Chapter 11

MULTIDIMENSIONAL SCALING MODELS FOR COMMUNICATION RESEARCH

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I. INTRODUCTION

All measurement systems, whatever their type, share as a basic goal the determination of *difference* or separation among the elements measured. The crudest measurement systems are able to detect only the presence or absence of gross differences, while the most sensitive and precise measurement systems can reliably detect the smallest of differences and relate their magnitude to any other such difference as ratios. All measurement systems in practice lie

MULTIVARIATE TECHNIQUES
IN HUMAN COMMUNICATION RESEARCH

ISBN 0-12-504450-X

along a continuum between these extreme points. These differences or separations among the elements scaled may be thought of as distances, and multidimensional scaling (MDS) capitalizes fully on the analogy to spatial distances implicit in the measurement model. MDS procedures construct a multidimensional space or map in which the objects scaled are arrayed such that the distances between any two objects in the map are functions of their measured distance from each other on the scaling instrument. To the extent that the measurement system used by the researcher yields outcomes toward the precise end of the measurement continuum, this spatial analogy becomes increasingly appropriate.

Although several variations of this analysis system exist (Carroll & Chang, 1970; Coombs, 1958; Harshman, 1972; Kruskal, 1964a, b; Lingoes, 1972; McGee, 1968; Pieszko, 1970; Shepard, 1962a, b; Torgerson, 1952, 1958; Tucker & Messick, 1963; Tucker, 1972), all share the central notion of a spatial coordinate system as a frame of reference within which symbols are arrayed and therefore "pictured." Insofar as they constitute projections of distances among points into a coordinate system, MDS procedures provide the closest analogy to mechanics in the social sciences.

The many variants of multidimensional scaling may be broadly classified as either "metric" or "nonmetric"; since Norton (in this volume) presents a chapter (Chapter 10) on one variant of nonmetric multidimensional scaling (smallest space analysis), no further elaborations will be made here.

II. METRIC MULTIDIMENSIONAL SCALING: THE CLASSICAL MODEL

The metric multidimensional scaling model was the first multidimensional scaling model developed and it is known as the "classical" approach. Following Young and Householder (1938) and Richardson (1938), among others, Torgerson (1952, 1958) is most well known for general improvements and dissemination of this approach. Unlike the nonmetric approach, the metric procedure begins with a precisely scaled $n \times n$ data matrix S (see Table I) and concludes with an identically precise multidimensional space. Any cell s_{ij} in this matrix represents the measured dissimilarity or difference between the ith and jth object or concept scaled. In a typical metric study as usually practiced in the communication field, two of the objects to be scaled are chosen as a "criterion pair" and the difference between them assigned a numerical value like 10 or 100. All other pairs are then compared as ratios to this criterion pair in a statement of the form: "If a and b are u units apart, how far apart are . . . and . . . ?" When there is more than one respondent, estimates of all samples responses are usually averaged within each cell s_{ij} across all sample members to yield the average dissimilarities matrix $\bar{S}$

(Gillham & Woelfel, 1977). The ı ideal type of such a matrix.

Though its foundation can b (Serota, 1974), the modern basis in 1938 when Young and Hou describing the location of point separations (distances) among t converted the matrix of interp products B, whose elements b_{ij} a

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where the point P is an arbitrary of the space.[1] Although there is the origin of the space, the separ remain invariant regardless of wh

Torgerson (1958) describes a centroid of the space. The centro points, and Torgerson's procedu plot will be centered on the page and Householder solution, Torg Any element b^*_{ij} in Torgerson's matrix is given by

$$b^*_{ij} = \ldots$$

where

$$s^2_{j} = \frac{1}{n}\sum_{i=1}^{n} s^2_{ij}, \quad s^2_{i.} = \ldots$$

That is, placing the origin of th the space is accomplished by column means leaving only (in a Geometrically, any b^*_{ij} element

$$b^*_{ij}$$

where cos θ_{ij} is the cosine of t vector length of point i from the of point j from the origin (centr

Once the B^* scalar products n system is fairly straight forward; matrix. This factorization is iden to most communication resear matrix is input instead of the us

[1] The notation B, b and B^*, and b^* are t

(Gillham & Woelfel, 1977). The matrix of intercity distances in Table I is an ideal type of such a matrix.

Though its foundation can be traced back to the Greeks and beyond (Serota, 1974), the modern basis for metric multidimensional scaling was laid in 1938 when Young and Householder (1938) presented a technique for describing the location of points in a spatial configuration given only the separations (distances) among the points. Young and Householder (1938) converted the matrix of interpoint separations S into a matrix of scalar products B, whose elements b_{ij} are defined as

$$b_{ij} = \tfrac{1}{2}\left(s_{iP}^2 + s_{jP}^2 - s_{ij}^2\right) \tag{1}$$

where the point P is an arbitrary point in the space and is used as the origin of the space.[1] Although there is a unique B matrix for each point selected as the origin of the space, the separation relations among the points in the space remain invariant regardless of which point is selected as the origin.

Torgerson (1958) describes a procedure for locating the origin at the centroid of the space. The centroid is the exact center of the configuration of points, and Torgerson's procedure simply ensures that the resulting map or plot will be centered on the page. While functionally equivalent to the Young and Householder solution, Torgerson's procedure is more commonly used. Any element b^*_{ij} in Torgerson's (1958) "doubled centered" scalar products matrix is given by

$$b^*_{ij} = \tfrac{1}{2}\left(s_{ij}^2 - s_{j}^2 - s_{i.}^2 + s_{..}^2\right) \tag{2}$$

where

$$s_j^2 = \frac{1}{n}\sum_{i=1}^{n} s_{ij}^2, \qquad s_{i.}^2 = \frac{1}{n}\sum_{j=1}^{n} s_{ij}^2, \qquad s_{..}^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}^2.$$

That is, placing the origin of the space at the centroid (geometric center) of the space is accomplished by subtracting out the grand row and grand column means leaving only (in analysis of variance terms) the "interactions." Geometrically, any b^*_{ij} element represents

$$b^*_{ij} = \cos\theta_{ij}|R^i||R^j| \tag{3}$$

where $\cos\theta_{ij}$ is the cosine of the angle between the two vectors, $|R^i|$ the vector length of point i from the origin (centroid), and $|R^j|$ the vector length of point j from the origin (centroid).

Once the B^* scalar products matrix is obtained, establishing the coordinate system is fairly straight forward; it simply consists of a factorization of the B^* matrix. This factorization is identical to the factor analysis algorithm familiar to most communication researchers—the only difference is that the B^* matrix is input instead of the usual correlation matrix. It consists essentially

[1] The notation B, b and B^*, and b^* are taken from Torgerson (1958).

TABLE I

SEPARATIONS IN SPACE AMONG 16 SELECTED U.S. CITIES[a] ([a] 1 UNIT = 1 KM)

City	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(1) Atlanta	0															
(2) Boston	1508	0														
(3) Chicago	944	1369	0													
(4) Cleveland	891	886	496	0												
(5) Dallas	1160	2496	1292	1649	0											
(6) Denver	1950	2846	1480	1974	1067	0										
(7) Detroit	869	986	383	145	1607	1860	0									
(8) Los Angeles	2310	4177	2807	3297	2005	1337	3191	0								
(9) Miami	972	2019	1911	1749	1788	2777	1354	3764	0							
(10) New Orleans	682	2186	1340	1487	713	1741	1511	2692	1076	0						
(11) New York	1204	302	1147	652	2211	2624	1258	3944	1757	1884	0					
(12) PHoenix	2562	3701	2338	2814	1427	943	2719	574	3189	2117	3451	0				
(13) Pittsburgh	838	777	660	185	1721	2124	330	3437	1625	1478	510	2941	0			
(14) San Francisco	3442	4343	2990	3485	2386	1524	3364	558	4174	3099	4137	1051	3643	0		
(15) Seattle	3511	3979	2795	3260	2705	1643	3118	1543	4399	3381	3874	1792	3440	1091	0	
(16) Washington	874	632	961	492	1907	2404	637	3701	1485	1554	330	3191	309	3929	3849	0

of finding the eigenvectors of the
be a $k \times r$ matrix $R(\mu = 1, r; \alpha$
projections of the αth concept or
set of r orthogonal basis vectors
shows it is a vector (no index w
represent a matrix, and so on). T
that it is not an index, but rather
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vector refers to *observations* or *m*
called "contravariant"; subscript
therefore, is a *contravariant vector*

Similarly, each of the e_μ repre
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indicates that each e_μ does not re
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projections of the cities on the fi
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which will be 0.0.

Each row of Table II represent
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of finding the eigenvectors of the B^* matrix. The output of this analysis will be a $k \times r$ matrix $R(\mu = 1, r; \alpha = 1, k)$ where each row $R^{\mu}_{(\alpha)}$ represents the projections of the αth concept or object (city, in the example of Table I) on a set of r orthogonal basis vectors e_{μ}. The fact that there is a single index (μ) shows it is a vector (no index would represent a scalar; two indices would represent a matrix, and so on). The (α) is placed within parentheses to show that it is not an index, but rather only a marker to describe which vector we are referring to. Furthermore, the index is superscripted to show that this vector refers to *observations* or *measured values*. (Superscripted quantities are called "contravariant"; subscripted quantities are called "covariant." R^{μ}, therefore, is a *contravariant vector*.)

Similarly, each of the e_{μ} represents a unit vector (the single index shows that it is a vector), and the fact that it is subscripted rather than superscripted indicates that each e_{μ} does not refer to observations of measured values but rather to an arbitrary reference vector onto which the measured or observed values ($R^{\mu}_{(\alpha)}$) are projected. Since it does not refer to measured values, it is a *covariant* vector and hence subscripted.

