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Procedures for controlling reference frame effects in the measurement of multidimensional processes

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Abstract. A wide array of perceptual mapping techniques have been developed which make it possible to describe the dissimilarities relations among datapoints as spatial arrays. While most of these present advantages and disadvantages for representing any single dataset, special difficulties arise when time-ordered data are available. These difficulties arise from the fact that the directional orientation of such techniques are (necessarily) arbitrary. When multiple datasets are scaled, therefore, the arbitrary orientations of each of the maps representing each of the time points render the description of motion and change difficult or impossible.

This problem can be solved by choosing a set of stable points within the process to serve as anchoring reference points for controlling the orientation of the individual "frames". A worked through example is provided, in which the positions of the end points of the hands of a clock are mapped over ten intervals of time using conventional methods and the method proposed. Results indicate that a satisfactory choice of stable referent points, along with a suitable choice of rotation and translation rules, can overcome the original difficulty.

The problem

It has been known since the time of Galileo that the choice of a frame of reference against which to array physical motion has a profound effect on the apparent trajectories of objects. Changes from one coordinate system to another ("Galilean transformations") are well known in the study of physical motion, and consist entirely of rotations and translations.

Modern multidimensional scaling representations of attitudes and beliefs share with measurements of physical motions the idea of projecting "objects" (in one case physical and in the other psychological) on a mathematical coordinate system which serves as a frame of reference for locating those objects. When attitudes and beliefs change, their measured location on multidimensional scaling coordinate systems also changes. These apparent motions, like their physical counterparts, are only defined up to arbitrary rotations and translations, so that repeated measures multidimensional scaling of changing attitude and belief structures actually yields an infinite set of potential trajectories.

Determination of which of this infinite family of apparent trajectories is optimal in any case depends on theoretical considerations. Psychometricians

Table 1. Distances among features on a clockface over time

	Pivot	Second	Minute	Hour	12	3	6	9
			Time = 12:00:00					
Pivot	0.00	100.00	80.00	60.00	100.00	100.00	100.00	100.00
Second	100.00	0.00	20.00	40.00	0.00	141.42	200.00	141.42
Minute	80.00	20.00	0.00	20.00	20.00	128.06	180.00	128.06
Hour	60.00	40.00	20.00	0.00	40.00	116.62	160.00	116.62
12	100.00	0.00	20.00	40.00	0.00	141.42	200.00	141.42
3	100.00	141.42	128.06	116.62	141.42	0.00	141.42	200.00
6	100.00	200.00	180.00	160.00	200.00	141.42	0.00	141.42
9	100.00	141.42	128.06	116.62	141.42	200.00	141.42	0.00
			Time = 12:07:18					
Pivot	0.00	100.00	80.00	60.00	100.00	100.00	100.00	100.00
Second	100.00	0.00	97.14	128.74	161.80	31.29	117.56	197.54
Minute	80.00	97.14	0.00	51.60	69.66	72.98	167.18	165.75
Hour	60.00	128.74	51.60	0.00	40.30	113.30	159.92	119.85
12	100.00	161.80	69.66	40.30	0.00	141.42	200.00	141.42
3	100.00	31.29	72.98	113.30	141.42	0.00	141.42	200.00
6	100.00	117.56	167.18	159.92	200.00	141.42	0.00	141.42
9	100.00	197.54	165.75	119.85	141.42	200.00	141.42	0.00

have considered the mathematical issues which underlie such "Procrustes" transformations, but have not addressed the role of theoretical constraints necessary to establish appropriate reference frames for describing temporal processes. In the present paper, a simple case of physical motion (the moving of clock hands against the face of a clock) is analyzed by repeated measures multidimensional scaling procedures. The paper shows that standard multidimensional scaling procedures in everyday use are unable to describe this simple process. It then shows simple modifications which make it possible to generate the "correct" solution.

Multidimensional scaling coordinates as frames of reference

Consider a clock face marked at the 12, 3, 6, and 9 o'clock positions. Each marker is positioned 100 units from the central pivot of the clock face. The clock has a second hand one hundred units long, a minute hand 80 units long, and an hour hand 60 units long. The center pivot, the end point of each hand, and the four hour markers make eight points whose interpoint distances at 12:00 are given in Table 1.

Table 1 shows the distances among the eight points which define the clock face (the pivot, the end points of the second, minute and hour hands, and

the four hour markers 12, 3, 6, and 9) at 12:00 and at 12:07:18 (7.3 minutes[1] later).

