PROCEDURES FOR THE PRECISE MEASUREMENT OF CULTURAL PROCESSES

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While noteworthy exceptions can be found, the work of social-science methodologists and mathematical sociologists has typically entailed efforts to develop optimal research procedures for measuring and evaluating existing social science theory. While this is obviously a main function of methodology, the clear bulk of existing theory has been written prior to the development of the high-level technical skills which are available to social science now, and also in the main by those who have not specialized in methodological or mathematical work. The result frequently has been that the most powerful mathematical techniques available seldom apply closely to existing theories, and those techniques optimally matched to existing theories are less powerful than the best available.

Another strategy, perhaps less frequently employed, involves the restructuring of existing theory or the creation of new theory itself designed to fit the most powerful measurement and research models currently available. This article attempts the latter procedure. Specifically it attempts in a fairly eclectic fashion to construct a theory of self-conception and its relationship to culture which is based as far as possible on interactionist perspectives, but whose principle concepts are continuous ratio-scalable measures which fit the classical multidimensional scaling model (Torgersen, 1958) as closely as possible. The result of this process (hopefully) is the generation of theory which, while reasonably close to the existing conceptions of the discipline, has the advantage of testability of sufficient precision to allow not only rejection where false, but precise modification to fit the pattern of data actually observed.

A. The Self Conception

The identification of ones' self, that is the process of in an everyday sense knowing who one is, inherently involves the establishment of relationships to objects in the conceptual environment other than the self. This proposition is meant in a very unassuming sense, and is intended to imply nothing more than the fact that an individual, as a part of the process of defining himself, must say things about himself which imply the existence of things other than himself, and must at the same time set himself apart from these other objects. To know who he is, a man must cite a relationship between self and other objects: he must know that he is taller than a dog, shorter than a tree, stronger than his brother, more intelligent that his cat but not so intelligent as his sister, and so on. It is the totality of these relationships that gives a man his identity, and it is the uniqueness of the totality of such relationships that distinguishes him from all other men, even in his own mind.

This is tantamount to saying that the self, notwithstanding its very special character as the identity and uniqueness of any given individual, constitutes an object in the conceptual world of the individual, and is defined by means of the same processes as any object is identified and defined. Nor is the term "object" meant to reify the self or deny its processual character; by "object" nothing more is implied than a psychological content of which the individual is aware (Blumer, 1967). To be sure, the self is a very special object, highly unique, changing and developing across time and situations, but this should not obscure the more fundamental fact that it is, nonetheless, an object of which the individual conceives, and an object that is defined essentially through the same processes as more prosaic objects are defined.

The process of definition is a process of relating objects of thought to each other. Fundamentally this involves taking note of similarities and differences between objects, or identifying the attributes of an object (or the self) with similar attributes of different objects, and differentiating the attributes of the object from those attributes of the objects' which are different.

This process of identification and differentiation has usually been considered a process of categorization. So deeply imbedded in the traditions of epistemology is the notion of category that some of the most perceptive of our current psychologists consider it self evident: (Bruner, 1957)

> The first, and perhaps most self-evident point upon reflection, is that perceiving or registering on an object or an event involves an act of categorization. We "place" things in categories. That is a "man" and he is "honest" and he is now "walking" in a manner that is "leisurely" with the "intention" of "getting some relaxation." Each of the words in quotation marks involves a sorting or placement of stimulus input on the basis of certain cues that we learn how to use.