Each of these e_{μ} ($\mu = 1, r$) vectors represents a unit reference axis orthogonal to each other such reference axis, and thus the set of these basis vectors constitutes an ordinary r-dimensional Cartesian coordinate system. These vectors are usually called dimensions (sometimes factors or eigenvectors) and the fact that more than one such vector is usually needed to represent the configuration gives rise to the term "multidimensional scaling." In fact, it is always the case that $r \leqslant k - 1$, since any k points can always be represented on $k - 1$ orthogonal coordinates. Any three points, for example, can always be fit on a (two-dimensional) plane, but may in some cases be on a (one-dimensional) line. Factoring the centroid scalar products matrix (B^*) derived from the intercity distance matrix in Table I yields the results given in Table II and Fig. 1. Each column of Table II represents the projections of the cities on a reference vector e_{μ}; the first column, therefore, represents the projections of the cities on the first unit vector e_1, the second, their projections on e_2, and so on. The reader can easily verify that these columns are orthogonal by calculating the correlations among pairs of columns, all of which will be 0.0.

Each row of Table II represents the projection of one of the cities' position vector $R^{\mu}_{(\alpha)}$ on the e_{μ} basis vectors, thus the first row $R^{\mu}_{(1)}$ represents the projection of Atlanta's position vector on the space, so that $R^{1}_{(1)} = -808.7$ $R^{2}_{(1)} = 481.3$, etc. Moreover, since the transformation by which this solution is achieved is distance preserving, these numbers are to be understood in the original units of measure—in this case kilometers. Plots based on the first two dimensions (columns e_1 and e_2) of Table II are presented in Fig. 1. Actually, the figures in Table II have first been reflected (multiplied by -1) because the algorithm generated an inverted mirror-image of our conventional representation of the earth's surface—the algorithm, of course,

TABLE II

GALILEO COORDINATES OF 16 SELECTED CITIES IN A METRIC MULTIDIMENSIONAL SPACE NORMAL SOLUTION

	1	2	3	4	5	6	7	8
(1) Atlanta	−808.701	481.329	−26.846	−71.783	16.023	16.599	12.634	10.807
(2) Boston	−1677.183	−745.227	−184.542	85.696	−10.499	−27.056	−13.778	−8.615
(3) Chicago	−406.610	−363.631	110.592	−119.280	−.893	−7.731	−19.989	15.535
(4) Cleveland	−891.643	−403.765	54.297	−32.279	33.483	27.932	−8.801	−6.017
(5) Dallas	322.591	699.325	42.781	−152.488	−54.108	25.889	−16.486	−8.547
(6) Denver	1042.064	−82.214	156.341	−114.150	−11.267	−23.404	23.232	−7.455
(7) Detroit	−767.895	−473.812	73.193	−66.076	−13.045	4.508	1.460	−23.434
(8) Los Angeles	2268.293	367.182	18.883	128.831	86.748	−3.487	.092	−2.216
(9) Miami	−1326.834	1253.670	−256.774	102.060	−11.724	.826	5.017	−10.785
(10) New Orleans	−329.342	866.871	−51.591	−92.584	21.478	−16.044	−5.036	9.862
(11) New York	−1517.500	−480.943	−51.155	79.323	−22.697	9.141	5.351	23.699
(12) Phoenix	1733.251	541.157	52.360	−4.181	−17.993	−27.342	−6.725	5.051
(13) Pittsburgh	−1053.005	−319.613	21.158	−29.486	−5.232	3.694	28.170	2.261
(14) San Francisco	2529.978	−119.627	33.792	190.582	−54.348	25.969	.114	2.814
(15) Seattle	2240.414	−1172.344	−272.454	−88.105	.814	−.161	.044	.055
(16) Washington	−1357.579	−148.357	279.964	183.919	−8.741	−9.436	−5.304	−3.014
Eigenvalues (foots) of eigenvector matrix								
	32900982.656	6480979.446	310095.395	189689.444	22651.153	4969.506	2586.446	1974.062
Number of iterations to derive the root								
	4	4	24	4	44	7	9	13
Percentage of distance accounted for by individual vector								
	82.428	16.237	.777	.474	.057	.012	.006	.005
Cumulative percentages of real distance accounted for								
	82.428	98.665	99.441	99.915	99.972	99.984	99.991	99.996
Cumulative percentages of total (real and imaginary) distance accounted for								
	83.086	99.453	100.236	100.714	100.771	100.783	100.790	100.795

Trace 39598514.222

Number of dimensions in real space 9

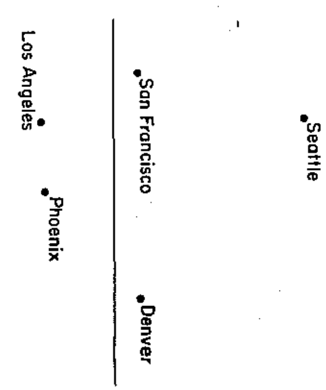

FIG. 1. Plot of factors 1

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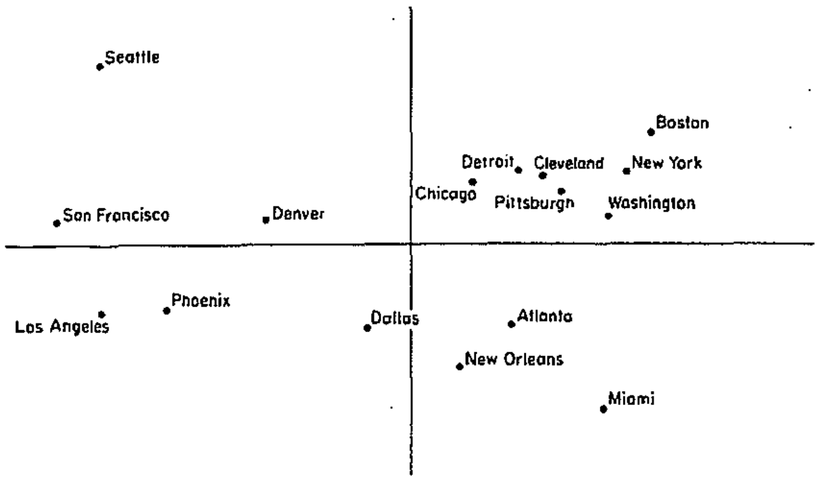

FIG. 1. Plot of factors 1 and 2 from the unstandardized analysis.

cannot know which half of the world we like to consider the "top." Given this reflection, $R^1_{(1)} = 808.7$ and $R^2_{(1)} = -481.3$ tell us that Atlanta is 808.7 km east and 481.3 km south of the geographic center of these 16 cities.

It is worth noting also that two dimensions are enough to give a reasonably complete representation of these data. The third dimension (not plotted) represents the (minor) curvature of the earth; all the others represent rounding errors (the original data are not perfectly error-free). The determination of how many dimensions to retain is based solely on the relative sizes of the projections on the dimensions. When the projections are too small to be worth considering, or are within the precision of measure, they are ignored. As an aid in determining when they are "too small," we note that the squares of the projections on each factor sum to their corresponding eigenvalue, $\lambda_{(\mu)}$. (Note that μ is in parentheses, indicating that it is not an index; since $\lambda_{(\mu)}$ has no index it is a scalar.) That is,

$$\lambda_{(\mu)} = \sum_{\alpha=1}^{k} (R^{\mu}_{(\alpha)})^2 \tag{4}$$

This value, $\lambda_{(\mu)}$, may be thought of in ANOVA terms as the amount of variance explained by the μth dimension. The total variance is called the trace (T) and is given by the sum of the eigenvalues, i.e., $T = \Sigma^r_{\mu=1}\lambda_{(\mu)}$. The proportion of variance explained by any single factor, therefore, is given by its eigenvalue divided by the trace, or

$$\%\text{VAR} = 100\lambda_{(\mu)}/T \tag{5}$$

Statistical tests for the significance of this ratio are not known in the MDS literature but the reader is referred to Barnett and Woelfel (1978) for a more thorough discussion of the question of dimensionality. These authors conclude that valuable information can be found in factors much smaller than typical practice usually retains, and they recommend retaining all or nearly all the dimensions, particularly when precise scaling procedures and large samples have been employed. (Most typical procedures now in use would ignore the small third dimension in this example and thus conclude that the world—or at least the United States—is flat.)

III. METRIC RATIOS, METRIC AXIOMS, AND "METRIC" SPACES

In the process of "measuring," the term metric usually refers to the initial *standard* for which other numerical values of separation are obtained by comparing these other magnitudes to the initial standard. As such, physical distances are *metric* ratios, proportions, or multiples of a consensually shared, prespecified distance: the meter. In the language of mathematics, however, the term "metric space" usually refers to a space which is isomorphic with certain prespecified axioms; for the metric Euclidean space, these axioms take the following form (cf. Blumenthal, 1961)

$$\begin{aligned} s_{ij} &= 0 \quad \text{if and only if} \quad i = j && \text{(Positivity)} \\ s_{ij} &= s_{ji} && \text{(Symmetry)} \\ s_{ij} + s_{jk} &\geqslant s_{ik} \quad \text{for all} \quad i, j, \text{and } k && \text{(Triangle inequality)} \end{aligned} \tag{6}$$

With the presence of positivity and symmetry, all that is needed to make the space "metric" is the requirement that any triangle formed by any three points be real; that is, that any side of a triangle not exceed the sum of the other two sides. These constraints are clearly met by the intercity distance of Table I. In communication research, however, the triangle inequality rule is usually violated by the original data set. Thus, when one is working with reliable metric ratios provided by human respondents, metric MDS frequently results in complex, non-Euclidean multidimensional spaces characterized by both real and imaginary eigenvectors in R.

The term "imaginary" has caused unfortunate misgivings among psychologists; some psychometricians have assumed that imaginary eigenvectors cannot be meaningful and, therefore, represent measurement error. Thus, if A is "close" to B, and B is "close" to C, the failure to find the logically expected "nearness" between A and C has usually been attributed to faulty data gathering procedures. Beginning with Shepard (1962*a*, *b*), Kruskal (1964 *a*, *b*), Guttman (1968), Lingoes (1972), and others have devised "nonmetric" procedures which eliminate triangle inequality violations by iteratively transforming data into a "metric space" of a prespecified number of dimensions.

