7 Attitudes as Nonhierarchical Clusters in Neural Networks*

Joseph Woelfel
Department of Communication
University at Buffalo

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I. THE PROBLEM

Whatever the nuances that distinguish them, virtually all attitude theories consider attitudes to be some sort of relationship between a person and an object. What kind of relationship is involved and what kind of an object is considered makes up the largest part of the difference in definitions from one theory to another.

Since Mead (1934), however, problems with the concept of "object" have multiplied. Mead questioned the unitary, absolute concept of any object, arguing instead that any object was a constructed amalgam of stimuli, assembled on the basis of situational and historical cues, changing from time to time and from situation to situation. Clearly questions of how attitudes toward objects are formed and changed are compounded if the definition of the object of the attitude is itself inconstant.

Most theories of attitude formation and change have simply disregarded this difficulty, particularly when formulating mathematical expressions (Saltiel & Woelfel, 1974; Woelfel & Hernandez, 1973; Saltiel & Woelfel; 1975; also see chapter 3 in the present volume). This is not a problem for the typically short-term experiments usually reported by these studies (see chapter 3 in the present volume) or long-term objects like career aspirations (Saltiel & Woelfel, 1975), but it detracts from the generality of any such theory. What is needed is a more general model that accounts not only for the formation of relationships between person and object, but also for the formation of the object itself.

II. INTRODUCTION

The concept of attitude was invented by Aristotle to resolve a philosophical dilemma that had frustrated Greek thinkers since Parmenides and Heraclitus. According to the principle of causality, nothing could come from nothing, which meant that everything must have a cause. Moreover, the principle of noncontradiction held that the cause must be like the effect—if the effect were motion, then motion must be in the cause; if the effect were redness, then redness must be in the cause. This thinking led Aristotle to believe that every act existed in potency before it was actualized and that these potential acts could be traced in an unbroken chain back to the original uncaused cause.

This notion, called *Aristotle's entelechy*, was meant to apply to all motion and change, so that the forms red and ripe existed in a green tomato from its first moment of existence, but became actualized as it ripened. In Aristotle's psychology, the idea of a preexisting potential act takes the form of an appetite or intention, which is a potential to respond in a certain way.

So deeply embedded in the Western mind is this Aristotelian entelechy that it underlies virtually every modern definition of attitude. Thus George Casper Homans, for example, unaware that he is simply recasting a 2,000-year-old theory in 20th century words, theorizes that "internal states" such as "drives, emotions, feelings, affective states, sentiments, attitudes" cause "activities," which are "things that people do" (Homans, 1950, pp. 34–38). Even George Herbert Mead, whose theory of self-concept is often considered a contemporary alternative to traditional attitude theory, was an Aristotle scholar, and held fast to the neo- Aristotelean concept of "impulse" in his own model.

Mead and his students did, however, undermine the simplistic Aristotelian model in several ways. First, they argued that attitudes were more than simple, individual impulses toward individual activity, but rather were socially organized orientations toward coordinated activity that arose from the interlocking role structure of a society. And second, behaviors themselves were understood as social objects, which were constructed out of many component activities by actors in situations interpreting their behavior in socially defined ways. Thus the boundaries of a specific behavior, such as "shopping" or "eating dinner" where both fuzzy and socially defined in an ongoing and creative process.

However advanced, Mead's notion of attitude, like Aristotle's and Homan's, remains abstract and unobservable, and this nonempirical characteristic is a serious problem for many, giving rise to the whole philosophy known as behaviorism, which argues that the concept of an unobservable disposition is unnecessary to the understanding of the interrelationships among various behaviors. For scholars of every persuasion, however, the absence of a mechanism that describes how behavior is generated from experience has remained a deep problem. By what mechanism are linkages between individual actors and actions made, and, perhaps more fundamentally, by what mechanism is any sequence of continuous and undifferentiated activities clustered into a named and definable act or behavior?