The main implication of categorization is that individuals are able to come to grips with, i.e., enter into relationships with, objects they have not yet confronted. The notion of categorization means that individual members of a class are grouped together on the basis of some shared characteristic or set of characteristics. Thus an orientation developed toward a category can be seen to govern the orientation an individual takes toward any of the constituent objects of that category. The importance of these kinds of conceptual linkages--whether or not they are aptly called categories with the discrete, nominal character that word implies--cannot be overstressed since it is this process that makes organized social life possible. Without categorization, each encounter an individual entered with an object would be wholly new and completely unique; each act would be a wholly creative process, with the definition of self and object, as well as the relationship between the two, emerging spontaneously and freely during the course of the act. To an extent, of course, this does happen, as Blumer would have us note (Blumer, 1967). To be sure, each act is in part unique, but the uniqueness of each situation should not focus attention away from the over-riding extent to which all human actions within similar contexts in the same society are similar. The most simple social transactions, such as boarding a bus, smoking a cigarette, buying a book, tying a shoe, or attending a class, imply to an overwhelming extent preknowledge of the basic structure of the transaction, the nature of the objects confronted in it, and the interrelations of the elements of the situation. Even when an individual confronts an object wholly new to him, he must construe and identify that object in terms of categories of its component characteristics with which he is familiar; he will note that it is blue or yellow, large or small, animate or inanimate, smells or does not, etc. In the case of an adult individual, the probability of encountering an object wholly unique in all its characteristics would be essentially nil. Even in that most remote case, the category "something unknown" is nevertheless a socially shared category, and has socially shared characteristics that will govern the individual's orientation and action toward the alien object.

Not all categories, it goes without saying, are of equal generality. In the strictest sense, the perception of a single object <u>as</u> a single object implies a process of categorization in that all the discrete stimuli which constitute the physiological mechanism of that perception are set apart from the totality of stimuli impinging on the organism at the time and designated as a single object of thought. The category renders discrete what is really a continuous process of exposure to stimulation by the environment, whereby an arbitrary segment of a continuum of stimulation is set aside and referred to as "a perception." Although this limiting case dramatizes the interposition of the concept between object and preception, this is perhaps the lowest level of categorization in the self. Each of these "object-categories" is itself in turn a member of one or more general category, and so on until the most general levels of categorization, such as "material" or "existing" or such. The higher a position in this hierarchy of categories a given category holds, the more pervasive will be its influence over the definitions an individual holds of himself and the world.

Such a view is compelling, but closer scrutiny reveals that it is only approximate. The notion of category, mathematically speaking, implies a two valued function and no more, i.e., an object is either perceived as a member of a category or it is not. Yet such precise two valued function seldom if ever occur in human perception. The categories Bruner italicizes in his earlier statement are illustrative: if the notion of "honest", for example is truly categorical, i.e., two-valued, then two men, both classified as "honest" by some observer, should be indistinguishable from each other in terms of their honesty. Even within the class "honest" however, a continuous range of variation is obviously recognized and indeed forms the basis of actions on the part of men. "Walking" of course also admits of considerable within-class variation as do "leisurely," "intention," "getting some relaxation," and so on. Even "man" admits if variation, and, ethical constraints aside, men frequently distinguish the extent to which various persons exhibit the characteristics of "manhood." What is clearly categorical, however, is not the process of perception, but the language--particularly the language of social psychologists--by which perception is described. Whorf (1956) defines this process as one of "lexation" or

> ...the level of the process of "lexation" or of giving words (names) to parts of the whole manifold of experiences, parts of which are thereby made to stand out in a semi-fictitious isolation. Thus, a word like "sky" which in English can be treated like "board" leads us to think of a mere optical apparition in ways appropriate only to relatively isolated solid bodies. "Hill" and "swamp" persuade us to regard local variation in attitude or soil composition of the ground as distinct things almost like tables and chairs. Each language performs this artificial chopping up of the continuous spread and flow of existence in a different way.

When an individual identifies two color chips as "yellow," for example, this classification does not imply that he perceives them to be the same, even with regard to color, but simply <u>similar enough to warrant description by means of the same linguistic category</u>. The visual color spectrum generally covers the range from about 4000 to 6400 Angstrom units, and research indicates that color differences of only a few Angstrom units are perceivable, yet ordinary language does not provide color terms for all these differences. Thus the ordinary language people speak (and for the most part the language of social science) allows only a crudely approximate description of the perception of color.

The point of departure for this discussion was the notion that

> ...The process of definition is a process of relating objects of thought to each other. Fundamentally, this involves taking note of similarities and differences between objects...

The notion of category enters the discussion in an attempt to lump together those objects which are "similar enough" with regard to some attribute to be considered the same. But no two objects are ever identical, even in a very limited sense, and it is probably very rare that two objects are perceived as identical in some regard, even allowing for the physiological limits of human sensation. Whatever the practical benefits of categorizing several similar objects or attributes as identical may be for everyday life, in science such a classification constitutes error of measurement and should not be tolerated.