Research by Danes and Woelfel (1975), Serota, Cody, Barnett, and Taylor (1975), and Woelfel (1977), among others, indicates, however, that respondents frequently and reliably report "inconsistent" separation judgments.

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IV. STRUCTURE OF MULT AND ATTRIBUTES

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or dimensions themselves, since
would prove to be independent
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Very substantial evidence sugg
MDS space should be thought of
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Collectively, these studies suggest that "inconsistent" separation judgments result from the differential interpretation of a given concept; that is, the meaning for a given symbol frequently varies with the context in which the symbol is presented. For example, "red" and "orange" may be conceived of as similar; "orange" and "tangerine" may be conceived of as similar; but "red" and "tangerine" may be viewed as very dissimilar. If such an example were quantified with metric ratio data, this would result in "inconsistent" separation and result in a complex multidimensional "metric" ratio space. In general, the spaces yielded by typical communication separation matrices are multidimensional and complex.

IV. STRUCTURE OF MULTIDIMENSIONAL SPACES: CLUSTERS AND ATTRIBUTES

Multidimensional scaling was conceived of primarily by psychologists to measure psychological structures, and early analyses usually were confined to efforts at identifying different characteristics that might be associated with different regions of the space. Five procedures for such analyses are most common. First is a simple "eyeball" approach, similar to the intuitive interpretation of factor analyses, where the investigators carefully scrutinize the plots of the configuration to determine obvious features. Very frequently the graphic simplicity of the multidimensional plot (e.g., Fig. 1) makes obvious facts concealed both by the separation matrix and other analytic procedures.

Second, many researchers frequently perform cluster analyses on either the separation matrix or the coordinate matrix R to identify meaningful clusters of elements. Those elements which cluster together are usually thought to possess some common characteristic(s). Interpretation of these analyses give precision to "eyeball" analyses, and the reader is referred to Chapter 9 for an explication of such procedures.

A third common analysis consists of attempts to locate linear arrays of "objects" which might express fundamental psychological attributes. If all the concepts in a space, for example, could be seen to lie on a line from "bad" to "good," a "good–bad" attribute might be inferred. Many early analysts, in fact, hoped or assumed that these attributes might correspond to eigenvectors or dimensions themselves, since they felt the basic attributes of experience would prove to be independent of each other, but few workers still hold to this view today (Rosenberg & Sedlak, 1972; Cody, Marlier, & Woelfel, 1976; Schmidt, 1972).

Very substantial evidence suggests rather that the orthogonal factors of the MDS space should be thought of only as a convenient reference frame (much like the numbered and lettered grids on street maps). Attribute lines may well take *any* orientation within this grid, and frequently the number of attributes found greatly exceed the number of eigenvectors.

This suggests a fourth common procedure for analysis of the structure of an MDS space. Locating such attribute vectors in the space can be accomplished very simply by capitalizing on the orthogonality constraints on the eigenvectors to yield the regression equation (see Gillham & Woelfel, 1977):

$$A = B_{(1)}R^{\mu}_{(1)} + B_{(2)}R^{\mu}_{(2)} + \cdots + B_{(r)}R^{\mu}_{(r)} \tag{7}$$

where A are the measured scores of the concepts scaled on any attribute scale, $B_{(i)}$ the standardized regression coefficients representing the cosines of the angles between the attribute vector A and the orthogonal $R^{\mu}_{(\alpha)}$ (due to the orthogonality constraint, the B_i are equal to the zero order correlations $r_{AR^{\mu}_{(\alpha)}}$), and $R^{\mu}_{(\alpha)}$ the eigenvectors (factors, axes, dimensions) of the solution. Even this procedure contains important flaws however. Among these are the assumptions implicit in (7) that each attribute or trait is equally salient or relevant for every element of the domain and that each trait is of infinite or at least indefinite length (Cody *et al.*, 1976; Cody, 1976).

Fortunately, a fifth procedure which overcomes these and other problems requires simply that the words which describe traits (e.g., friendly, warm, sharp, unobtrusive, etc.) be included as concepts in the original separation judgments. Line segments between semantic "opposites" (e.g., good–bad) can be taken as finite attributes whose position and orientation vis-á-vis the other concepts in the domain are completely given by the scaling solution itself. Except for the difficulties due to respondent burden and other economic factors which occur when k (the number of concepts scaled) becomes large, this approach seems to be free of the problems inherent in the other methods. This procedure does not require that any empirical parameters be constrained in advance, but rather determines the number, length, and orientation of attributes by measurements.

V. THE COMPARISON OF MULTIDIMENSIONAL SPACES

As interesting and informative as these techniques are, by far the most interesting use of multidimensional scaling is for the comparisons of spaces across groups and across time, since these transformations provide the basis for projections of future events, causal analyses, and ultimately engineering applications.

Since the axes in a multidimensional space have an arbitrary orientation, some scheme of rotation and translation is necessary to "match" the spaces as closely as possible before such comparisons are undertaken. The transformation required is one which will minimize the discrepancy between spaces while leaving the measured distances within each space invariant. These transformations (frequently called "Procrustes" rotations to distinguish them from the analytic rotations—like "varimax" or quartimax"—common in factor analysis) are of great theoretical significance, since they establish a

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problem is presented by Lissitz,
Lissitz solution does not include
real coordinates.

VI. COMPARISON OF SPACE SQUARES EXAMPLE

For the cross-sectional compa
paring the identical concepts and
different initial metrics (separatio
dure given above with each conc

common frame of reference across respondents, observers, and time periods. Alternative choices of such transformations will result in different reference frames which will determine the form of regularities observed (Woelfel, 1977). In its most general form, this problem was solved independently by Cliff (1966) and Schonemann (1966). The general solution involves rotating a matrix of coordinates $R^{\mu}_{(a)}$ at $t + 1$ about its center until the sum of the squared distances of each point in $R^{\mu}_{(a)}$ at $t + 1$ from its counterpart in another space $R^{\mu}_{(a)}$ at t is at a minimum. This transformation conserves position in that it minimizes total motion when t and $t + 1$ refer to times of measurement, or total difference if t and $t + 1$ refer to any arbitrary groups. Whenever two or more scaling solutions—or even factor analyses—are to be compared, Procrustes rotation is *required*; rotating all the spaces to a criterion like varimax or any other analytic solution will not minimize artifactual differences.

Under many circumstances it is desirable to weight these rotations. In an experiment, for example, in which some concepts are manipulated and others controlled, one would try to conserve the position of the unmanipulated concepts, but would expect the manipulated concepts to move freely. Under these conditions, the control concepts should be assigned unity weights and the manipulated concepts should be assigned zeros. Under more complicated conditions—such as those in which concepts were known to be measured with differential reliability—continuous variable weights may be assigned. The key function of these weights is to assign differential stability to the points across the rotations.

Coupled with the idea of rotation is the notion of translation. Translation means parallel displacement of the space, or relocation of the origin of the space. Translations represent changes in viewpoint in the space and are of great theoretical significance. It can easily be shown that distances within each data set remain invariant under both rotation and translation. Solutions to be weighted rotation problem, including translation to different origin, are presented in Woelfel *et al.*, (1979). While the Woelfel *et al.* solution is an iterative solution, it includes translation and is defined over complex coordinates as well as real. A direct, noniterative solution to the weighted Procrustes problem is presented by Lissitz, Schonemann, and Lingoes (1977), but the Lissitz solution does not include translation of origin and is defined only for real coordinates.

VI. COMPARISON OF SPACES: AN EQUALLY WEIGHTED LEAST SQUARES EXAMPLE

For the cross-sectional comparison of nine groups of subjects when comparing the identical concepts and using different criterion pairs (anchors) and different initial metrics (separations), Gordon (1976) used the rotation procedure given above with each concept given an equal weight.

The intent of Gordon's (1976) study was to evaluate whether the ratio judgements of separation scaling model (i.e., scales of the form: "if a and b are μ units apart, how far apart are x and y?; Danes and Woelfel, 1975) would yield equivalent solutions when subjects used different criterion pairs with different initial separation values. Four groups were given the larger criterion pair "children's comedy–crime drama" (CC) with an initial separation value of either 10, 25, 50, or 100 units; four groups were given the smaller criterion pair "family drama–medical drama" (FM) with an initial separation value of either 10, 25, 50, or 100 units. The ninth group who rated the identical concepts was instructed to "· · · keep a ten point scale in mind—