Because we share a common cultural understanding of what a clock is and how it works, the trajectories represented by these two distance matrices seem fairly simple. The end point of the second hand will have travelled 7.3 times clockwise around the clock face, and will stand at an angle of $0.3(360) = 108°$ from the vertical, while the minute hand will have moved $7.3(360/60) = 43.8°$ clockwise, and the hour hand will have moved $7.3(360/(60 \times 12)) = 3.65°$. The pivot and the four hour markers, of course, will not have moved at all.

As simple as this seems, it masks a very complicated Galilean transformation that we have learned to perform subliminally, and which depends on a culturally embedded "theory of clock motion" which guides our analysis of the raw observations beneath awareness. The raw data as given in the two distance matrices in Table 1 could just as easily bear other interpretations. Among these, for example, is the notion that the second hand moved $108 - 3.65 = 104.35$ degrees clockwise, the minute hand moved $43.8 - 3.65 = 40.15$ degrees clockwise, the hour hand remained motionless, and the clock face itself rotated 3.65 degrees counterclockwise. Even this simple alternative, however, as well as all the other possible rotations we might consider, rests on the greatly simplifying assumption, drawn from our subliminal clock motion theory, that the pivot has remained motionless.

Theory-free analysis

The importance of these considerations becomes clear when we attempt to describe processes in domains about which we know little or nothing; that is, domains for which, unlike our clock, we do not have an implicit theory of motion to guide our choice of rotation and translation strategies. In such cases, we do not have a conventionally agreed upon reference frame against which to project changes. This is precisely the kind of situation we would face if we measured, for example, the political positions of eight persons (or countries) relative to each other at two points in time. The results (ignoring uncertainties of measurement) of such measurements would be two 8×8 matrices identical in form to those describing the revolving clock hands. But, in the absence of any theory which defines a "preferred" frame of reference, there is no way to choose among the infinity of possible sets of trajectories describing the changes in the matrices over time.

Table 2. Coordinates of clock features at two points in time

Time	12:00		12:07:18	
feature	Dim I	Dim II	Dim I	Dim II
Pivot	30.0	0.0	13.4	17.6
Second hand	–70.0	0.0	–86.2	8.1
Minute hand	–50.0	0.0	–14.4	–57.4
Hour hand	–30.0	0.0	33.7	–38.8
12	–70.0	0.0	53.2	–74.1
3	30.0	–100.0	–78.4	–22.2
6	130.0	0.0	–26.5	109.3
9	30.0	100.0	105.1	57.5

Table 2 provides the results of a typical multidimensional scaling (MDS) analysis of the distance matrices shown in Table 1. The data were analyzed using the normal solution provided by the Galileo Version 5.4 computer program at the University of Buffalo (Woelfel & Fink, 1980). In its default configuration, Galileo calculates the principle axes of the centroid scalar products of the original distance matrices as described by Torgerson (1958). This solution is a simple linear transformation of the distances to their principle axes, so that the original metric is preserved in the solution. It should be noted, however, that the original distance matrices describe two dimensional physical distances, which meet the triangle inequality constraints, and so virtually any multidimensional scaling program, metric or nonmetric, would produce an equivalent solution up to a scaling constant for these data (Woelfel & Barnett, 1982). (Any program which normalized the data to z-scores, such as a common factor analysis, or provided any other non-linear renormalization would produce a distorted result which would further complicate efforts to identify the underlying temporal process.)[2]

As Table 2 shows, a normal multidimensional scaling analysis hopelessly obscures the "simple" motions underlying these two matrices. As a first indication of this confusion, points 1, 5, 6, 7, and 8, which represent respectively the pivot, and the 12, 3, 6, and 9 markers on the face, do not move at all in our "normal" clock motion reference frame, and should be identical across the interval. In Table 2, however, they differ substantially from time to time.

Plotting the eight features of the clock face for the first time period (12:00) yields a picture of the clock face at 12:00. Since the scaling algorithm knows nothing of "clock theory", it does not know the preferred orientation for clocks, and places the twelve o'clock marker at the left of the horizontal axis and the six o'clock marker at the right of the horizontal axis. It also produces a mirror image of a typical clock face, by placing the three o'clock marker at the bottom of the plot and the six o'clock marker at the top. Nor does