Fortunately, such errors of classification within scientific theory need not occur, at least not due to the clumsiness of a merely categorical theoretical language. Mathematics provides a language capable of describing differences small without limit, and can describe differences much smaller than may be discriminated by human perceptual apparatus. But if the process of definition requires "...taking note of the similarities and differences among objects...", then the continuous set of positive real numbers offers a potentially error-free language for the definition of any set of social objects with a level of precision for greater than the limits imposed by human chemical sensory apparatus.*

*This point may seem overstressed, but it is quite important, particularly in response to those who argue that human and social phenomena are too subtle and complex to be described mathematically. The point, of course, is that mathematics as a language is much more precise and subtle than categorical languages like English, and in fact has a wider range of potential variation than human perception.

Dissimilarities among objects (whatever those objects may be) may be represented by a continuous numbering system such that two objects considered to be completely identical are assigned a paired dissimilarity score or distance score of zero (0), and objects of increasing dissimilarity are represented by numbers of increasing value. Assuming that the definition of an object or concept is constituted by the pattern of its relationship to other objects, the definition of any object may be represented by a 1Xn vector where d_{11} represents the distance or dissimilarity

$$d_{11}, d_{12}, d_{13}, \ldots d_{1n}$$

of object 1 from itself (thus $d_{11}=0$ by definition), d_{12} represents the distance or dissimilarity between objects 1 and 2, and d_{1n} represents the distance between the <u>1st</u> and the <u>nth</u> objects. Similarly, the second object may be represented by a second vector

$$d_{21}, d_{22}, d_{23}, \ldots, d_{2n}$$

and the definition of any set of concepts or objects may therefore be represented in terms of the matrix

$$\begin{matrix} d_{11}, & d_{12}, & \ldots, & d_{1n} \\ d_{21}, & d_{22}, & \ldots, & d_{2n} \\ \vdots & \vdots & & \vdots \\ d_{n1}, & d_{n2}, & \ldots, & d_{nn} \end{matrix}$$

where any entry d_{ij} represents the dissimilarity or distance between <u>i</u> and <u>j</u>.

As we suggested at the beginning of the discussion,

> ...to know who he is, a man must cite a relationship between self and other objects...it is the totality of such relationships that distinguishes him from all other men, even in his own mind.

Clearly the matrix D has the abstract capacity to describe all the possible interrelationships among any number of objects, and just as clearly, there is room for infinite variety. First of all, D can represent anything that can be said in categorical terms with no error of translation as the following matrix makes clear:

	apple	banana	red	yellow	round	fruit	good
apple	1	0	1	0	1	1	1
banana	0	1	0	1	0	1	1

The first row of the matrix shows that apple has the quality apple (by definition), red, round, fruit, and good. The second row shows that banana has the quality banana (again, by definition), yellow, fruit and good. Since it uses only a categorical logic of classification, however, the entries in the matrix are restricted to zeros (absence of a quality) and ones (presence of a quality). Obviously, such a restriction eliminates a great deal of information about both bananas and apples that is available even in terms of just these qualities. No apple, for example, is completely round, yet is certainly rounder than a banana. While both bananas and apples are seen to be "good" in this example, the categorical logic does not allow any difference in "goodness" to be expressed.

If the entries in the matrix were allowed to take on any positive real value, however, and the number of objects to which apple and bananas were compared were to be increased without limit, then clearly the range of sublety and complexity--the nuances of meaning that could be conveyed in this matrix, would be adequate to far beyond the range of complexity of human perception. The richness of description made possible by this model is made clearer still when it is understood that D represents the static structure of the interrelationships among the set of N objects at any instant in time, and that, as time passes, the processual character of these relationships can be captured in successive matrices D_{t0}, D_{t1}, ...D_{tn} where the intervals between time periods, 0, 1, 2, ...n, can be made as small as desired. + feasible.

