. Three points may be connected by a plane. No information as to their differentiation would be lost by indicating the objects' scale values on the two dimensions. The same holds for four points in a pyramid and n points in a hyperspase of $n - 1$ dimensions. It should be noted that if any three or more points lie along a line, fewer dimensions would be needed to precisely describe the system. Note of course that in a dissimilarity matrix the major diagonal contains all zeros, which by definition results in a space of rank $n - 1$.

2 This assumes that the data matrix is not symmetrical. If it meets this criterion then the columns and the rows are equal and only one need be considered.

3 The reason why the absolute values of the eigenroots are used is because metric scaling often results in a non-positive semidefinite matrix. When this matrix is orthogonally decomposed negative roots result. As a result the absolute value rather than signed-value should be used in the scree test.

4 For $r = 2$, the ratio of the degrees of freedom for the stimuli to the degrees of freedom for the solution is 2.65. For $r = 3$, it equals 1.88. For $r = 4$, 1.50 and $r = 9$, 1.0.

5 The same analysis was performed with the results from the KYST program. $R = .73$ and $R^2 = .54$. This multiple was produced from dimensions 4, 5, and 6 whose $r > .15$, the same criterion as in the metric case. Dimension 4, $r = .25$; 5, $r = .50$; 6, $r = .41$. It is interesting that the metric solution provides much better recovery of the language attribute than does the non-metric.

6 A Metric MDS scaling program (Galileo 4.0) which is specifically designed to deal with time-sequenced MDS spaces is available from the authors.

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