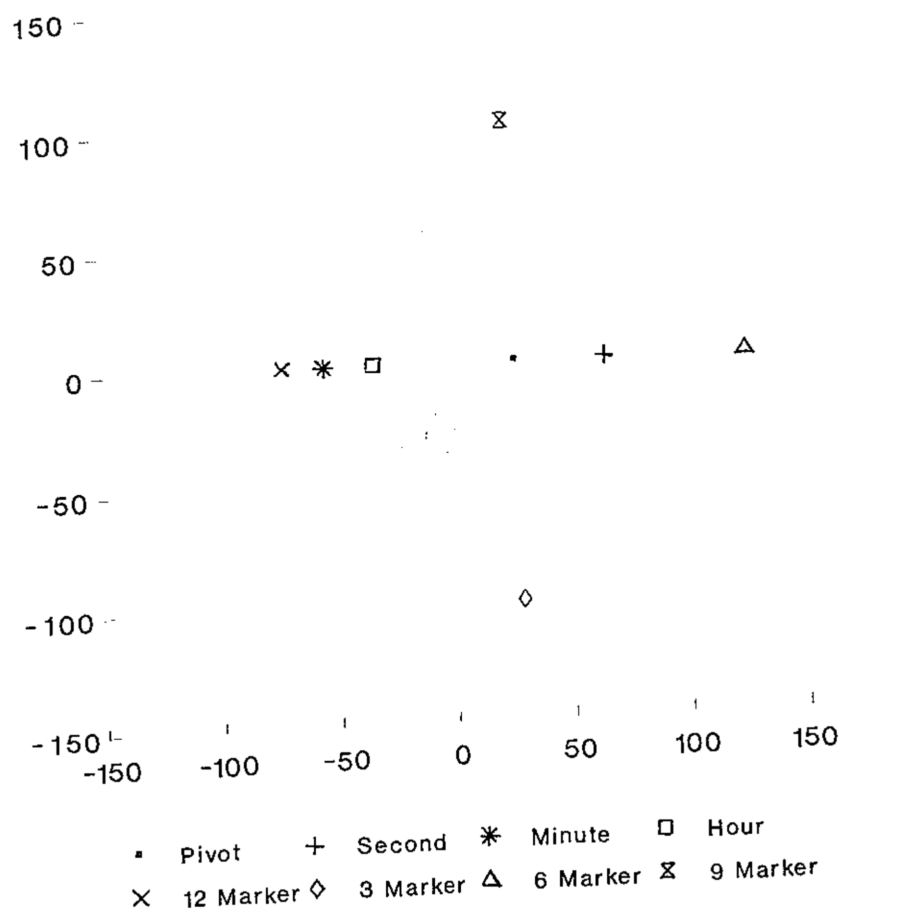

Fig. 1. Clockface at 12:00.

the program place the pivot at the center of the plot, but rather, as is nearly universal in MDS programs of whatever type, places the origin at the geometric center of the set of eight points, thirty units to the left of the pivot. (These results are presented graphically in Figure 1.)

This representation, although different from standard clock presentation practice, really presents no serious problem to the analyst, and it is likely that careful scrutiny of the plot would lead most analysts to recognize the off center mirror image as the face of a clock. The problem occurs when the second instant (12:07:18) is analyzed. Once again, the scaling algorithm has no knowledge of implicit clock theory, so it once again chooses its own reference frame in which to array the data. It might be appropriate to say that the algorithm chooses this reference frame on the basis of an implicit "multidimensional scaling theory." Although there are variants in implicit MDS theory, the most common model places the origin of the coordinate axes at the geometric center of the points, and arrays the coordinates so that the sum of squares of the projections (coordinates) on the horizontal dimension are maximized, and the successive dimensions maximize the residual sum of squares (Barnett & Woelfel, 1979).

As a result of the motion of the clock hands, the geometric center of the eight features of the clock face has shifted across the two time periods, so the pivot appears to have moved. The hour markers (which have "actually" remained at rest) appear to have rotated substantially from their positions at 12:00, and the second hand, which moved the largest distance, seems to have moved the least. Examination of Figure 2, which plots both the first and second time points superimposed on the same coordinates, shows a picture in which each of the eight features of the clock face have moved quite substantially – so much so that most analysts would have little chance of inferring the underlying simple motion.

The importance of this example can be made clear by noting that it is a common practice for analysts to compare the results of MDS analyses performed and published by different authors on different samples taken at different times, often with different item sets. The present example shows that such comparisons can be completely misleading, since even very small changes in the configuration of the data can lead to huge artifactual differences in the orientations of the resulting coordinates.

Adding additional time periods does not ameliorate the situation. Figure 3 shows the results of scaling ten periods of 7.3 minute intervals via ALSCAL. Careful scrutiny of Figure 3 reveals occasional reflections and substantial shifts of "stable" objects, and generally produces an incoherent picture which gives the impression of very substantial and unsystematic change over time. Figure 4 plots the apparent positions of only the pivot marker across the ten time periods. While a "correct" solution would show no movement at all, the marker appears to move widely across the entire configuration.