Even more basic is the question of how any set of multiple and disparate sensations is combined to produce a single object of perception toward which one can be said to have an attitude. Most definitions of attitudes consider them to be some sort of orientation toward an object or objects, but leave unanswered the question of what is an object and how objects come to be formed. The question of how multiple stimuli or sensations are separated from the rest of experience and defined as a single object or behavior is, in this chapter, *clustering*.

III. CLUSTERING

Since Aristotle, clustering[1] has involved assigning objects into categories based on one or more shared characteristics. All those who share the characteristics rational and animal are assigned to the category men by Aristotle, and all those cars that have four doors and cost less than $10,000 are classified as low-priced sedans by automakers.

In everyday life, however, people categorize intuitively, often without any explicit understanding of the basis of their own classification scheme. When a person shops for a new car, for example, he or she forms a category called the "consideration set" that includes only those cars actually considered for purchase. Quite often, the consideration set of the buyer includes vehicles that manufacturers do not generally see as members of the same category, such as sport/utility vehicles and luxury cars, or sedans and pickup trucks. More often than not, not even the prospective buyers themselves have any idea what characteristics these vehicles share, other than they are all cars they might consider buying.

Although comparing the average motorist's selection of a new vehicle to Aristotle's essentialistic view of category formation may seem like moving from the ridiculous to the sublime, each illustrates one of the two most common theories of clustering: (a) stimuli form into clusters because they share one or more essental characteristics, or (b) stimuli form into clusters because they have come to be related to each other due to arbitrary, perhaps random, factors.

IV. NEURAL NETWORKS

Neural networks provide a fundamentally new approach as well as a powerful new way to think about categorization and clustering. This new and fundamental aspect of neural networks follows from the fact that they represent a synthesis of new discoveries about how clustering occurs (Rumelhart et. al., 1986; Woelfel, 1993).

Neural networks do not work by maximizing or minimizing some criterion as to how clusters should be optimized, as do conventional algorithms. Rather, neural networks work by examining examples of existing clusters and "learning" to produce clusters like those studied. More precisely, neural networks become increasingly similar to their environment, and, if the environment finds a set of objects closely linked for whatever accidental reasons, these objects will be closely linked in the network. If those car buyers who consider buying Lincoln Mark VIII, Cadillac STS, and Acura Legend also consider Jeep Cherokee, then the net will link these vehicles together even if it has no clue as to what attributes they might share that makes them similar.

In practice, this means that analysts need not understand what criterion is being maximized or minimized to produce clusters of the sort needed—and it is not even necessary that there even be such criteria—it is only necessary to produce some examples of clusters that already exist. These "cases" are then studied by the neural network, which learns how to produce others like them.

A. A Backpropagation Example

There are two major kinds of neural networks: supervised and self- organizing. The best known are backpropagation supervised models, which have been well-defined elsewhere (Rumelhart et al., 1986; Woelfel, 1993). The basic back-propagation network consists of three or more layers of neurons, an input layer, one or more hidden or middle layers, and an output layer. In the simplest model, each neuron can be either "active" or "inactive"; basic biological neurons generally take on three values,[2] "off" or inactive, latent or "ready," and "active." The most powerful networks have neurons that can take on continuous values within a defined range. Initially, each neuron in the input layer is randomly connected to the neurons in the hidden layer, and each neuron in the hidden layer is randomly connected to the nodes in the output layer. The connections can generally take on any positive or negative value within a defined range.

In operation, each neuron in the input layer is identified with an input characteristic (equivalent to an independent variable in regression analysis.) Each output neuron is similarly associated with an output characteristic (equivalent to a dependent variable in regression analysis.) The network then reads a case (exactly as in regression analysis) and assigns to each input neuron the value of its corresponding input characteristic or independent variable for that case. These activation values are then propagated forward to the hidden layer through the (random) connections. Each hidden neuron sums up all inputs it receives from the input layer (these consist of the activation values of the input neurons multiplied by the connection weights), transforms them (usually by a logistic function), and, if the result exceeds a given threshold, takes on its own activation value. These activation values are then communicated in the same way to the output layer neurons, which take on certain activation values.

