A MATHEMATICAL PROCEDURE FOR OPTIMIZING POLITICAL CAMPAIGN STRATEGY*

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The development of public opinion polling has had substantial impact on contemporary political campaigns. Even so, the information obtainable by such procedures has been importantly limited by the unidimensional ordinal scales typically employed. Recently, however, Barnett, Serota & Taylor (1974) presented procedures by which the far more powerful and informative ratio-scaled metric multidimensional scaling techniques could be applied to the context of political campaigns. Briefly, the Barnett, Serota & Taylor procedures are as follows. First, a sample of respondents from the appropriate electorate is drawn and interviewed to determine those issues most voters perceive to be central to the campaign in question. Secondly, a larger sample is asked to provide complete pair-comparison estimates of the dissimilarities among a set of concepts consisting of issues, the candidates, and a concept called "me," which represents the voter's own position. These estimates are made following a procedure described by Woelfel (1973) and Danes & Woelfel (1975) called ratio judgments of separation, and yield a matrix S for each respondent whose entries S_{ij} comprise continuous ratio measures of perceived distances or differences among the issues, candidates and voter's own position. These matrices are averaged to yield the aggregate matrix $\overline{S}$, which represents the average position of the electorate vis-a-vis candidates and issues. This matrix is then orthogonally decomposed to yield a multidimensional spatial coordinate system R in which candidates, issues and the "me" of the average voter are arrayed. A principle characteristic of this metric space is that the distances between issues, candidates and self are identical to those in the matrix $\overline{S}$. In at least one election which they measured, Taylor, Serota & Barnett (1975) found that one candidate closest to the average voter's position represented by the "me" won the election, and that the proportion of the vote each candidate received was an inverse linear function of his distance from the "me." (The candidates in this election were all male.)

Perhaps most important, however, is the dynamic nature of the procedure. Traditionally, political campaign efforts have usually consisted of attempts to portray the position of a candidate or set of candidates as similar or close to the voter's own position. This can be accomplished graphically and conveniently in the multidimensional space R by moving the candidate through the space toward the "me," which represents the average voter's position. To do this, Taylor, Serota & Barnett identify a subset of the issues which lie in the direction of the "me" when viewed from the candidate's position. Formally, this can be represented by a coordinate system R whose columns R_j represent orthonormal basis vectors, and whose rows R^i represent the projections of the issues, candidates and "me" in the R_j columns.[1] Figure one illustrates such a space. The desired motion is given by the vector $R^m - R^c$ represented by the dashed line-segment $\overline{R^c R^m}$ in figure one.

Taylor, Serota & Barnett do not provide an analytic procedure for selecting the optimal subset of issues or concepts. Instead they rely on

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[1] It is convenient for what follows to denote column vectors with subscripts and row vectors with superscripts. Some additional notational conventions will be introduced later.

a visualization based on a set of three-dimensional sub-spaces of the general space R. This intuitive approach is imprecise and liable to serious distortion as the dimensionality or rank of the space R becomes large. These difficulties are compounded further by the empirical geometry of the cognitive spaces within which election processes take place, which seems to be non-Euclidean.

This paper presents an exact mathematical algorithm by which the maximally effective subset of issues can be selected, based on identifiable assumptions. To show how this can be done, we first center the coordinate system R on the concept representing the candidate for whom the strategy is to be devised, by the translation of coordinates

(1) $$\cancel{R^i = R^{i'} - R^{c'}} \qquad R^i_j = R^{i'}_j - R^{c'}_j$$

$$j = 1, 2 \ldots k$$

where R^i = the position vector of the ith concept after recentering

$R^{i'}$ = the original position vector of the ith concept

$R^{c'}$ = the original position vector of the candidate

Due to this recentering, the candidate's position vector R^c is now the null vector $|R^c| = 0$, and the position vector R^m representing the location of the "me" or average voter's position also represents the vector along which the conception of the candidate is intended to move. This vector is represented in figure two as R^m "target vector."