Although psychometricians have considered the problem of comparison of multiple MDS spaces from a mathematical point of view for several decades, ordinary "Procrustes" rotations of the type found in the literature will not solve this problem. Several writers have recognized the problems of arbitrary orientation in repeated measures multidimensional scaling, and proposed various algorithms for rotating multiple datasets to "best fit" one on the other (Cliff, 1966; Lissetz *et al.*, 1976). Although the specific algorithm by which this is accomplished varies, in general all procedures involve rotating

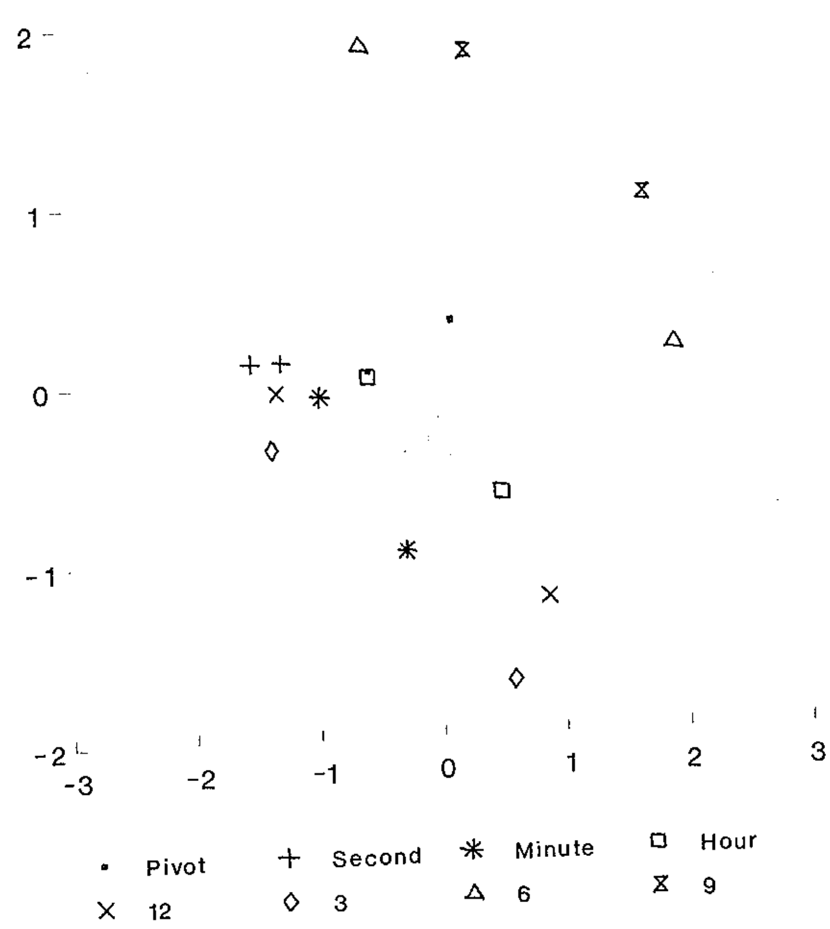

Fig. 2. Clock at 7.3 minute interval.

one or more sets of MDS coordinates about their origin until some measure (typically a least-squares criterion) of the global difference between them is minimized. Some procedures include a provision for change of scale, which allows the stretching and shrinking of one or more of the MDS spaces along with the rotation.[3]

Table 3 presents the results for the first two time periods of a typical Procrustes rotation of the coordinates in Table 2. Coordinate axes each of the configurations were rotated to least squares best fit against the coordinates of the preceding configuration using Galileo Version 5.4. Once again, since the data are Euclidean and exactly two dimensional, no significantly different