Because the initial connection weights in the network are random, the values taken by the output neurons will themselves be random. But the network can compare these activation values to the actual values for the case. It is straightforward from this point to write the function that relates the errors of the output neurons to the activation values of the input and hidden neurons, as well as the connection weights. The derivatives of this function are calculated, and the weights are adjusted via a quasi-steepest descent algorithm until the differences between the activation values of the output characteristics and the values of the corresponding variables in all the cases are minimized (Rumelhart et al., 1986; Werbos, 1974).

The backpropagation supervised neural network may be considered a generalized regression model that can approximate any function, regardless of nonlinearity and interactions, and is often used as a replacement for regression models (Cybenko, 1989; Dispenza & Dasgupta, 1992; Funahashi, 1989; Kurkovά, 1992). In this sense, the network is a kind of model of the attitude formation process, in that it "learns" to form a relationship between one (input) set of objects and another (output) set of objects; the relationship, of course, representing the attitude. When the network encounters a particular pattern of input objects, it will output a particular set of responses toward the output objects (i.e., it will "act" toward the objects in a certain way).

To show how a backpropagation supervised neural network can form categories without referring to a defining attribute or attributes, a simple problem was constructed using the concepts P51, P38, B17, B29, BOMBERS, FIGHTERS, ALLIES, and AXIS. Of these, the first seven were used as input characteristics (the equivalent of independent variables in regression analysis), while the last four were used as output characteristics.

Data consisted of "cases," analogous to the regression model case, where each case consisted of a specific set of values of both input and output characteristics. The first case, for example, gave these values (dependent or output characteristics below the line):

P51	1
P38	0
ZERO	0
ME109	0
B17	0
B29	0
HEINKEL	0
FIGHTER	1
BOMBER	0
ALLIES	1
AXIS	0.

This case indicates that the P51 is an Allied fighter.

Note that it is not necessary to restrict ourselves to a single "active" input characteristic per case, as the following case shows:

P51	1
P38	1
ZERO	0
ME109	0
B17	1
B29	1
HEINKEL	0
FIGHTER	1
BOMBER	1
ALLIES	1
AXIS	0.

This case says that the P51, P38, B17, and B29 are associated with fighters, bombers, and the Allies.

The network developed to deal with these cases consisted of seven input nodes, three hidden nodes, and four output nodes. It trained in 4,600 "training events" and produced a solution that allows convenient classification of any input. For example, if one were to input the following values of the input characteristics, the trained network would estimate the output characteristic values as follows:

P51	1
P38	0
ZERO	0
ME109	0
B17	0
B29	0
HEINKEL	0
FIGHTER	.89
BOMBER	.11
ALLIES	.90
AXIS	.10.

This output means that, when faced with a P51, the network classifies it as an Allied fighter. But consider the following input:

P51	1
P38	1
ZERO	1
ME109	1
B17	0
B29	0
HEINKEL	0
FIGHTER	.90
BOMBER	.10
ALLIES	.09
AXIS	.11.

This means that, when faced with two Allied fighters and two Axis fighters, the network decides they are members of the category "fighter" and declines to say whether they are Allies or Axis. Although this is a very simple example (deliberately designed to be so) it shows that there is a sense in which the neural network is nonhierarchical. The neural network does not assign each stimulus into its one best category, but assigns each input stimulus into one or another (or several) categories depending on the context in which it is seen.

If we accept the general notion that an attitude is a relationship between a person and some object, the implications of neural clustering become evident: What one considers an "object" to be at any given moment and in any situation depends on the configuration of stimuli active in that situation. Earlier considerations of attitude formation tend to attribute a unitary and permanent existence to objects and have been justly criticized for failing to recognize the situational definition of objects. What has been called Linear Force Aggregation Theory (Woelfel & Hernandez, 1973; Woelfel & Saltiel, 1978), for example, argues that the stability of an attitude toward an object is a function of the amount of information (operationalized as number of messages) that formed the attitude. But that model fails to deal with the very likely assumption that the definition of the object of an attitude changes situationally. Neural networks provide a satisfying mechanism for explaining how situated meaning works. But they go further and show how the relationship between the person and the object are simultaneously defined in the same situation: An individual defines both the object and his or her relationship to it simultaneously in real time.