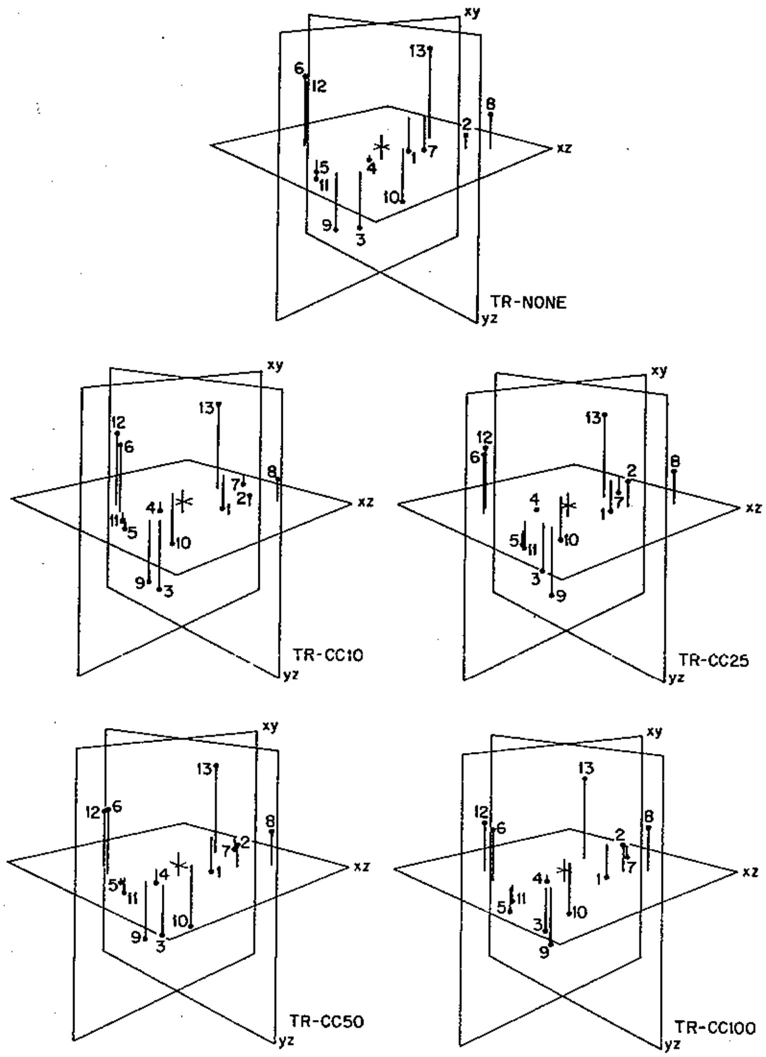
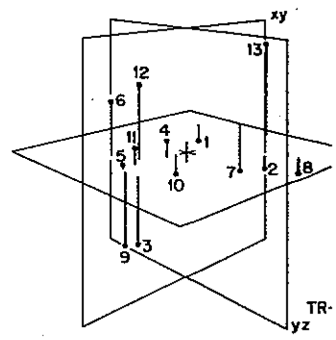
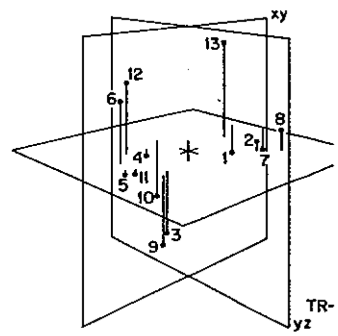


FIG. 2. Individual plots of each treatment.

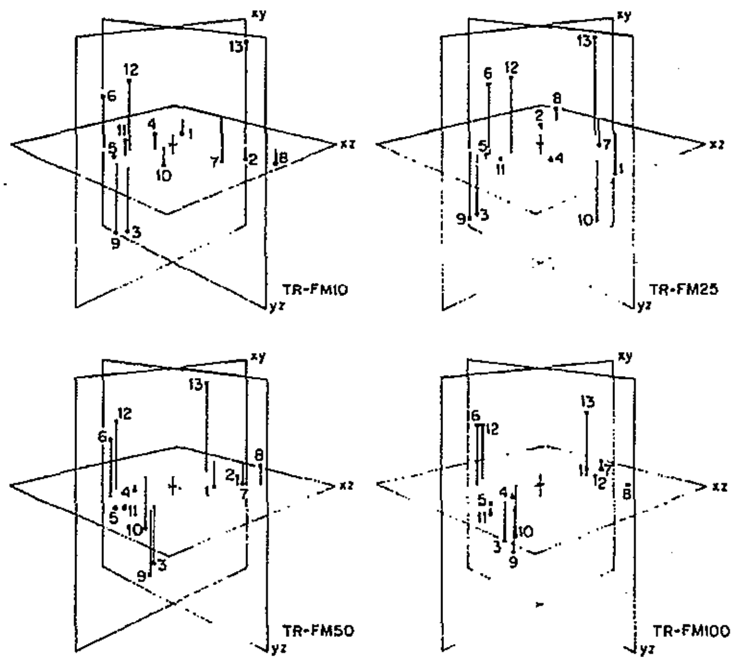




some concepts may be less tha
[p. 9]." Group sample size rang
participated in their study. Ea
following 13 concepts:

1. Children's comedy
3. Soap opera
5. Medical drama
7. Fat Albert
9. General Hospital
11. Medical Center
13. Me

On the basis of trace size, i.e.,
matrices were rank ordered fr
following order was obtained:
CC100, and FM100. These resu
(1) that the smaller the separati
space (CC > FM), and (2) that
the smaller the space. Leaving
spaces were then computed and
of these eight spaces appears in



some concepts may be less than ten units apart and others may be more [p. 9]." Group sample size ranged from 92 to 112; a total of 863 subjects participated in their study. Each group rated the separations among the following 13 concepts:

1. Children's comedy
2. Adult situation comedy
3. Soap opera
4. Family drama
5. Medical drama
6. Crime drama
7. Fat Albert
8. All in the Family
9. General Hospital
10. The Waltons
11. Medical Center
12. The Streets of San Francisco
13. Me

On the basis of trace size, i.e., the total variance of each space, the nine matrices were rank ordered from the low 238.98 to high 45,100.99; the following order was obtained: CC10, None, FM10, CC25, FM25, FM50, CC100, and FM100. These results confirmed Gordon's (1976) expectations: (1) that the smaller the separation between the criterion pair, the larger the space (CC > FM), and (2) that the larger the numerical separation metric, the smaller the space. Leaving the "none" treatment out, multidimensional spaces were then computed and rotated to least squares congruence; the plot of these eight spaces appears in Fig. 2. The plot of the three principle planes appears in Fig. 3, the plot of the "none" and the CC10 group appears in Fig. 4.

Aside from the rotation illustration, the Gordon (1976) study illustrated two basic findings important for the ratio judgment of separation measurement procedure: (1) apparently subjects do perceive differential magnitudes of initial metric separations; that is, larger spaces were obtained when the criterion pair was smaller although the number assigned to that criterion pair remained the same; (2) although the spatial structure obtained from the "none" treatment was similar to the spatial stucture obtained from the CC10 treatment, the variance for the "none" group was almost three times as large as that obtained from the CC10 treatment group, a finding which indicates that the use of a criterion pair reduces the potential amount of "noise" in separation judgments as well as supplying a basic metric for the space.

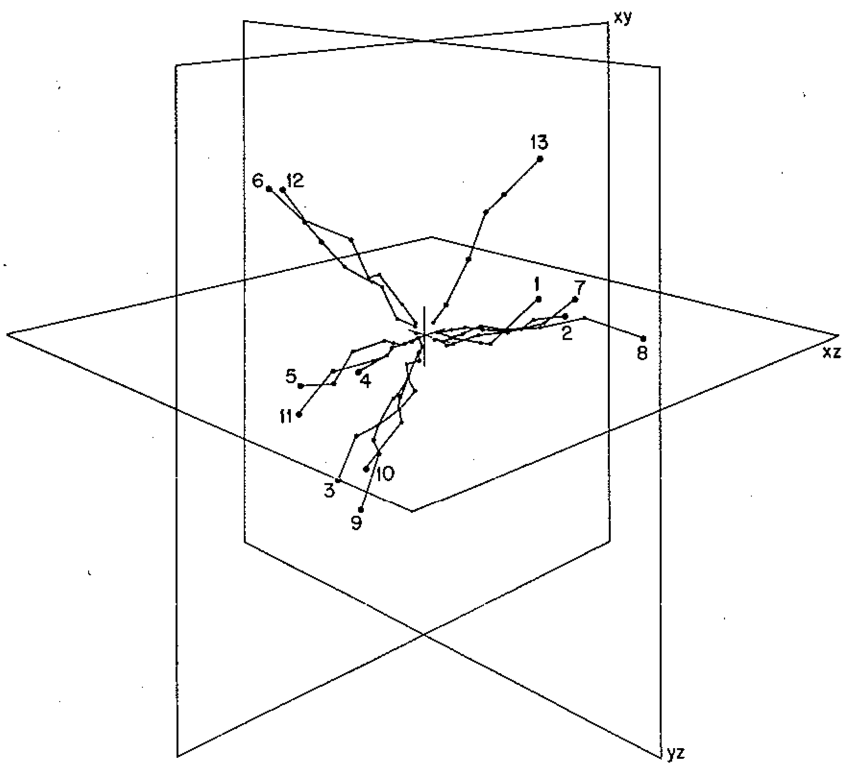


FIG. 3. Comparison of treatments. Beginning at concept number, each point represents the judgment of that concept using a different criterion pair. The order of treatments from outer to inner is: FM100, CC100, FM50, CC50, FM25, CC25, FM10, CC10 ("none" treatment not included, see Fig. 4).