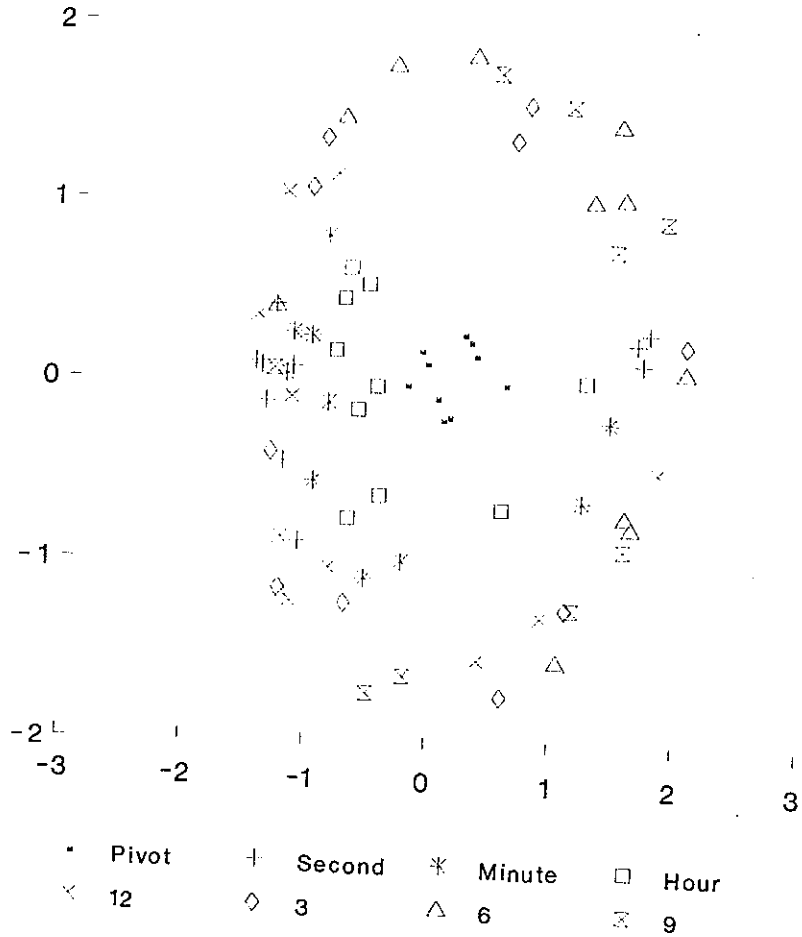

Fig. 3. Clock at 10 7.3 minute interval.

results would be likely regardless of the software used. Moreover, in the metric, Euclidean data set included in this paper, the Galileo rotation procedure produces results identical to the algorithm provided by Lissitz *et al.*, which is the classical form of Procrustes rotation known to psychometrics. No change of scale was included, since the fully metric data in the example maintained their scale exactly, and no expansion or dilation of metric could improve the fit in any case.

The results of this analysis are presented in Table 3. There is no difference, of course, in the coordinates of the 12:00:00 data, since they served as the "target" for the rotation and are therefore unaffected. The coordinates of

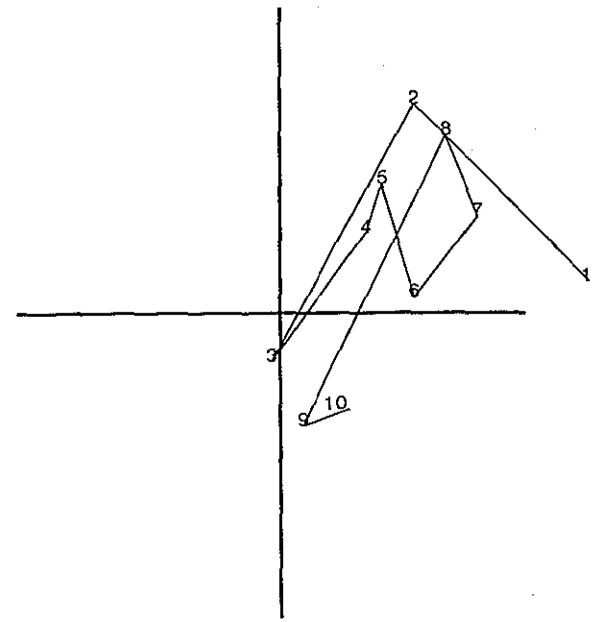

Fig. 4. Apparent motion of pivot point.

Table 3. Coordinates of 12:07:18 data after ordinary least squares procrustes rotation

Feature	Dim I	Dim II
Pivot	14.8	16.5
Second hand	24.4	−83.0
Minute hand	−53.6	−25.1
Hour hand	−44.5	25.7
12	−82.8	38.1
3	−6.9	−81.9
6	112.4	−5.1
9	36.4	114.1

the 12:07:18 data, on the other hand, have changed considerably from their unrotated values. Once again, in a "correct" solution, points 1, 5, 6, 7, and 8 should show no change whatever, but again they show substantial change.

Plotting these data shows that they still do not reveal the simple underlying clock hands motion. What they show instead is a slight motion of every feature of the clock face. Once again, adding additional time periods does not ameliorate the situation. Figure 5 shows the results of an ordinary Procrustes rotation of ten time periods, each representing an interval of 7.3 minutes.