1. Unsupervised Neural Networks as Clustering Algorithms

There are still problems with the supervised approach to classification and clustering, however. These networks are still hierarchical, because they treat one set of stimuli (usually elements or members of categories) as inputs and others (usually the category names) as outputs. Although it is possible to define a problem within a supervised network so that this is not so, there is an easier and more direct way to deal with the problem using unsupervised networks. Moreover, the supervised approach requires that the list of possible categories be known in advance, because they must be coded as the output characteristics. A further problem is that the backpropagation algorithm is computationally intensive, and can be very slow—and sometimes completely intractable—for large problems (Jacobs, 1988; Rigler, Irvine, & Vogl, 1991; Tolleneare, 1990; van Ooyen & Niehaus, 1992; Vogl, Mangis, Rigler, & Zink, 1988).

In an unsupervised network, however there need be no distinction between input and output nodes. (Some unsupervised plans do make such a distinction, but this is not necessary.) It is also unnecessary to define the categories in advance or even to know how many there are. It is not even necessary that each case have the same number of variables. Moreover, the interactive activation and competition (IAC) network that is described here is very fast and can train in a single pass.

2. Structure of a Simple Unsupervised Network

Consider a network of 11 nodes, none of which are connected to any of the others. Let each node represent one of the stimuli in the preceding example—that is, P51, P38, B17, B29, BOMBERS, FIGHTERS, ALLIES, and AXIS. Now we can expose this network to the data by the following rule. When it reads a "case," each stimulus that occurs in the case will activate its corresponding neuron. If the network "reads" a case that says "P51, P38, FIGHTER, ALLIES," for example, the nodes that correspond to those four stimuli will become active.

3. A Simple Learning Rule

Now we adopt a second rule (called a Hebbian learning rule, which is mathematically equivalent to Pavlov's law of association) that says that the connection among any nodes that are simultaneously active will be strengthened, whereas all others are weakened (Hebb, 1949). Clearly, after reading several cases, those stimuli that co-occur in the cases will tend to become positively interconnected in the network, whereas those that seldom or never co-occur will become negatively interconnected. The net result will be a square similarities–dissimilarities matrix that expresses the interrelations among the stimuli in a concise fashion.

4. Operation of the Network

A very wide variety of operational rules exist, but the following is one of the most powerful. Whenever a node is active, it transmits its activation to all the other nodes to which it is connected with a force that is proportional to the activation value multiplied by the connection strength. Thus, if we consider the activation value of a given node to be 1, and it is connected to a second node with a connection strength of .5, it will transmit an activation force to that node of .5. If it is connected to a third node with an activation value of −.7, it will communicate to that third node an activation force of −.7. Because this force is negative, we can consider it a force that attempts to turn that third node off rather than on.

Now, what happens to the nodes that are not active? Each of these receives activation forces from all the other nodes to which they are connected. Some of these forces are positive, and others may be negative. In a typical network, each node sums up the incoming forces (usually in a nonlinear summation function, typically a logistic) and, if the resulting sum is greater than some present threshold value, the node itself becomes active. In some networks, the activation values of the nodes may be binary—on or off, zero or one, or plus or minus one—but in the network considered here, the activation value is continuous, allowing any positive or negative number.

What results is an interaction among the nodes, each competing to turn on or off others in the network (hence this kind of unsupervised network is often called an IAC network.) In practice, in terms of our simple network, we might activate the node called "P38," which will in turn attempt to turn on some other nodes and turn off some others, depending on the connection strengths among them. As some others turn on, they will in turn attempt to turn on or off still others, and so on. (Each of these stages is called a "cycle.")