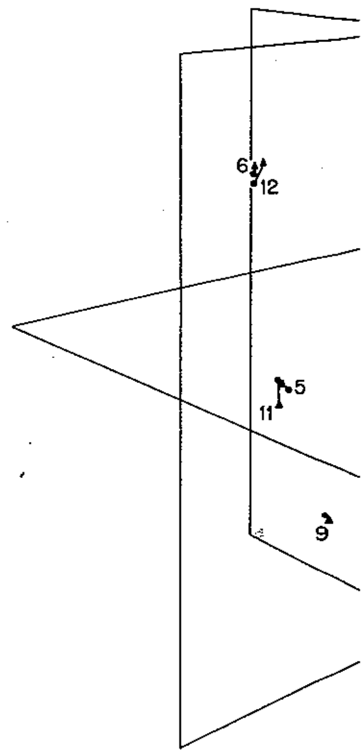


FIG. 4. Comparison of spa

VII. COMPARISON OF SPAC EXAMPLE

For the study 42 subjects esti the same ratio judgments proce concepts mapped were

1. Sleeping
3. Daydreaming
5. Marijuana high
7. Depression
9. Relaxation
11. Alpha wave media
13. Reliable
15. Message Source A

Two days after the first (t_0) me study received a letter from a v advocated frequent daily practic

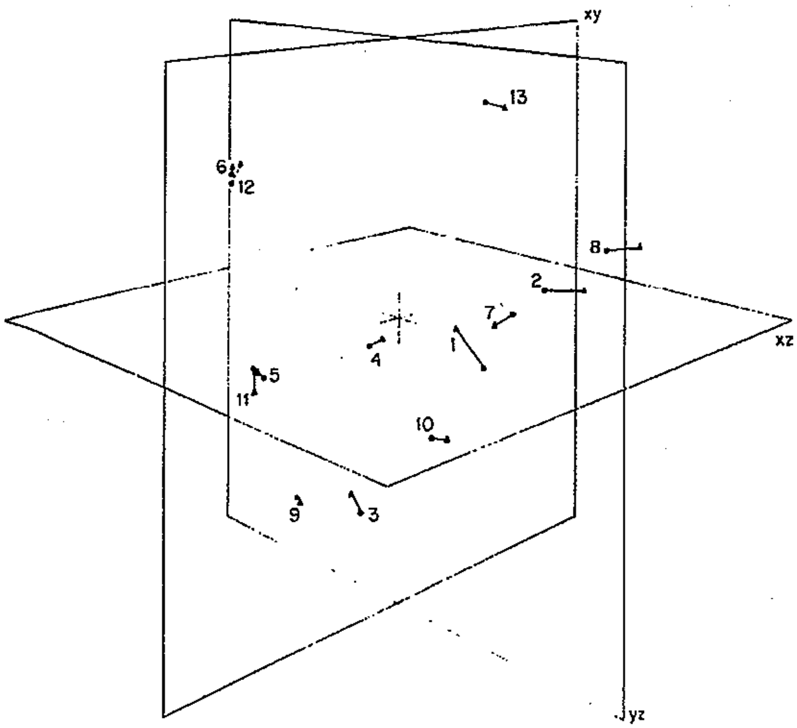


FIG. 4. Comparison of spaces for treatments "none" (▲) and CC10 (●).

VII. COMPARISON OF SPACES: AN UNEQUALLY WEIGHTED EXAMPLE

For the study 42 subjects estimated the separation among concepts using the same ratio judgments procedure used in the Gordon (1976) study; the concepts mapped were

1. Sleeping	2. Dreaming
3. Daydreaming	4. Intense concentration
5. Marijuana high	6. Good
7. Depression	8. Alcohol high
9. Relaxation	10. CTP
11. Alpha wave mediation	12. Transcendental meditation
13. Reliable	14. Message Source B
15. Message Source A	16. Me

Two days after the first (t_0) measurements were made, the subjects in this study received a letter from a well-known credible source (Source A), who advocated frequent daily practice of CTP, a deliberately undefined fictitious psychological activity—the "cortical thematic pause." After reading the letter from Source A, the subjects were then asked to estimate the concept relations again (t_1). Five days later a similar letter from a less credible source (Source B), who also advocated frequent CTP practice was delivered and the concepts scaled once again (t_2). Finally, a fourth (t_3) wave of data was collected two days later. It was expected that (1) two concepts CTP and Source A should converge after the reception of the first message, and that (2) the three concepts CTP, Source A, and Source B should converge after the reception of the second message. Those concepts not mentioned in either letter were expected to remain invariant.

Using separation matrices that consisted of averaged values for each measurement session, the second space was rotated to the first and in doing so, CTP and Source A were given weights of zero while the remaining concepts were assigned weights of unity. Further, the third space was rotated to the second and in doing so, CTP, Source A, and Source B were assigned weights of zero while the remaining concepts were again assigned unity values. Last, the fourth space was rotated in the same way to the third (see Fig. 5 and Table III). As Fig. 5 shows, at the t_2 measurement there is a triple convergence of Source A (15), the CTP (10), and the Me (16). At the t_3

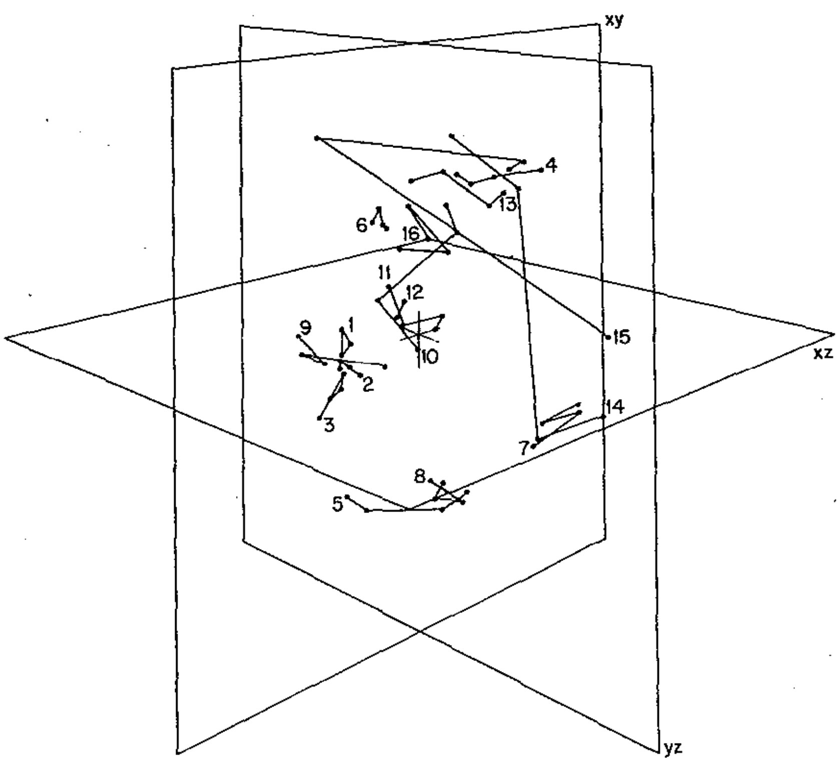


FIG. 5. Stable concepts rotation.

TABLE III

MEAN CHANGE SEPARATION VALUE (N
UNEQUALLY WEIGHTED LEAST SQUAR

Concept
(1) Sleeping
(2) Dreaming
(3) Daydreaming
(4) Intense concentration
(5) Marijuana high
(6) Good
(7) Depression
(8) Alcohol high
(9) Relaxation
(10) CTP
(11) Alpha wave meditation
(12) Transcendental meditation
(13) Reliable
(14) Message Source B
(15) Message Source A
(16) Me

[a]Denotes the concepts that were s

measure, there is a convergence of So (10), but a divergence (boomerang ef no new manipulations, little interp reflects the magnitudes of these moti motion (like Brownian motion) of substantive interpretations of this ex chapter the main point is to show hov can aid in the interpretation of multid

VIII. MESSAGE DESIGN SYSTEM

Among theoretical and applied co agreed that linking one's message top increases the effectiveness of one's propriate" message appeals? This s analytic development for the design within a multidimensional analysis f turns to procedures designed speci message appeals for the design of *me*

Ego as a Concept

In the example given earlier, the c position, my purchase, my support,

TABLE III

MEAN CHANGE SEPARATION VALUE (MOTION) AS A FUNCTION OF AN UNEQUALLY WEIGHTED LEAST SQUARES CONCEPT–CONCEPT ROTATION

Concept	$t_0 - t_1$	$t_1 - t_2$	$t_2 - t_3$
(1) Sleeping	20.923	15.752	19.067
(2) Dreaming	14.539	18.902	19.285
(3) Daydreaming	22.033	12.331	17.785
(4) Intense concentration	20.110	17.462	14.972
(5) Marijuana high	21.858	24.396	16.136
(6) Good	30.317	23.736	13.200
(7) Depression	20.354	16.093	12.331
(8) Alcohol high	25.877	11.497	14.937
(9) Relaxation	31.861	16.341	26.256
(10) CTP	56.868[a]	67.708[a]	36.790[a]
(11) Alpha wave meditation	18.728	25.431	12.237
(12) Transcendental meditation	19.175	15.446	16.713
(13) Reliable	22.446	21.220	27.047
(14) Message Source B	27.933	71.548[a]	37.551[a]
(15) Message Source A	124.533[a]	74.952[a]	43.308[a]
(16) Me	24.766	25.259	23.889

[a] Denotes the concepts that were set to weights of zero; i.e., $m_i = 0$.

measure, there is a convergence of Source B (14), Source A (15), and the CTP (10), but a divergence (boomerang effect) of the Me (16). At time four, after no new manipulations, little interpretable motion is evident. (Table III reflects the magnitudes of these motions.) Note especially the small random motion (like Brownian motion) of the unmanipulated concepts. Although substantive interpretations of this experiment are beyond the scope of this chapter the main point is to show how the stable concepts rotation procedure can aid in the interpretation of multidimensional experiments.

VIII. MESSAGE DESIGN SYSTEMS

Among theoretical and applied communication specialists, it is commonly agreed that linking one's message topic with the appropriate message appeals increases the effectiveness of one's communications; but what are the "appropriate" message appeals? This section of this chapter presents a new analytic development for the design of optimum message strategies; that is, within a multidimensional analysis framework, the topic of discussion now turns to procedures designed specifically for the selection of the "best" message appeals for the design of *message content*.

Ego as a Concept

In the example given earlier, the concept "ego" (me, myself, my vote, my position, my purchase, my support, etc.) was mapped into the space as were the other concepts of concern. Within the multidimensional analysis framework, the concept "ego" and other such similar concepts appears to have special properties vis-à-vis behavior. In a political mass communication study, for example, Barnett, Serota, and Taylor (1975) have found the political candidate-ego separation to be inversely related to voting behavior; that is, that candidate which was nearest to "ego" received the largest share of the vote in a congressional election. Furthermore, earlier research by market researchers (e.g., Green & Carmone, 1972; Steffler, 1972) has found products nearest to the ego (e.g., my purchase or my choice) yield greater sales than those that are distant from the ego. Additionally, Jones and Young (1972) have found the separation between graduate students and graduate faculty to be predictive of communication frequency, as indicated by the formation of graduate committees.