The ordinary Procrustes rotation in Figure 5 results in a marginal improve-

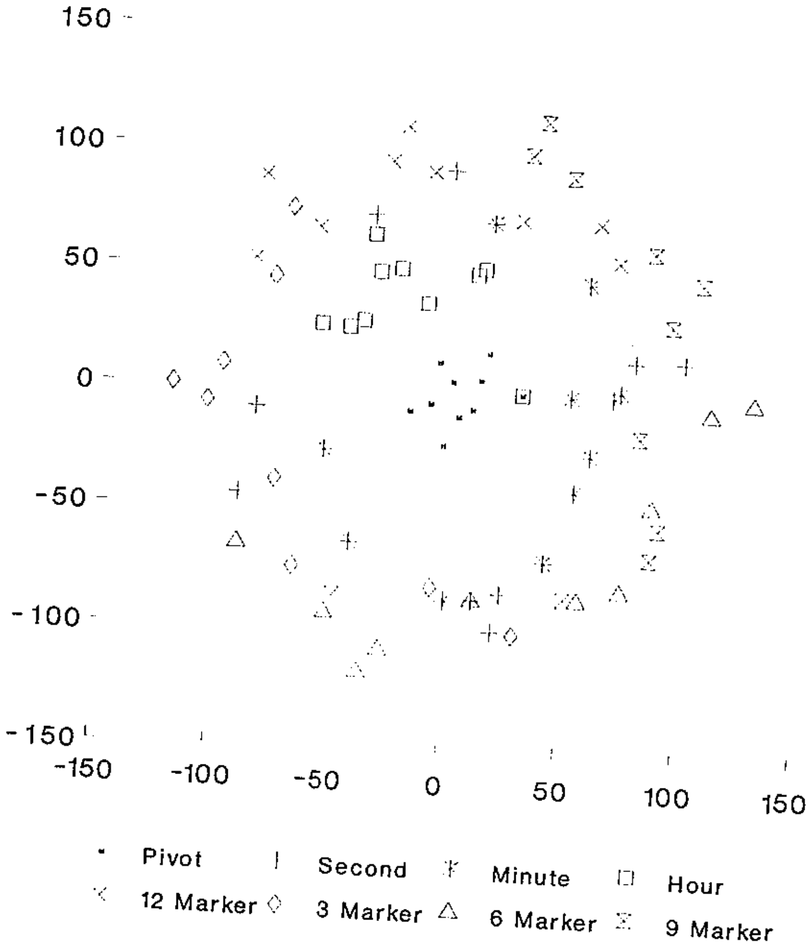

Fig. 5. Clock at 10 7.3 minute intervals.

ment over the unrotated data in Figures 2 through 4, since reflections of coordinate axes are eliminated. Nevertheless, the artifactual motions generated by the inappropriate choice of a coordinate reference frame obscure the simple rotary motions very substantially. Figure 6 shows the apparent migration of the pivot marker over the ten time periods, and once again substantial "pseudo" motion is apparent in a point which should remain completely stable.[4]

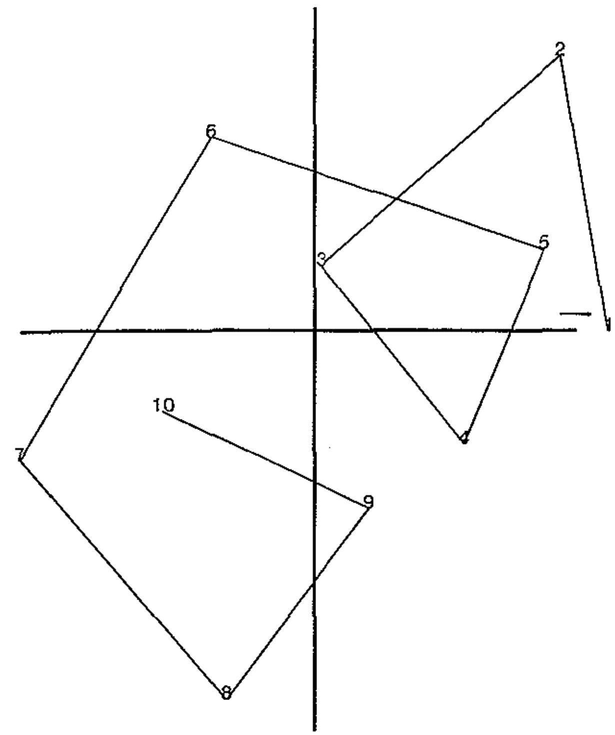

Fig. 6. Apparent motion of pivot point.