It is the point of this discussion to make emphatic that no structure or process, physical or psychological is so complex that it cannot be represented in such a set of matrices. Without doubt, such a set of matrices is inherently capable of representing the full complexity of the self conception of any individual, or of any set of individuals. To be sure, in practice no one could, or would want to, represent the full complexity of any single self conception, since the number of objects in the set n which constitutes any single self conception is so large that even the reading of the matrix would be impossible were it to be known in the first place. Nor, for that matter, is it even conceivable that any individual himself could entertain an awareness of the totality of his own self-concept at any point in time. What is of importance, however, is this: if the matrix D is capable of rendering accurately the totality of the self, then any subset of D is certainly capable of properly describing any subset of the self, and can serve as an accurate tool for the description of any part of the self conception, such as an attitude or belief or set of beliefs, etc.

These considerations aside, the self conception in a static state, i.e., a person's instantaneous definition of self at a point in time, is defined theoretically as the NXN matrix D, where any entry dij = the distance or dissimilarity between objects i and j as perceived by the individual, and where N - the number of objects of which that individual holds a definition, however tentative. Elements of the self, such as attitudes, beliefs and the like are similarly defined as subsets of D.

B. Culture

While this view is clearly a psychology of individual cognition and not a treatment of "culture", it may be seen to have strong implications for a theoretical definition of culture that is at the same time conceptually powerful and precisely measurable.

The meaning of "culture" in social science is unclear, although the range of phenomena designated by the term is generally confined to those aspects of belief, attitude, ritual and patterned activity widely shared by members of a social system. (See Gillham, 1972) While the word "shared" is emphatic in any consideration of the belief patterns which make up a culture, it is also somewhat misleading, since of course no two members of a culture will (or can) share exactly the same view on any topic, much less maintain such consensus across all topics. Nonetheless, if the concept of culture is to have any meaning, then there must be some central tendency of opinion around which individual beliefs may be seen to cluster themselves more or less cohesively. Not only is this core of opinion central to the culture, but clearly it is thought to exercize a constraining effect upon those persons exposed to it, such that deviation from the central belief or practices of a culture results in forces compelling the deviant to readjust his belief or activity toward the central tendency once again.

Unfortunately, many treatments of this compelling character of culture tend to give the impression that the culture "intends" to reduce deviation and deliberately exerts corrective pressures when deviance is noted, but such anthropomorphism serves mainly poetic purposes. In fact, this compulsion is assumed in this article to be a natural phenomenon divorced entirely from any intentionality as the following citation of Durkheim (19) implies:

> On the one hand, all internal life draws its primary material from without. We cannot reflect our own consciousness in a purely undetermined state; in this shape it is inconceivable. Now consciousness becomes determined only when affected by something not itself.

In more primitive terms, this means simply that one cannot think without something to think about, and the information which provides the basis for thought is always gained from outside the individual mind. In the most fundamental and important sense, therefore, the culture compells individual thought because it provides the body of information out of which and about which thought is generated. If two members of the same culture, then, receive essentially the same basic corpus of information from the same sources, no extraordinary mechanism need be postulated to explain why they should think and act similarly. In fact, at any given point in time, the larger cultural environment of the whole set of members of a society will be largely similar. All will be faced with a common language, a general climate of religious belief, a common set of architectural styles, a common pattern of law and of basic notions about the proper activities of individuals in general and in specific cultural locations. It is impossible that any member of the society should go unaffected by these general cultural components or collective representations, and to this extent the core psychological structures of all members of the society will be similar. As the content and character of the collective representations shift, so too will the character of the population.

It is the position of this work that the collective consciousness, i.e., that aggregate psychological configuration which constitutes the culture of a society and toward which individual beliefs may be seen to tend, may be represented accurately as the average matrix $\bar{D}$, where any entry $\bar{d}_{ij}$ is the arithmetic mean conception of the distances or dissimilarities between objects i or j as seen by all members of the culture. To be sure, this is a position with which Durkheim would not agree, since he believes the collective representations must be more than a simple average, as witnessed by their power to compel individual compliance. It is, in fact, this power of constraint that makes the collective representations interesting to Durkheim, and at the same time makes them mysterious and "larger than life", i.e., too large and powerful to be "mere" averages.