While our understanding of the dynamics of such spaces is very rudimentary, the original Taylor, Serota & Barnett procedure is motivated by a simple dynamical assumption: when two concepts in the space are associated (formally, when they are linked in an assertion of the form x is

y) they converge relative to one another along the vector connecting them. In figure two, the sentence "the candidate is friendly" should therefore result in a motion of the candidate concept along the vector R^i in the direction of R^i. This vector is labled R^i (predicted vector) in figure two. As yet, insufficient data are available to warrant predictions of the magnitude of this motion, but its direction is clearly given from our starting assumption.

Based on this assumption, determination of a single optimal issue may be simply accomplished: first, the angle α_{im} between any predicted vector and the target vector can be conveniently calculated from the scalar product

(2) $$\alpha_{im} = \cos^{-1}\left(\frac{R^i R^m}{|R^i|\ |R^m|}\right)$$

$$i = 1, 2, \ldots k\text{-}1$$

That concept whose position vector forms the smallest angle with the target vector will represent the concept that will draw the candidate most nearly in the direction of the "me." The amount of change advocated by this message strategy is given straightforwardly by the length of the predicted vector $|R^i|$, which is given by

(3) $$|R^i| = \left(\sum_{j=1}^{r} R_j^{i\,2}\right)^{1/2}$$ $$|R^i| = \left(\sum_{j=1}^{r} R_j^{i2}\right)^{\frac{1}{2}}$$

At this point the non-Euclidean characteristics of the space R must be taken into account. Non-Euclideanism in R results from "inconsistencies" in the distance judgments of the subjects, i.e., we may find a set of reported distances such that the triangle inequalities

$$S_{ij} + S_{ik} \geq S_{jk}$$

$$S_{ij} + S_{jk} \geq S_{ik}$$

$$S_{ik} + S_{jk} \geq S_{ij}$$

are not satisfied for all values of i, j and k. (This situation has been common early in campaigns measured previously, and diminishes as the campaigns go on.) These inconsistencies result in negative eigenroots and imaginary eigenvectors in R. A non-Euclidean Galileo Space has the form

$$\frac{R}{K \times R} = \begin{pmatrix} R_1^1, & R_2^1 & \ldots & R_p^1 \underline{i}, & R_{p+1}^1 \underline{i}, & \ldots & R_r^1 \underline{i} \\ R_1^2, & R_2^2 & \ldots & R_p^2 \underline{i}, & R_{p+1}^2 \underline{i}, & \ldots & R_r^2 \underline{i} \\ \cdot & \cdot & & & & & \\ \cdot & \cdot & & & & & \\ \cdot & \cdot & & & & & \\ R_1^k, & R_2^k & \ldots & R_p^k \underline{i}, & R_{p+1}^k \underline{i}, & \ldots & R_r^k \underline{i} \end{pmatrix}$$

where the boldface $\underline{i}$'s indicate imaginary numbers. Both the scalar product in (2) and the vector length in (3) will occasionally result in a product of two imaginary numbers or the square of an imaginary number, and therefore one should be careful with the signs of particularly these terms. It is convenient, therefore, to adopt the tensor notation for the scalar product

$$(3) \qquad B_{ij} = g_{\mu\nu} R_\mu^i R_\nu^j$$

as well as the Einstein convention that repeated subscripts are to be summed over.[2] For a positive space (i.e., Euclidean) the metric tensor

[2]This avoids the more cumbersome conventional notation for (3) as

$g_{\mu\nu}$ is given by

(4) $$g_{\mu\nu} = \delta^{\mu}_{\nu} = \begin{cases} 1 & \mu = \nu \\ 0 & \mu \neq \nu \end{cases}$$

Where the pth through rth roots are negative (representing a non-Euclidean space) the $g_{\mu\nu}$ are given by

(5) $$g_{\mu\nu} = \begin{cases} 0 & \mu \neq \nu \\ 1 & \mu = \nu < p \\ -1 & \mu = \nu \geq p \end{cases}$$

Given these considerations, the angle between the predicted vector and the target vector is now given by