Theory guided multidimensional scaling analysis

When ordinary individuals look repeatedly at a clock, they subliminally constrain their observations by setting certain values consistent with values given by their implicit theory of clock motion. In the case of clock motion, these constraints are simply that the net motion of the central pivot and hour markers be set to zero, so that all apparent change is attributed to the motion of the hands of the clock. Once this theory has been made explicit, it can be applied as well to the multidimensional scaling solution. In the Galileo program, this is easily accomplished in two equivalent ways: one may specify any subset of the objects set as "free", or, alternatively, the remaining set may be set as "stable". (Which option is chosen is a matter of convenience.) Whichever option is chosen, the program resets the origin of the coordinate system to the geometric center of the "stable" set, and rotates all the objects in both sets about this new origin until the sum of squared distances among only the stable set is minimized. Although the "free" objects are transformed

Table 4. Coordinates of clock features at two points in time under theoretical constraints

Time feature	12:00 Dim I	Dim II	12:07:18 Dim I	Dim II
Pivot	0.0	0.0	0 0	0.1
Second hand	−100.0	0.0	31.8	− 94.8
Minute hand	− 80.0	0.0	− 57.2	− 55.9
Hour Hand	− 60.0	0.0	− 59.8	− 4.4
12	−100.0	0.0	−100.0	0.9
3	00.0	−100.0	0.9	−100.0
6	100.0	0.0	100.0	0.9
9	0.0	100.0	0.9	100.0

by the same rotation as the stable concepts, the discrepancies between their positions at one time and the next are not taken into account in establishing the least squares minimum. The program thus does not attempt to find a global minimum distance between two sets of data, but a minimum subject to the constraints imposed by the theory (Woelfel and Fink, 1980; Woelfel *et al.*, 1989).

Table 4 shows the results of applying these constraints to the Galileo solution for the clock data in Table 1. Since the algorithm still has no information about the preferred orientation of clocks, it still produces a sideways, mirror image of a clock, and on this mirror image, hand motion is counterclockwise. In all other respects, however, the program "correctly" identifies the motions implicit in Table 1 as the motions of the hands of a clock against a fixed clock face.[5] Figure 7 shows the results of carrying out the same operations beginning with 12:00:00 for ten time points at 7.3 minute intervals. As Figure 7[6] shows, the result is a fixed clock face around which the hands move appropriately.

Although the pattern of theoretical constraints applied to the Galileo solution in the present example was able to produce a solution in which clock hands moved lawfully relative to a stable clock face, one must not assume that this is a simple consequence of "self fulfilling prophecy". It is not the case that the result can be made to come out any way one likes by applying appropriate theoretical constraints. If the objects chosen as part of the stable set have indeed moved relative to each other, stipulating them as fixed objects will not hold them fixed, but merely as nearly stable as possible. If they have moved a great deal relative to each other, then a solution which tries to hold them fixed will not be able to prevent them from exhibiting a great deal of relative motion. In fact, had the clock hands been designated as stable concepts in the present example, the solution would not have been able to hold them fixed. Only if the theory from which the constraints have