A reanalysis of the data collected by Danes and Woelfel (1975) revealed a strong association between ego-concept separations and evaluation ($r = .93$) such that those concepts nearest to ego were rated the most favorable. However, Green, Maheshwari, and Rao (1969) did not find support for an ego-concept evaluation association. A casual reading of their work suggests that the lack of an association between preferences and the ego-concept separations may be due to the notion that many of the products scaled were out of economic reach; thus, although the subjects in their study preferred or liked certain products, they may have felt distant from them because they could not afford them. Nonetheless, there is ample empirical evidence indicating that the ego-concept separation relationship is predictive of approach behavior; the message strategy discussed next capitalizes upon this relationship.

IX. THEORY

We begin by defining the vector space $R^{\mu}_{(\alpha)}$ where each of the contravariant vectors $R^{\mu}_{(\alpha)}$ represents the projections of the αth concept on a set of covariant (basis) unit vectors e_{μ}. In practice we expect the $R^{\mu}_{(\alpha)}$ to be the result of a multidimensional scaling analysis of a set of proximities data for k concepts where r is the number of dimensions retained. Therefore, we allow α to range over the number of concepts from 1 to k and μ over the number of dimensions from 1 to r.

We further designate the concept to be moved or manipulated (the "start" concept) as $R^{\mu}_{(s)}$ and the ideal point toward which it is to be moved as the "target" concept $R^{\mu}_{(t)}$. The object of the analysis thus becomes one of moving the start concept along the target vector $R^{\mu}_{(t)} - R^{\mu}_{(s)}$. For convenience, we first recenter the coordinate system with the start concept $R^{\mu}_{(s)}$ on the origin by the translation

$$R^{\mu}_{(\alpha)} = \overline{R}^{\mu}_{(\alpha)} - \overline{R}_{(s)} \tag{8}$$

where $R^{\mu}_{(\alpha)}$ is the position vecto
original position vector of the a
the concept to be manipulate
$\mu = 1, 2, \ldots, r$. Since $R_{(s)}$ (by
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simply accomplished: First, the
and the target vector $R^{\mu}_{(t)}$ can b

$$\theta_{(p)(t)} = \text{co}$$

where

$$R_{(p)} = |$$

$$R_{(t)} = |$$

and where the quantities $g_{\mu\nu}$ are
basis vectors, i.e.,

The $g_{\mu\nu}$ can be shown to be
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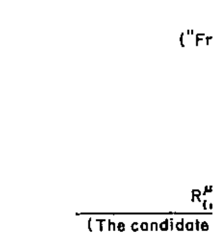

FIG. 6. Hypothetical repre

where $R^{\mu}_{(\alpha)}$ is the position vector of the αth concept after recentering, $\bar{R}^{\mu}_{(\alpha)}$ the original position vector of the αth concept, $R^{\mu}_{(\alpha)}$ the original position vector of the concept to be manipulated (the "start" concept), $\alpha = 1, 2, \ldots, k$, and $\mu = 1, 2, \ldots, r$. Since $R_{(s)}$ (by definition the magnitude or length of $R^{\mu}_{(s)}$) now is zero, the target vector is given by $R^{\mu}_{(t)}$, which is represented in Fig. 6 as the "target vector."

While our understanding of the dynamics of such spaces is very rudimentary, the original procedure is motivated by a simple dynamic assumption: When two concepts in the space are associated (formally, when they are linked in an assertion of the form "x is y"), they converge relative to one another along the line segment connecting them. (This assumption is motivated solely by assuming each concept will move toward the other by the shortest path.) In Fig. 6, the sentence "the candidate is friendly" should therefore result in a motion of the candidate concept along the vector $R^{\mu}_{(p)}$ (predicted vector) in Fig. 6. As yet, insufficient data are available to warrant predictions of the magnitude of this motion, but its direction is clearly given from our starting assumption.

Based on this assumption, determination of a single optimal issue may be simply accomplished: First, the angle $\theta_{(p)(t)}$ between any predicted vector $R^{\mu}_{(p)}$ and the target vector $R^{\mu}_{(t)}$ can be conveniently calculated as

$$\theta_{(p)(t)} = \cos^{-1}[g_{\mu\nu} R^{\mu}_{(p)} R^{\nu}_{(t)} / R_{(p)} R_{(t)}] \tag{9}$$

where

$$R_{(p)} = |R^{\mu}_{(p)}| = [g_{\mu\nu} R^{\mu}_{(p)} R^{\nu}_{(p)}]^{1/2} \tag{10}$$

$$R_{(t)} = |R^{\mu}_{(t)}| = [g_{\mu\nu} R^{\mu}_{(t)} R^{\nu}_{(t)}]^{1/2} \tag{11}$$

and where the quantities $g_{\mu\nu}$ are given by the scalar products of the covariant basis vectors, i.e.,

$$g_{\mu\nu} = e^{j}_{\mu} e^{j}_{\nu} \tag{12}$$

The $g_{\mu\nu}$ can be shown to be a covariant tensor of the second rank which defines the metric properties of the space and is therefore referred to as the fundamental or metric tensor. If the covariant basis vectors e_{μ} are real and

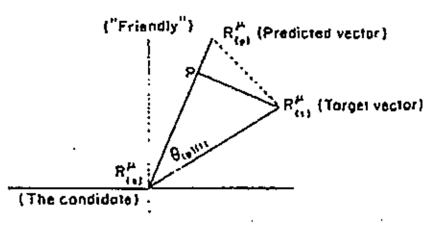

FIG. 6. Hypothetical representation of a multidimensional scaling space.

orthogonal, then the $g_{\mu\nu}$ take on the familiar form

$$g_{\mu\nu} = \delta^{\mu}_{\nu} = \begin{cases} 1 & \text{if} \quad \mu = \nu, \\ 0 & \text{if} \quad \mu \neq \nu \end{cases}$$

The numerator of the parenthetical expression in (9) is the tensor notation for the scalar product of $R^{\mu}_{(p)}$ and $R^{\nu}_{(t)}$; this alone may be seen as the product of the two vector lengths and the cosine of the angle between them. Dividing through by the vector lengths [given by (10) and (11)] leaves the cosine of the angle between them. That concept whose position vector forms the smallest angle with the target vector will represent the concept which lies most nearly in the *direction* of the "me" or ideal point. The amount of change advocated by this message strategy is given straightforwardly by the length of the predicted vector $R_{(p)}$, which is given by Eq. (10).

Although it is common practice for psychometricians to retain only real eigenvectors or dimensions, it is the prevailing practice of many communication researchers to perform metric analyses of ratio-scaled data averaged over very large samples, and most frequently all or nearly all eigenvectors are retained, including the imaginary eigenvectors (Woelfel, 1977). As we suggested earlier (Section II) these imaginary eigenvectors are the result of violations of the triangle inequalities usually found in empirical data sets. Since these violations occur reliably and frequently, communication researchers are usually unwilling to transform them away by nonmetric procedures, and must, therefore, make provision for them in analytic algorithms such as these.

Where the pth through rth roots are negative (corresponding to imaginary eigenvectors), the $g_{\mu\nu}$ are given by

$$g_{\mu\nu} = \begin{cases} 0 & \text{if} \quad \mu \neq \nu, \\ 1 & \text{if} \quad \mu = \nu < p, \\ -1 & \text{if} \quad \mu = \nu \geqslant p \end{cases} \tag{13}$$

Given these considerations, and Eqs. (9)–(11), we can now solve any part of the triangle $R^{\mu}_{(s)}R^{\mu}_{(t)}R^{\mu}_{(p)}$ in Fig. 6. If the message equating the start concept $R^{\mu}_{(s)}$ with the concept $R^{\mu}_{(p)}$ were completely successful such that $R_{(s)}$ moved to $R_{(p)}$, then the distance between the start concept and target concept (after the message) would be given by the distance

$$S_{(pt)} = |R^{\mu}_{(p)} - R^{\mu}_{(t)}| \tag{14}$$

Such an outcome is very unlikely in most cases, since we could at best assume the point represented by $R^{\mu}_{(s)}$ would move only part of the distance toward $R^{\mu}_{(p)}$. The point P in Fig. 6 represents the orthogonal projection of $R^{\mu}_{(t)}$ on $R^{\mu}_{(p)}$ and gives the point of closest approach to $R^{\mu}_{(t)}$.

The length of this line segment is given by

$$|PR^{\mu}_{(t)}| = R_{(t)} \sin \theta_{(p)(t)} \tag{15}$$

where $\theta_{(pt)}$ is as given in (9). Similarly, the distance along $R^{\mu}_{(p)}$ that the start

concept must travel to reach P is

$PR^{\mu}_{(s)} = |Pl$

The percentage of change a
message to have its maximum ef

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These calculations, along with
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Multiconcept messages are ve
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Equation (18) can easily be g
number of such combinations of
becomes large.[2]

Evaluation of the degree of su
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The target vector across Δt is de

$R^{\mu}_{(t)}$

[2]In practice, the Galileo™ compute
messages with up to four concepts to de

concept must travel to reach P is given by

$$PR^{\mu}_{(s)} = |PR^{\mu}_{(s)}| = |PR^{\mu}_{(t)}|/\tan \theta_{(pt)} \tag{16}$$

The percentage of change advocated that must be achieved for this message to have its maximum effect is given simply by

$$\Delta\% \max = 100|PR^{\mu}_{(s)}|/R_{(p)} \tag{17}$$

These calculations, along with an empirically measured estimate of the proportion of advocated change actually to be expected, provide ample data on the basis of which the optimal single issue may be chosen.

Multiconcept messages are very easily (and similarly) determined on the basis of an additional assumption: Messages average like vectors in the space. This is equivalent to the assumption that order effects (like primacy-recency) are negligible over the life of the message campaign. Based on this assumption, the position vectors or any two or more issues may simply be averaged to yield a resultant vector given (for two vectors) by

$$R^{\mu}_{(p)} = (R^{\mu}_{(\alpha)} + R^{\mu}_{(\beta)})/2 \tag{18}$$

This resultant vector is then taken as the predicted vector and the procedures just described are repeated.