Yet while Durkheim frequently argued that the emergent character of the collective representations made them more than the simple sum or average of the individual conceptions, it is the mean that he not only uses in his own empirical work, but prescribes as proper sociological analysis.

> ? ...Currents of opinion, with an intensity varying according to the time and place, impel certain groups either to more marriages, for example, or to more suicides, or to a higher or lower birth rate, etc. These currents are plainly social facts. At first sight they seem inseparable from the forms they take in individual cases. But statistics furnish us with the means of isolating them. They are, in fact, represented with considerable exactness by the rates of births, marriages and suicides, that is, by the number obtained by dividing the average annual total of marriages, births, suicides, by the number of persons whose ages lie within the range in which marriage, births, and suicides occur . . . the average, then, expresses a certain state of the group mind (l'ame collective) (Durkheim,).

No doubt this view of culture as the arithmetic mean of the self-conceptions of all members of the culture will be viewed as an absurd oversimplification by many investigators, but it is precisely this simplicity which constitutes its main advantage. If we assume only for the purposes of argument an individual, previously unsocialized, who receives at random messages $k_1, k_2, \ldots k_n$ about the dissimilarity between any two objects i and j, and assume further than some "cognitive consistency" mechanism like dissonance operates, then as n becomes large, the individual's definition of the distance dij might be expected to converge on the cultural average $\bar{d}_{ij}$, since $\bar{d}_{ij}$ has the powerful property that $\sum_{K=1}^{n} (d_{ijk} - \bar{d}_{ij}) = 0$. Of course these assumptions are unrealistic: individuals do not communicate at random, they may not weight each communication from each other person equally, and so on. But nevertheless, $\bar{d}_{ij}$ may be seen to operate as a central tendency much in the way cultural beliefs are typically thought to work: it is a position toward which individuals may be seen to tend, but (due to deviations from the assumptions of random communication and equal weighting of sources) with which few if any individual's beliefs would be expected to conform exactly.

This argument can be made even more explicit. Based upon the initial (unrealistic) assumptions above, any given person's conception of the dissimilarity between two concepts *i* and *j* would be given by the average of the dissimilarities and judgments he has encountered, or

$$1. \quad d_{ij} = \frac{\sum_{K=1}^{n} d_{ijk}}{n}$$

If we designate this attitude now as dij (recalling that dij is a mean score based on *n* cases) and assume that the individual receives *p* additional judgments about the dissimilarity between objects i and j with a mean given by $\tilde{d}_{ij}$ (thus relaxing the assumption that the person is previously unsocialized), then the same logic yields the equation*

$$2. \quad d_{ij}^{1} = \frac{n d_{ij} + p \tilde{d}_{ij}}{n + p}$$

where d_{ij}^{1} = the person's new conception of the dissimilarity between *i* and *j*.

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*This equation is the equation for the change in a mean given additional values.

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We can see immediately that dij (the new opinion of the individual) will be substantially different from his old opinion to the extent that the absolute value of pdij is large relative to that of ndij. This can only by the case if the average of new judgments he receives is substantially different from his old opinion, and if p is large relative to n. This expression states explicitly that the force generated to change an attitude or belief in an individual is a function of (a) the amount of discrepancy (deviance) between the attitude of the individual and the culture of the society and (b) the amount of information exchanged about that discrepancy.* This allows for the possibility that very small forces would be generated even in the case of major deviations if those deviations are not the subject of interaction between the deviant and the larger society. This expression can therefore account quite easily for the tendency of societies to "allow" large deviations from standard cultural practice if those deviations are not important or visible. More precisely, if such deviations are not relevant to day-to-day commerce so that they are not raised as topics for information interchange during the course of everyday interaction, then the net force toward conformity will be small even though the deviation is measureably large.