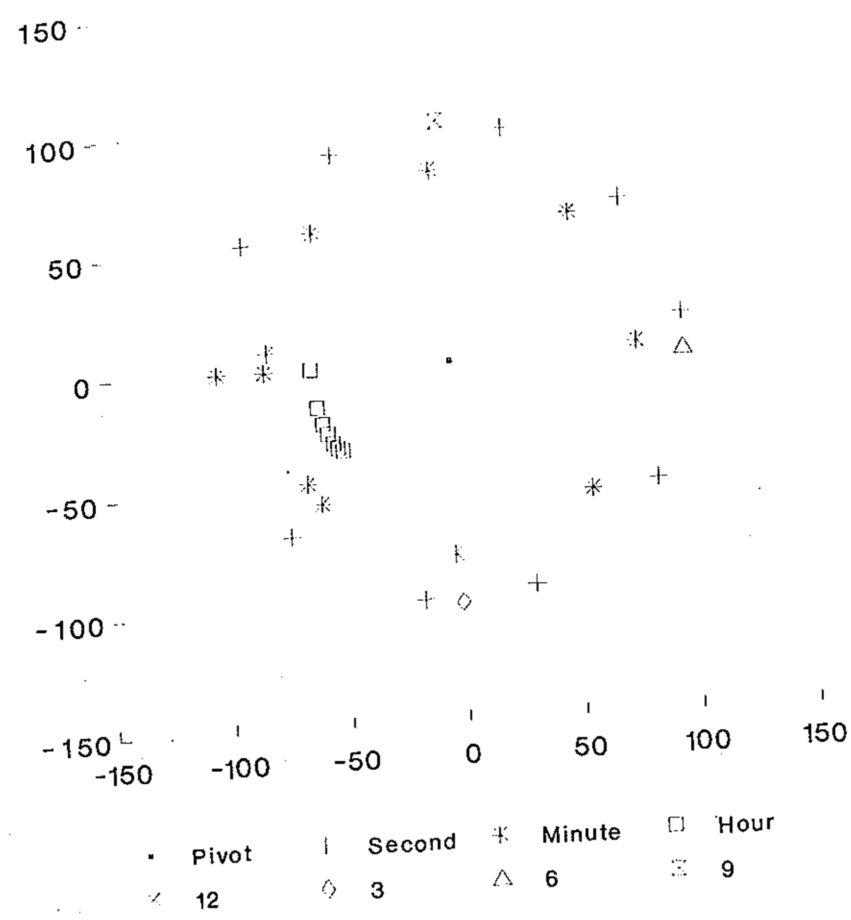

Fig. 7. Clock at 10 7.3 minute intervals.

been taken is "correct" – that is, if it is a possible interpretation of the relative interpoint motions – will the Galileo program confirm its predictions. (The same process takes place when ordinary observers look at a clock periodically. Their first effort is to apply the subliminal theory of clock motion to the repeated observations. But if the clock has fallen, or if one or more of the hour markers has come loose, the simple clock motion theory will not fit the observations, and the observer will have to select another theory to make sense of the observed pattern.)

Conclusions and implications

The results of the example provide ample evidence that reference frame effects must be considered in the description of processes, whether physical, psychological or cultural. Moreover, even in very simple processes, artifactual orientation effects of multidimensional scaling solutions can be of the same order of magnitude as the effects of the actual processes themselves.

Ordinary Procrustes transformations frequently recommended and applied to comparative MDS studies are not able to resolve the problems of artifactual orientations, and even after the application of Procrustes rotations, artifactual effects of orientation can still be as large as the effects of the processes themselves. These considerations apply not only to processes, but to any case in which two or more MDS solutions are to be compared.

No "automatic" procedure appears likely to be able to resolve these problems. Rather appropriate choices of reference frames for arraying processes appears to depend on theoretical and conventional choices. Some theory or conventional agreement which fixes stable points of reference for anchoring multiple reference frames is required to establish an appropriate orientation scheme which renders comparisons from one frame to another meaningful.

Assuming appropriate theoretical or conventional stable reference points can be identified, not all MDS software allows simple application of such procedures. The Galileo program has the capacity for easy control of reference systems for process data.

Notes

1. The reason for using a fractional interval, i.e., 7.3 minutes, is to assure that the second hand does not always point to the '12' marker.
2. These data were also analyzed using KYST and ALSCAL. KYST offers two approaches to ways to handle multiple datasets like that which describes time on the face of the clock. They may be analyzed independently, one at a time, or they may be combined into a single data set and then scaled.

   ALSCAL provided a more precise solution than KYST for any given time point; each individual time was approximately correct. However, none of the ALSCAL options provides for controlling the orientations between each of the adjacent points in time. It only provides a two-dimensional configuration for each separate clock face, each of which is oriented arbitrarily relative to all others.

   Although the results are not reported in detail here, neither KYST nor ALSCAL could be made to produce a reference frame in which the motion of the hands of the clock could be reproduced, nor is it likely that any analyst, however skilled, could guess that the data represented the motion of clock hands from the KYST or ALSCAL solutions regardless of choice of options.

3. Such "three-way" solutions as are available (such as Tucker's three-mode factor analysis and Carroll and Chang's INDSCAL model) are inappropriate to the reference frame model at hand not so much because they are atheoretical, but rather because they incorporate a theory which cannot be made to fit to the process implicit in the clock example. These models assume that there is a "common" or "joint space", and that each individual space differs from the joint space in a definable way. Thus, three way models typically provide the coordinates of the joint space, along with parameters which relate each of the individual spaces to the joint space (Woelfel and Danes, 1980).
4. It is significant that the apparent motion of the pivot exhibits the same epicyclic motion shown by the planets in a geocentric model of the solar system, perhaps the most well known illustration of an inopportune choice of reference frame in human history.
5. Constraints as to the preferred orientation of the solution can easily be entered into the Galileo program in the form of a "target" matrix, although this has not been done in the present paper.
6. Note that the mark at (−100,0) does not represent an erroneous location of the end point of the minute hand, but rather is the (correct) superposition of the "12" marker and the end point of the second hand at 12:00.

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