Equation (18) can easily be generalized for n vector sums, although the number of such combinations of possible messages grows very rapidly as n becomes large.[2]

Evaluation of the degree of success of the message strategy is also simply a matter of determining the angles included between the predicted, target, and observed vectors over the time interval Δt. In practice, however, it is difficult to hold the origin of the space at $t + \Delta t$ precisely where it was at t, and so it is convenient to choose yet a different origin. In our work, we establish an origin at the centroid of the set of concepts not included or implicated in any message, and rotate the t and $t + \Delta t$ spaces to least squares best fit among only those unmanipulated concepts. This procedure may be seen as an effort to use the unmanipulated concepts to determine a stable frame of reference against which the relative motions of the manipulated concepts may be gauged. Time one variables transformed into these stable coordinates will be represented by barred tensors [e.g., $R^{\mu}_{(s)(t1)} = \bar{R}^{\mu}_{(s)}$] and time two variables will be represented by hats [e.g., $R^{\mu}_{(s)(t2)} = \hat{R}^{\mu}_{(s)}$] as shown in Fig. 7. (The reader should be careful to note that these bars do not mean "means," and the hats do not mean "predicted values" as is common in statistical usage.) Given these definitions we may define the predicted vector across the interval Δt as

$$R^{\mu}_{(p)} = \bar{R}^{\mu}_{(s)} - \bar{R}^{\mu}_{(p)} \tag{19}$$

The target vector across Δt is defined as

$$R^{\mu}_{(t)} = \bar{R}^{\mu}_{(t)} - \bar{R}^{\mu}_{(s)} \tag{20}$$

[2]In practice, the Galileo™ computer program with which we work computes all possible messages with up to four concepts to determine an "optimal message."

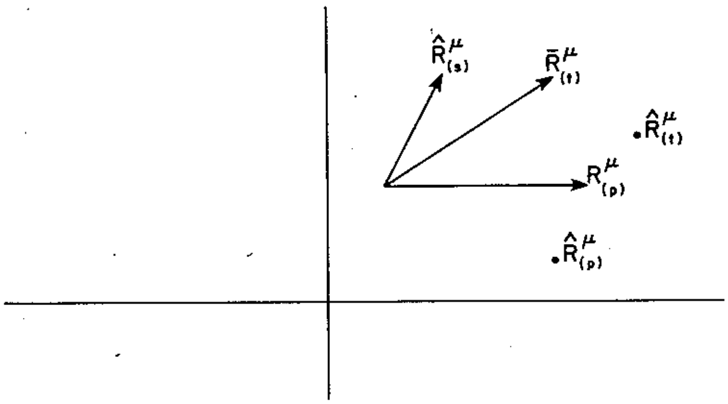

FIG. 7. Multidimensional scaling space at t and $t + \Delta t$ represented on stable coordinates.

Similarly the observed motion vector is given by

$$R^{\mu}_{(0)} = \hat{R}^{\mu}_{(s)} - \bar{R}^{\mu}_{(s)} \tag{21}$$

Evaluation of the extent to which the start concept has moved as predicted is given simply by the angle between the predicted and observed vector, which is given by

$$\theta_{(p)(0)} = \cos^{-1}\left(g_{\mu\nu} R^{\mu}_{(p)} R^{\mu}_{(0)} / R_{(p)} R_{(0)}\right) \tag{22}$$

Also of interest is the extent to which the start concept has moved in the direction of the target, which is given by

$$\theta_{(t)(0)} = \cos^{-1}\left(g_{\mu\nu} R^{\mu}_{(t)} R^{\mu}_{(0)} / R_{(t)} R_{(0)}\right) \tag{23}$$

There are further considerations. While these equations are sufficient to indicate the basic structure of the procedures, many valuable modifications can be derived easily by the interested reader. One such example is the unweighted summation of vectors in multiconcept messages given by Eq. (18), which assumes each concept to be equally effective. This assumption may be relaxed by providing weights $\beta_{(\alpha)}$ such that (18) is replaced by

$$R_{(p)} = \sum_{\alpha} \beta_{(\alpha)} R^{\mu}_{(\alpha)} / \sum_{\alpha} \beta_{(\alpha)} \tag{24}$$

where $\beta_{(\alpha)}$ are estimated empirically by the regression equation

$$R^{\mu}_{(0)} = \sum_{\alpha} \beta_{(\alpha)} R^{\mu}_{(\alpha)} + e \tag{25}$$

where e is a least squares error term.

The equations presented here, it may be noted, are all difference equations, reflecting the "before–after" or "treatment–control" designs typical of current practice. Clearly the emphasis on process implicit in this paper suggests a much heavier emphasis on longitudinal designs. When such data sets become available, the transformation of these equations into differential form is

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X. THREE-WAY MULTIDIM

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straightforward, particularly when orthogonal MDS routines are chosen. Thus the infinitesimal displacement of the start vector $ds_{(s)}$ is given by

$$ds_{(s)} = \left[g_{\mu\mu'} \, dR^{\mu}_{(s)} \, dR^{\mu}_{(s)} \right]^{1/2} \tag{26}$$

where the $dR^{\mu}_{(s)}$ represent coordinate differentials. Similarly, the instantaneous velocity of the start vector at t is given by

$$v_t = ds_{(s)}/dt \tag{27}$$

and the instantaneous acceleration of the start concept at t is

$$a_t = ds^2_{(s)}/dt^2 \tag{28}$$

X. THREE-WAY MULTIDIMENSIONAL MODELS

The discussion thus far has focused on the $n \times n$ (where n = the number of concepts) or two-way multidimensional model; now attention is given to the $n \times n \times d$ (where d = the number of data sources) or three-way multidimensional model. This section considers multidimensional analyses which relate data sources, individuals, experimental treatments, groups, etc., to the universe of which they are a part.

These "three-way" models require special treatment because the relationship between the spaces of individuals and an aggregate space made from those of the individuals is not obvious. As Durkheim (1938) suggests:

> Currents of opinion, with an intensity varying according to the time and place, impel certain groups either to more marriages, for example, or to more suicides, or to a higher or lower birthrate, etc. · · · Since each of these figures contain all the individual cases indiscriminately, the individual circumstances which may have had a share in the production of the phenomenon are neutralized. · · · *The average, then, expresses a certain state of group mind* [italics added, p. 10].

Thus, if one is interested in the actions of this aggregate—i.e., who it will vote for, whether or not it will go to war, repress minorities, buy a product, etc., then analyses ought to be performed on an averaged separation matrix.

This aggregate, however, may have properties which are different from the properties (and attributes) of the individuals who comprise it, since, as Durkheim makes clear, aggregation tends to obscure individual or subcultural "points of view." Given two major "points of view" from, for example, racists and nonracists, aggregating the data obtained is likely to produce ambivalence—a viewpoint that is neither racist nor nonracist. From a change over time perspective, if the population or "culture" that the aggregate represents is initially homogeneous (i.e., characterized by a common point of view) and messages are introduced into that culture which polarize them into two groups, the averaged point of view obtained may misleadingly present to the researcher no discernible changes in that culture. The use of the average gives limited and sometimes misleading information regarding the conceptual changes occurring within a collection of individuals *as individuals*.

Individual Differences

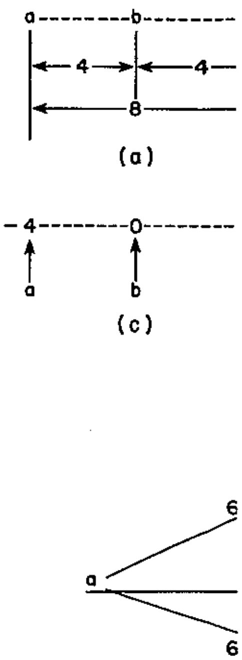
One of the first individual difference MDS procedures was supplied by Tucker and Messick (1963), and their model is known as "points of view." In this method, a matrix of observations with rows representing pairwise separation judgments and columns representing individuals is orthogonally decomposed via the Eckart and Young (1963) procedure. The resulting factor space gives the number of "viewpoints" of those subcultures that are relatively homogeneous with respect to the separation judgments of the concepts scaled. A "viewpoint" is a factor (dimension) in the factor space of individuals, and a subculture consists of those individuals with high loadings on this factor.

Presenting a different point of view about "points of view," a number of three-way metric models have been created; the most widely used of these is Carroll and Chang's (1970) INDSCAL.

Carroll and Chang (1970) began the construction of their procedure by assuming that a set of r dimensions underlying the n stimuli are common to each data source, and while assuming commonality, they reasoned that some data sources would use some dimensions for distance judgments among stimuli, that others would not, and that some would differentially weight the dimensions of the common space. Diffferential weighting of a dimension, Carroll and Chang (1970) argued corresponds to the importance or salience of that dimension for a given data source.

The result of INDSCAL is two spaces: a common space and a subject space. The common space is quite similar to an aggregate space obtained by a two-way procedure (with the exception of possible nonorthogonal dimensions and the orientation of the dimensions). The subject space yields coordinate values for each data source which represents their weights on the dimensions of the common space. For each data source, the correlation between the reconstructed separation derived from the "private" space and the original separations is used as a goodness-of-fit measure. For the solution as a whole, the average of these correlations is used.