While quite simple in its formulation, then, this theory is not at all over-simplified in terms of the empirical complexity it may embrace. In fact, much more empirical complexity can be described in the continuous logic of dissimilarities of the matrix $\bar{D}$ than in the categorical verbal logic which recognizes only the presence or absence of attributes, i.e., membership or non-membership in categories. Moreover, more than merely defining culture as a pattern or set of patterns toward which social events are in some fashion compelled to conform, this view allows an exact calculation

*It is also clear from this equation that the stability of the old attitude dij is wholly a function of n, or the amount of information the individual has previously accumulated about the topic. Thus long-standing attitudes important to the individual will be more stable than new attitudes about which the individual has received relatively little information.

of the extent any subset of any self-conception or set of self-conceptions deviates from the cultural pattern, as well as an exact calculation of the amount of force which will be exerted toward conformity.*

C. The Measurement of Culture and Cultural Change

The theory presented so far has as its most primitive concept the notion of distance or dissimilarity, and the problem of measuring the variables in the theory therefore, reduce to the problems of measuring distances. It is a fundamental belief of this work that the measurement of these psychological or cultural distances is more closely analogous to the measurement of physical distances than is usually supposed. In fact, as Einstein argues (Einstein, 1961)

> For this purpose [the measurement of distance] we require a "distance" (Rod S) which is to be used once and for all, and which we employ as a standard measure. If, now, A and B are two points on a rigid body, we can construct the line joining them according to the rules of geometry; then, starting from A, we can mark off the distance S time after time until we reach B. The number of these operations required is the numerical measure of the distance A B. This is the basis of all measurement of length.

Similarly, the measurement of the distance among objects of cognition can be accomplished simply by arbitrarily designating the distance between any two cognitive objects as a standard and comparing the distances (i.e., dissimilarities) between

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*For purposes of easy exposition we made the unrealistic assumptions that a) individuals communication at random without limit with other numbers of the culture, and b) that information from all sources is weighted equally by all persons. Neither of these assumptions need be made, and in fact, if they were true, all members of a culture would become identical over time. It is precisely the failure of these assumptions that allows heterogeneity in culture in this view. Realistically, this theory would imply that culture is a compelling but not overwhelming force toward conformity, and such a view corresponds well with everyday experience.

any other pair of objects to this standard.*

It is not a distance between cognitive objects in some abstract sense which is to be measured, of course, but perceived distance, i.e., the judgments of distances made by individuals and cultures. Consequently, what is needed are judgments of dissimilarities among objects made by respondents but expressed as ratios to some standard unit provided by the experimenter. This can be accomplished quite directly by a question worded in the form:

"If _x_ and _y_ are _u_ units apart, how far apart are _a_ and _b_?"

Such an item wording requests a dissimilarities judgment from a respondent ("...how far apart are _a_ and _b_?"), but requests that this judgment be made as a proportion of a standard distance provided by the experimenter ("If _x_ and _y_ are _u_ units apart...").

This technique has several key advantages: First and foremost, no restrictions are placed upon the respondent, who may report any positive real value whatever for

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*While there is truly a great range of freedom from within which the comparative standard may be chosen, certain criteria for making such a choice may be specified. First, the standard should be relatively stable. Changes in the standard over time can confound time series measurements and prevent meaningful comparisons of measurements made at different times. Secondly, the standard should be the same for all observers regardless of reference point, i.e., two independent observers must both agree on the length, for example, of a meter of a kilometer. Less important, but nonetheless worthy of consideration, good practice for minimum error suggests using a standard approximately midway between the largest and smallest measurement likely to be encountered, (measurement of astronomical distances in miles, for example, is cumbersome, as would be measurement of terrestrial distances in fractions of light-years). These criteria, however, are never achieved in any science. No distance, for example, is truly invariant, no clock emits signals so that "...the duration between any two signals is (exactly) the same...". Secondly, at least within the framework of relativistic physics, viewers in referent systems moving at differential velocities with regard to one another will not agree on distances or durations of time when viewing the same events. Whatever consequences failures to meet these criteria exactly may be for philosophy, they are not insuperable barriers to science.

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See page 6 "[illegible]"

any pair. Thus the scale is unbounded at the high end and continuous across its entire range. Secondly, because the unit of measure is always the same, (i.e., the unit is provided by the investigator in the conditional, "If x and y are u units apart," and thus every scale unit is $\frac{1}{u}$ units) and because the condition of zero distance represents identity between concepts and is hence a true zero, not at all arbitrary, this scale is what social-scientists usually call a ratio scale, which allows the full range of standard arithmetic operations. Third, since the unit of measure is provided by the experimenter it is possible to maintain the same unit of measure from one measurement to another, both across samples and across time periods, which is crucially important since time is one of the primitive variables of scientific theory. These three characteristics taken together provide the capacity for comparative and time-series analyses at very high levels of precision.