To show how such a simple IAC network can be used as a clustering machine, 35 cases were read into ORESME, a commercial implementation of the IAC network just described (Terra, 1993a, b, c).

Typical cases are shown in Figure 7.1. (Notice that there is no need for the number of variables or items in each case to remain constant from case to case.) As the

p51
p38
zero
me109
fighter
-1
b17
b29
heinkel
bomber
-1
zero
heinkel
me109
axis
-1

Figure 7.1. Typical Cases for an IAC Network
(−1 indicates the end of a case)

network reads these cases, it adjusts its connection strengths according to the Hebb rule discussed earlier. Once it has read all the cases, the final connection strengths or weights are arrayed in a square matrix that has the formal properties of a similarities matrix. This matrix can be treated statistically as if it were generated by any of a variety of conventional statistical methods.

Figure 7.2 shows a perceptual map made by the Galileo program (Woelfel & Fink, 1980) based on the similarities matrix generated by the network. In this map, we can see that planes toward the top of the space tend to be Axis planes, and those at the bottom tend to be Allied planes. Those to the right of the screen tend to be fighters, and those to the left are bombers.

In a typical conventional cluster analysis, the analyst would attempt to derive some criterion that would draw circles or spheroids or perhaps nonregular geometric surfaces that would separate the several stimuli into clusters. But the unsupervised neural net works quite differently. Instead of developing bins filled with elements, the neural net may be queried. One can enter a stimuli or set of stimuli into the network, and it will respond with the most closely related stimuli. Table 7.1 shows the results of entering various stimuli into the IAC.

As Table 7.1 makes clear, the common practice of deriving an exhaustive set of clusters, with each element occurring in one and only one cluster (as we do in a typical dendogram or Venn Diagram) is not sufficiently flexible to represent the depth of information available from the neural network. The IAC network, for example, is able to show that the stimulus *B17*, taken alone, is part of the category of all the aircraft and indeed elicits the names of all the other aircraft and

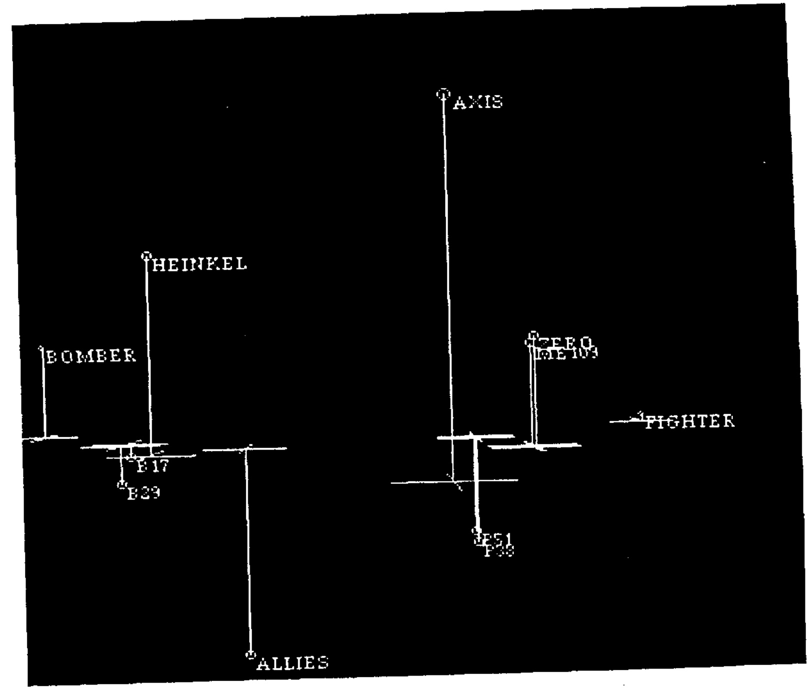

Figure 7.2. Perceptual Map of Similarities Matrix

Table 7.1 Inputs to the IAC Network (Left Column) and Its Responses (Right Column)

Input	Output
Bomber	Bomber, B17, B29, Heinkel
Fighter	Fighter, P38, P51, Zero, ME109
Axis	Axis, Zero, ME109, Heinkel
P38	Fighter, P38, P51, Zero, ME109, Heinkel
Axis, Fighter	Axis, Fighter, Zero, ME109
Axis, Bomber	Axis, Bomber, Heinkel
B17	Bomber, P38, P51, B17, B29, Heinkel
P51, B17	P38, P51, B17, B29

the category "Bomber" when input into CLUSTER. But when the same term, B17, is input into the network along with P51, only the Allied aircraft are elicited.