The INDSCAL solution, however, has some disadvantages. As is typical of psychometric practice, the centrality of the individual is paramount, and INDSCAL implies a set of different spaces, each one of which has an individual at the centroid from which originate a set of (generally oblique) attribute lines of varying length. Differential weighting of attributes is given by differential stretching and shrinking of these attribute lines. What INDSCAL cannot account for, however, are order inversions of concepts across data sources. Consider two hypothetical people who conceive of three concepts that are aligned differently on the *same* attribute. Person 1 conceives the separations shown in Fig. 8a, and person 2 conceives the separations shown in Fig. 8b. A metric analysis for person yields the unidimensional space shown in Fig. 8c, and a metric analysis for person 2 yields the unidimensional space shown in Fig. 8d. However, averaging the separations



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Marlier (1976) offers a soluti
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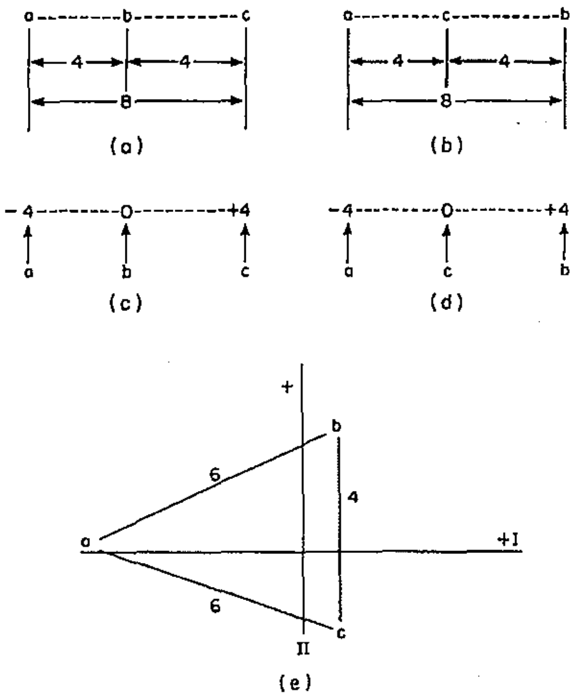

FIG. 8.

for these two individuals, and then analyzing the average $\bar{S}$ matrix does not yield a unidimensional space, as is illustrated in Fig. 8e.

Dimension I of the aggregate space does bear resemblance to the original attribute; however, an additional dimension emerges. In this space, note that no pattern of stretching or shrinking axes can account for this order inversion without the additional dimension. Why this is so can be shown by relaxing the requirement that each space be "subject centered" so that the different data sources may be free to locate at different points in the space as shown in Fig. 9.

Figure 9 shows two observers, x and y, situated among three stimuli, A, B, and C. From the viewpoint of x, these are arrayed along the continuum A_x in the order A, B, C. But to observer y, they appear to be arrayed along the A_y continuum in the order A, C, B. This can only be the case if A, B and C are *not* collinear in the joint space.

Marlier (1976) offers a solution to the "ordinal inversion" problem; the essence of Marlier's approach is the assumption that the aggregate space or cultural perception is "logically prior" to the individual viewpoint, and that individual perceptions (distortions) of the cultural "true" space are accounted

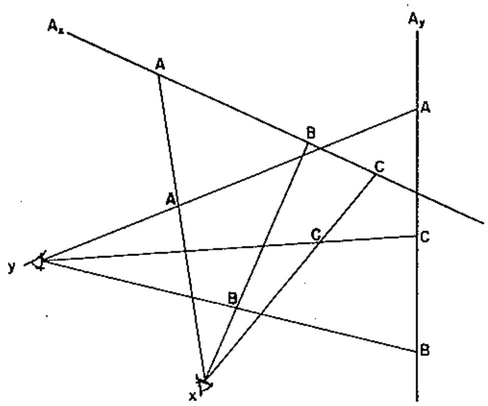

FIG. 9. Three objects **A**, **B**, **C** (boldface) appear in the order A, B, C to the observer at x, but in the order A, C, B to the observer at y. For the observer at x, A, B, C represent the unidimensional continuum A_x; for the observer at y, A, C, B represent the unidimensional continuum A_y. The joint space, however, is two dimensional.

for by different locations of individuals in the space, much like the view of Tucker and Messick (1963). Given this assumption, Marlier's approach is simple: First, a set of individuals make separation judgments among the n concepts, one of which is "ego." Second all pairwise separation judgments among the $n - 1$ concepts exclusive of "ego" are averaged into an aggregate matrix $\bar{S}$, which is $(n - 1) \times (n - 1)$. Then m additional matrices $S_1, S_2, \ldots, S_m$ are constructed by augmenting $\bar{S}$ by adding the row and column of estimates of each individual's separation of each concept to him or herself. These m matrices are then orthogonally decomposed into multidimensional spaces and then rotated using the unequally weighted rotation procedure described earlier. Weights of zero (0) are assigned to the "egos" and weights of one (1) are assigned to the common, aggregate concepts. The result is one space which portrays the aggregate common concepts along with the individual "points of view"—points which represent the view of each data source (ego). The result is an analytic paradigm which produces an aggregate space in conjunction with potentially differing "points of view."

XI. APPLICATIONS

The MDS procedure may be applied in any situation where a set of objects may be described in terms of the dissimilarities among the members of the set. Any such set may be described by means of some kind of spatial representation, and the number of communication applications which fit this model is quite large. Whenever the reliability of the measures of dissimilarity

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is high (i.e., random error of measure is small), metric MDS is the appropriate procedure.

Nonmetric procedures may be called for when two conditions are fulfilled: (1) The data are relatively unreliable, i.e., the random component in the measurements is large, and (2) the configuration of the points is known a priori to be Euclidean, that is, the configuration of the domain meets the criteria of additivity, associativity, and triangle inequalities in Section III. This is so because all nonmetric procedures consider any failure of these axioms among the measured values (data) to be solely the result of error of measure, and iteratively and monotonically transform the measured values until they meet the metric axioms as closely as possible. (Differences in the actual monotonic transforming algorithm and differences in the operational definition of "as closely as possible" distinguish the various nonmetric procedures from each other.) If *either* of these conditions is not fulfilled, nonmetric procedures will *introduce* rather than remove error from the resulting configuration. On the other hand, while the metric procedure does not constrain the data to fit the metric axioms, they will yield a metric outcome if the (measured) data themselves fit those axioms. Put yet another way, nonmetric scaling places low confidence in the original measures and, therefore, *forces the solution* to fit the metric axioms; *metric* scaling places high confidence in the metric of the original measures and, therefore, *does not constrain the solution* to fit the metric axioms.

The metric MDS procedure is appropriately used, therefore, whenever the domain of inquiry can be expressed as a relatively reliable matrix of dissimilarities among a set of objects. There are several general classes of problems within the field of communication which meet this model quite well. The intercities distance matrix of Table I is an ideal type of a class of such data where the cities are examples of any node in general (in this case locations) and the distances are examples of some physical measure like distance. Network-type problems (see Farace and Mabee, Chapter 12 of this volume) are a special case of this type, particularly when interaction rates are measured in real (clock) time. Frequencies of interaction can often be measured with high reliability, and there is no reason currently in the literature why the spatial structure of communication networks should be expected to meet the metric axioms (i.e., to constitute a real Euclidean space).

Another generic type of communication domain for which metric MDS can be an effective tool is the domain of cognitive and cultural structures and processes. Cultural or cognitive domains have often been described as sets of objects (concepts, ideas, attributes, etc.) among which people discriminate. A cultural domain which has been investigated carefully in communication is the domain of voting, where the set of objects consists of issues within which are arrayed both candidates and voters (Serota *et al.*, 1977). Linguistics and language behavior constitutes another example in which the domain may be described as a set of symbols which differ in meaning among each other. Once again, estimates of the perceived differences among psychological or cultural objects can frequently be made with good precision, particularly in the case of cultural differences, which can be averaged over a large number of cases. And there is little theoretical basis for the assumption that the structure of cognitive or cultural elements should be Euclidean—in fact most current theory would probably oppose the rationality implicit in such a model.

Special cases of such cultural domains include markets and market segments, where products, services, candidates, etc., may be arrayed among attributes, issues, and other relevant cultural objects. Metric MDS has found important applications in both business and marketing.

Because of the availability of the rotation algorithm for least squares matching of spaces, metric MDS has found important comparative applications within all these domains. Quantitative measures of the similarity of multiple communication networks, for example, are straightforward, and comparisons of subcultures (e.g., male–female, black–white) or cultures are similarly made by means of the least squares rotation algorithm. Metric MDS has thus found important applications in cross-cultural communications.

Since the same procedures apply as well to multiple times of measure (as well as cross-group comparison) metric MDS has been used extensively for longitudinal or *kinematic* studies. Thus changes in communication networks, cultural beliefs, market opinions, and so on, have been extensively analyzed by metric MDS procedures. Whenever ongoing processes are interrupted or modified by some treatment (such as television viewing, or political debates, etc.), effects may be observed by means of metric MDS, and several such studies have been done (Stoyanoff; Barnett). Since processes are central to communication, the range of application is very wide.

When hypotheses about motion are added to process or kinematic MDS studies, *dynamic* models emerge, and such models have engineering applications in both prediction of future states or processes (like elections, cultural changes, changes in structure of communication networks) and active intervention in those processes. Thus the message generation model described in Section VIII has found useful application in marketing and political campaigning as well as in theoretical studies of the dynamics of cultural change (Cody, 1977) and attitude formation (Gillham & Woelfel, 1977).

ACKNOWLEDGMENTS

We are grateful for the assistance of Peter Monge, Joseph Cappella, John Tukey, John Hunter, Robert Norton, Edward L. Fink, James H. Watt, Bob Krajcik, and Ray Cirmo in the writing of this chapter, and to the Department of Communication at Michigan State University, the Department of Speech at the University of Connecticut, and the Communication Institute at the East-West Center, Honolulu, Hawii, for assistance in the preparation of this manuscript. The authors are solely responsible for any errors.

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Chapter 12

COMMUNICATION N METHODS

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I. INTRODUCTION

The concept of a "commun broader literature on social n interpreted patterns of social pr a network of human relations. in manipulating the communic measured the frequency of char

Some years later, Barnes (19 importance of networks in anth cal methods to trace and com

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