(6) $$\alpha_{im} = \cos^{-1}\left(\frac{(g_{\mu\nu} R^{i}_{\mu} R^{j}_{\nu})}{|R^{i}|\ |R^{j}|}\right)$$

and the length of the predicted vector (the amount of change advocated) is given by

(7) $$|R^{i}| = (g_{\mu\nu} R^{i}_{\mu} R^{i}_{\nu})^{1/2}$$

where the values of the $g_{\mu\nu}$ are given as in (6).[3]

Given the length $|R^{1}|$, the angle α_{im} and the length $|R^{m}|$, where

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$B = \Sigma^{r}_{\mu=1} \Sigma^{r}_{\nu=1} g_{\mu\nu} R^{i}_{\mu} R^{j}_{\nu}$ or even $B = \Sigma^{r}_{\mu=1} \Sigma^{r}_{\nu=1} g_{\mu\nu} R_{i\mu} R_{j\nu}$.

[3]The superscripted i's in (7) do not represent tensor indices in this case, but identify the ^{i}th vector. This was also true of the i and j superscripts in (3), (6) and (8), but is confusing in (7) and (8) because of the summation convention.

(8) $$|R^m| = (g_{\mu\nu}\, R^m_\mu\, R^m_\nu)^{1/2}$$

we can solve any part of the triangle $R^c R^m R^i$ in figure two. The line segment $\overline{PR^m}$, for example, represents the closest approach the candidate can make to the voter's position while moving along R^i, and $|PR^m|$ is given by

$$|PR^m| = |R^m| \quad \sin \left(\cos^{-1}\left(\frac{(g_{\mu\nu}\; R^i\; R^j)}{|R^i|\;\; |R^j|}\right)\right) =$$

(9) $$|PR^m| = |R^m| \sin \alpha_{im} .$$

Similarly, the distance along R^i that the candidate concept must travel to reach P is $|R^c P|$ which is given by

(10) $$\frac{|PR^m|}{\tan \alpha_{im}} = |R^c P|$$

The percentage of change advocated[4] that must be achieved for this message to have its maximum effect is given simply by

(11) $$\Delta\% \max = \frac{|R^c P|\; 100}{|R^i|}$$

These calculations, along with an empirically-measured estimate of the

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[4]While there are not sufficient data to make predictions about the percentage of change advocated that will be attained, it seems from conventional attitude change studies that, within reasonable limits for at least modestly credible sources, the absolute amount of change obtained will be almost linearly proportional to the amount advocated, and this ratio can be obtained empirically by measurements made during the campaign.

proportion of advocated change actually to be expected, provide ample data on the basis of which the optimal single issue may be chosen.

Multi-issue messages are very easily (and similarly) determined on the basis of an additional assumption: messages in the space add like vectors. This is equivalent to the assumption that order effects (like primacy-recency) are negligible over the life of the campaign. Based on this assumption, the position vectors of any two or more issues may simply be added to yield a resultant vector $\hat{R}$ given (for two vectors) by

$$(12) \qquad \hat{R}_\mu = R^i_\mu + R^j_\mu$$

This resultant vector is then taken as the predicted vector R^i and the procedures just described are repeated.

Equation (12) can easily be generalized for n vector sums, although the number of such combinations of messages grows very rapidly as n becomes large. In practice, the Galileo program computes all possible one through four concept messages to determine a "best message." Several additional criteria for the evaluation of potentially "best" messages can easily be derived by the interested reader, such as the distance between the endpoints of the vectors R^i and R^m, or $|R^i-R^m|$, which might frequently exceed the present length $|R^c-R^m|$--an undesirable outcome unless the % attainment of change advocated is sufficiently low.