Moreover, Table 7.1 also shows that the IAC network is completely nonhierarchical; one can enter an element and retrieve its category name (and the other members of the category), or one can enter the category name and retrieve its elements. Indeed, the network does not treat category names differently than it does element names, and there is no distinction between input and output nodes; all are simply "objects" or "stimuli" that are more or less similar to others. This is highly consistent with the complex nature of human attitudes.

V. A BEHAVIORAL EXAMPLE

So far the examples chosen have consisted only of "things" and their abstract attributes, but objects can be any object of attention at all, even behaviors. Because they are completely nonhierarchical, neural networks can cluster behaviors along with any other objects, so that the occurance of a large enough subset of objects that are linked to a behavior can elicit that behavior. In the present case, we show how certain attributes can be attached to car purchases. To show how such a network might be used to predict vehicle choice, a simulation based on 54 cases of auto purchase behavior was run through ORESME. Each case consisted of selected demographic characteristics of a car buyer, along with the car purchased. Three typical cases are shown in Figure 7.3.

After ORESME read the 54 simulated cases, it was able to answer queries of several types:[3]

- What kind of cars would a professional, high-income male in his 50s consider? (Answer: Cadillac STS, Lincoln Mark VIII, Jeep Grand Cherokee, Pontiac Grand Am, Ford Bronco, Buick Riviera, Acura Legend)
- What kind of person might consider buying a Chevrolet Cavalier? (Answer: twenties, married, high school education, low or middle income.)
- What other cars might that same person consider? (Answer: Escort, used car, Civic, Neon)

VI. CONCLUSION

Neural networks provide a new and different way to think about attitudes and behaviors. Without contradicting the notion of an attitude as a predisposition to respond in a certain way to a certain object or situation, neural networks provide a convincing mechanism whereby behaviors can be "bundled into" a cluster or concept that includes objects, situations, and behaviors. Consistent with Mead's model, individuals encounter stimuli in recurrent situations, and the neurons that are activated in perceiving these stimuli become connected in such a way that the activation of a sufficient subset of them results in the activation of all.

TWENTIES
FEMALE
SINGLE
HIGH SCHOOL
LOWER MIDDLE
PROBE
–1
TWENTIES
MALE
MARRIED
3 CHILDREN
LOW INCOME
HIGH SCHOOL
USED CAR
–1
MALE
FIFTIES
HIGH INCOME
PROFESSIONAL
SINGLE
MARK VIII
–1

Figure 7.3. Typical Car Buyer Cases
(–1 indicates the end of a case)

This mechanism can completely explain the formation of "objects," which Mead considered to be sets of stimuli considered a unitary entity according to the interest of the perceiver. In the neural network, an object is simply a set of neurons sufficiently connected to one another that the activation of a sufficient subset will activate the remainder. Moreover, it can also explain the linkages among objects, some of which may be behaviors. Although research into the area is still very sketchy, neural networks are a very plausible mechanism for what has heretofore been a completely abstract concept.

VII. NOTES

1 In what follows, clustering, classification, and categorization are considered synonyms.
2 The term *activation value* refers in biological neurons to the rate at which the neuron is sending signals to other neurons to which it is connected. These signals are usually sent in the form of rapidly cyclical spikes of electrical energy.
3 The simulation included only brands of automobiles and a few demographic variables, so the lists are necessarily truncated. But the answers are indeed reflective of the cases in the simulated dataset and show the kinds of capability the unsupervised neural network has.

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