Evaluation of the success or failure of the predictions is given straightforwardly by the cosines (correlations) of the angles between the predicted vector R^i and the vector observed R^o across the interval of the message Δt. Given measures at two points in time t, $t+\Delta t$, we define the predicted vector across Δt as

$$R^{i2} - R^{i1} = R^{i}$$

where R^{i2} = the coordinates of R^{i} at $t + \Delta t$

R^{i1} = the coordinates of R^{i} at t

Similarly, the observed vector across Δt is given by

$$R^{c2} - R^{c1} = R^{o}$$

where R^{c2} = the coordinates of R^{c} at $t + \Delta t$

R^{c1} = the coordinates of R^{c} at t

But, due to the centering operation, $R^{c1} = 0$, so $R^{i} = R^{i2}$ and $R^{o} = R^{c2}$. Since we make no prediction about the magnitude of either R^{i} or R^{o}, then it is sufficient to confirm the prediction that $\cos \alpha_{io} \approx 1.00$, $\alpha \simeq 0.00$.

In practice, however, it is difficult to hold the center of the coordinate system precisely on the spot where the candidate concept was at t for $t + \Delta t$, and so frequently a third origin may be chosen, generally at the centroid of the issues and concepts considered stable or least likely to move across the interval based on some criterion (see Woelfel, et al., 1975). In this event the components of R^{c} at t cannot be neglected, and we require

$$(13) \qquad \frac{(R^{c2} - R^{c1})\,(R^{i2} - R^{i1})}{|R^{c2} - R^{c1}|\;|R^{i2} - R^{i1}|} = \cos \alpha_{io}$$

Once again we must consider the non-Euclidean characteristics of the space, so we let

$$R^{c2} - R^{c1} = R^{o}$$

$$R^{i2} - R^{i1} = R^{i}$$

and write

(14) $$\cos \alpha_{io} = g_{\mu\nu} R^{i}_{\mu} R^{o}_{\nu} \Big/ |R^{i}| \; |R^{o}|$$

where $|R^{i}|$ and $|R^{o}|$ are given as in (8).

Implications

The method specified above has several unique properties which make it especially efficacious for political research. First, it allows the investigator to specify the precise messages which are likely to yield the most positive audience response. The chosen message, emerging from the analysis, may be more or less complex depending on the number of concepts which yield the best approach vector.

Secondly, because the system allows for longitudinal data arrays, the investigator is able to determine how effectively a given strategy approximates a predicted angle of motion through the space. To the extent that motion deviates from a prediction, assumptions may be drawn about the resistance of the target concept (usually a candidate) to motion. In addition, as a candidate's motion deviates systematically from a predicted vector, the ratio of the observed angle and the predicted angle is a good estimation of the relative forces exerted by different concepts on the target concept.

Figure three shows an ~~possible~~ example. The predicted vector is the sum of the vectors of two message concepts. We observe that a message yields an approach which deviates from prediction which favors message concept B. Subsequent messages would thus emphasize A over B as a ratio of the regression coefficients b_2/b_1 in the equation ~~of the difference between the two angles a and b,~~ $R' = b_1 A + b_2 B + \epsilon$

The algorithm implies a degree of control heretofore unattainable in political research. First, because the data is constructed of ~~[illegible]~~ $n(n-1)/2$

pairs of n-concepts, the interrelationships between all the issues is represented in this factor solution. This means that the candidate is no longer simply interested in the effect of one variable taken independently upon his other position with a set of values. Instead, the candidate observes the interdependent consequences of simultaneously delivered strategies encompassing a variety of issues.

Currently the algorithm, called informally the automatic message generator, is being employed in conventional marketing efforts. Tests of the reliability of the predictions are being conducted on a number of populations of varying homogeneity. It should be recognized that the determining of a space is a function of the degree of similarity among members of the population. We observe that negative eigenroots grow large as the variances around the dissimilarity estimate for any pair grows large. We have been breaking populations into sub-sets when important pairs exhibit high variances. The result is independent strategies for population sub-groups with markedly different definitions of the same construct.

We have found, for example, that the concept "helpful" varies radically in its definition among two groups of educational administrators. Thus, independent spaces were generated for each sub-group and messages were created to maximize the amount of a desired change in <u>both</u> target groups.

FIGURE ONE

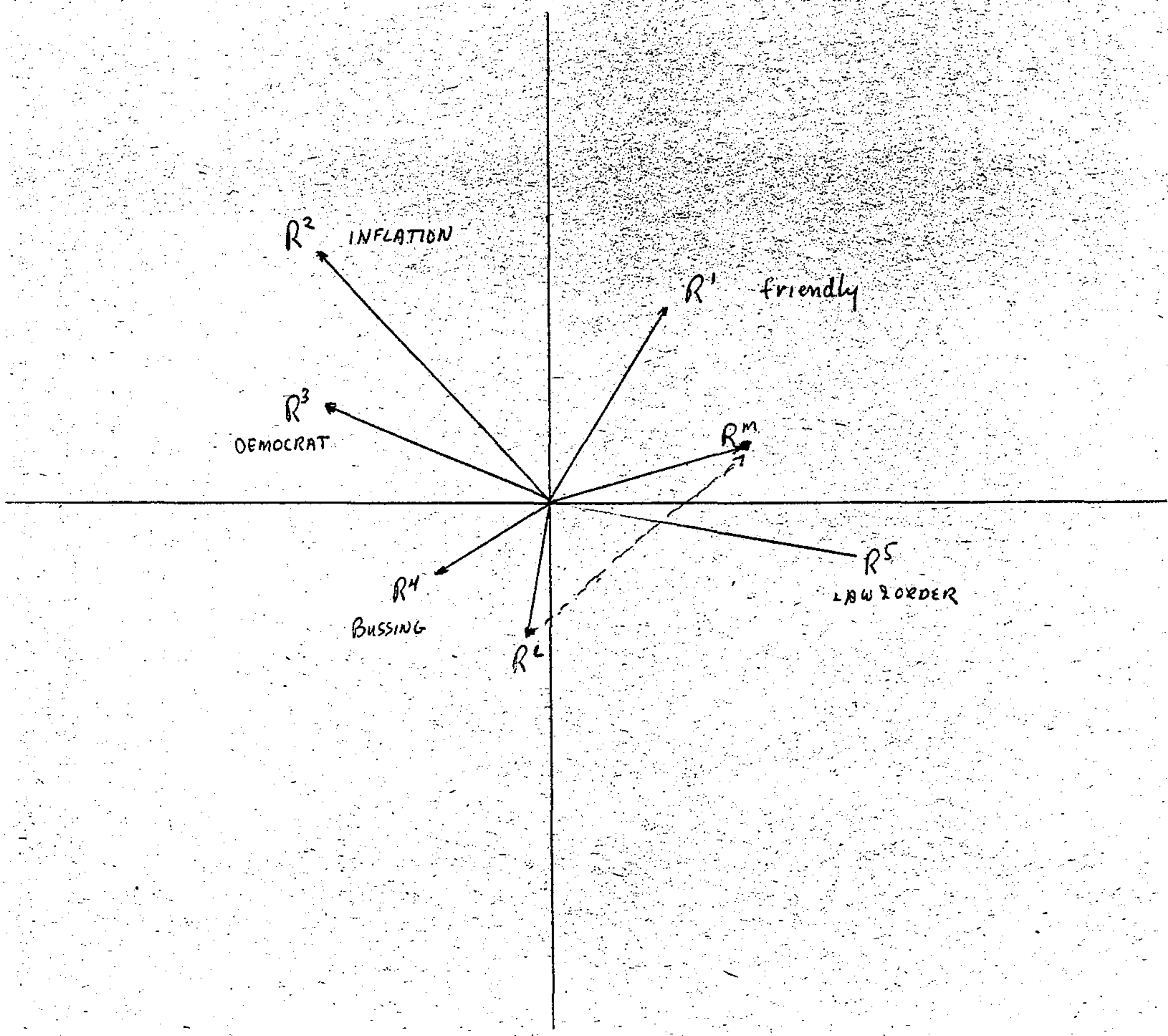

R^2 INFLATION
R^1 friendly
R^3 DEMOCRAT
R^m
R^5 LAW & ORDER
R^4 BUSSING
R^L

FIGURE TWO

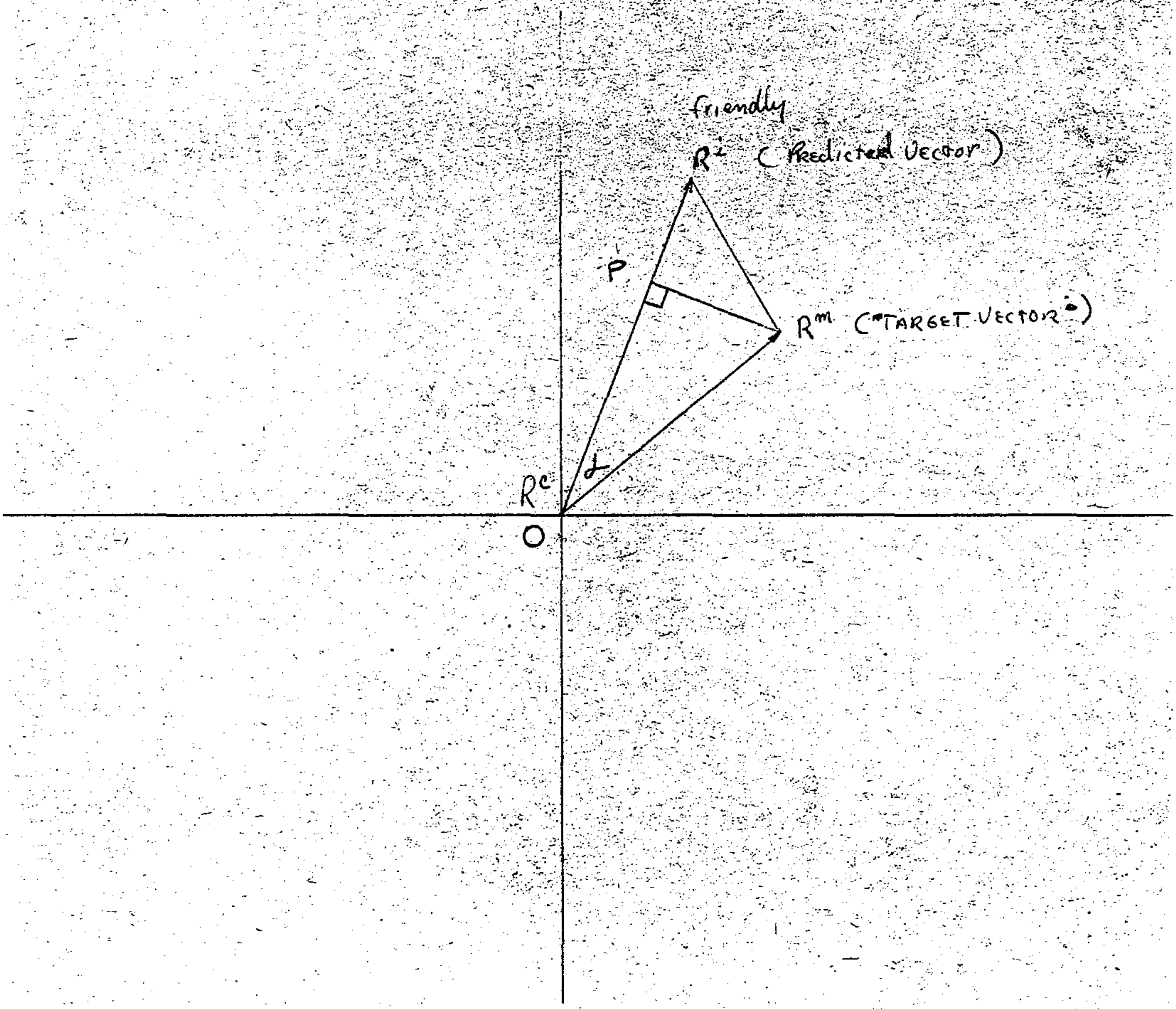
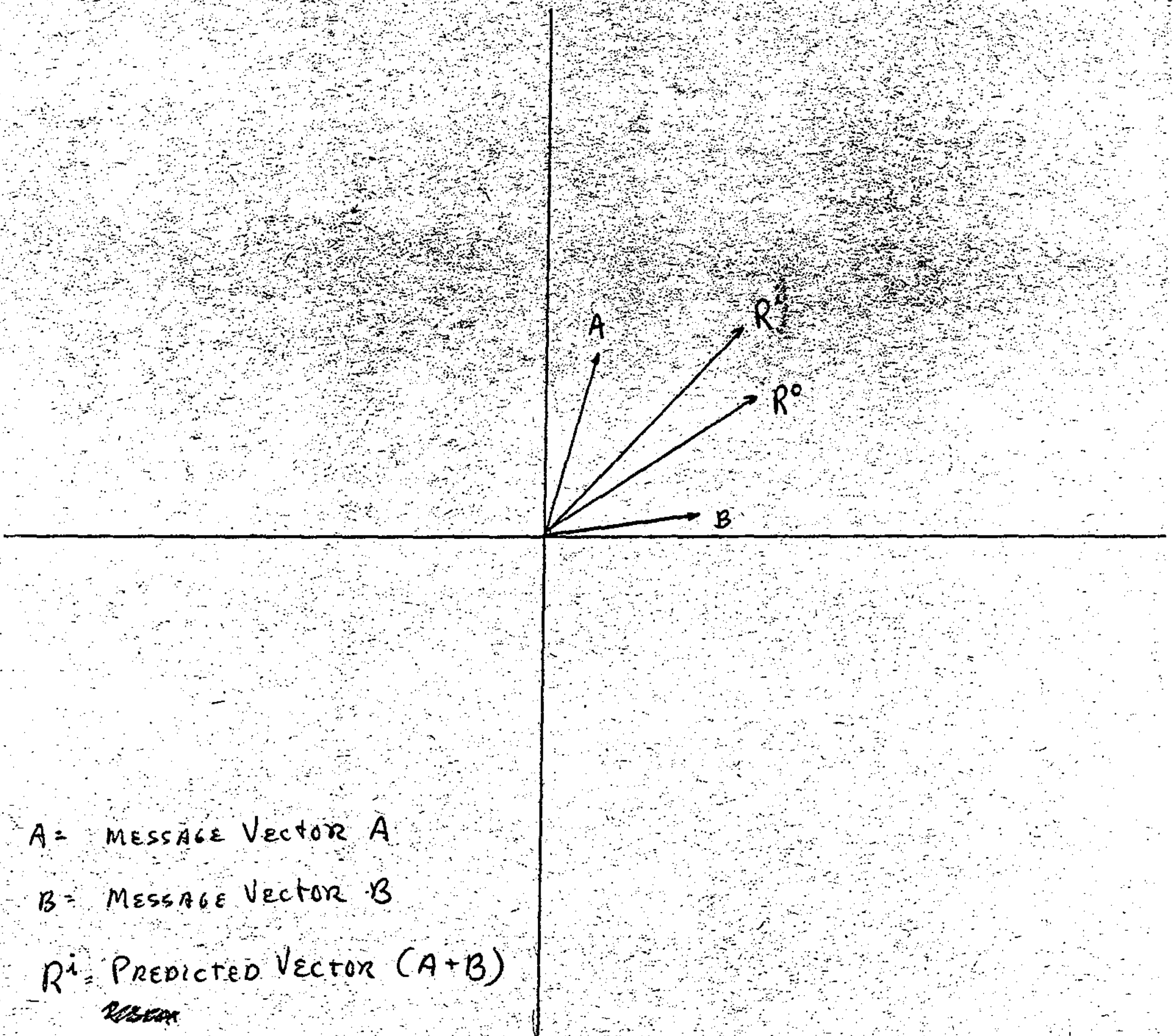

friendly
R^i (Predicted Vector)
$\vec{p}$
R^m ("TARGET VECTOR")
α
R^c
O

Fig. Three



A = Message Vector A

B = Message Vector B

R^i = Predicted Vector (A+B)

R^o = Observed Vector