While the technique suggested meets the criterion for scaling quite exactly, and in fact will be the technique of choice in the measurement of aggregate cultural patterns, problem of unreliability make it unsuitable for the measurement of individual self-conceptions. It is axiomatic in psychometrics that the reliability of any scale is approximately proportional to the degree to which the scale is structured and inversely proportional to the complexity of the judgmental task required of the respondent. The technique of direct paired distance estimates requires a highly complex set of judgments from the respondent while providing virtually no structure, and is consequently unreliable for measurement of individual psychological contents (typical test-retest reliability correlations range in the .70's). Fortunately, techniques for establishing distance relations among psychological contents are among the most well known and most carefully researched in psychometrics, but unfortunately all involve some sacrifices in precision as a trade-off for reliability.*

*As of this writing, the best source for such estimates is probably Torgerson (1958).

What error does occur in the measurement of individual self-conception, however, has the overwhelming advantage of being random error, as is all unreliability (as opposed to invalidity) of measure. Such random errors will be distributed normally in any series of measures. Should any number of persons n respond to a paired-comparison question like the one just specified, the law of large numbers assures that the scores obtained will be normally distributed about a sample mean score, and that that sample mean will converge on the population true score as n becomes large. But this population true score, that is the true mean dissimilarities estimate for all the members of a culture, is exactly the theoretical definition of culture suggested earlier. Operationally, therefore, culture is defined as the matrix $\bar{D}$ where any sentry $\bar{d}_{ij} = \sum_{K=1}^{n} \frac{d_{ijk}}{n}$ where d_{ijk} = the distance between the ith object and the jth object as estimated by the kth person using the method of direct paired distance estimates, and n = the number of persons making such an estimate.

Several qualifications must be made, of course. First, clearly the matrix $\bar{D}$, to be exhaustive, would be a c x c matrix where c = the number of objects defined by the culture, which is a very large, but finite, number. What is at issue, of course, is the measurement of subsets of the matrix $\bar{D}$ corresponding to segments of the culture under investigation. Secondly, the boundaries of the culture itself need not be so clearly drawn as is implicit in this discussion, and the investigation of subcultures is simply a matter of appropriate sampling. Third, as again was the case with the measurement of individual self-conceptions, the matrix $\bar{D}$ represents a static picture of the state of a culture at a given point in time. The processual character of the culture as it moves through time must be measured by successive matrices $\bar{D}_{t0}$, $\bar{D}_{t1}$, ... $\bar{D}_{tn}$ where each new matrix represents a further point in time. The difference $\bar{D}_{t1} - \bar{D}_{t0}$, for example, would represent the cultural change taking place over the interval from t_0 to t_1. The rate at which any culture is changing can be found by the derivitive

$$\lim_{t_1 - t_0 - 0} \frac{D_{t1} - D_{t0}}{t_1 - t_0}.$$

While more speculation about the meanings of these definitions will follow, for now it should be clear that the continuous logic of the mathematical function has the analytic capacity to describe the full complexity of human self-conception and cultural phenomena as matrices of distances among social objects, and, particularly in the case of cultural phenomena, this analytic capacity can be very closely translated into actual empirical measurement.

The utility of this conceptual system can be made even more graphic when we recall that any matrix (in this case the matrix D which describes the self-conception and the matrix $\bar{D}$ which describes the cultural system) describes an implicit vector space V_k where k (the dimensionality of the space) $\leq$ N - 1 where N is the order of the original matrix.

Although any matrix describes its underlying vector space fully, as the order of the matrix becomes large, calculations based on these matrices can become quite cumbersome, and the visualization of such spaces becomes impossible as the dimensionality (k) exceeds three. Given the condition that k $\leq$ n - 1, however, such operations are seldom necessary.

V_k, of course, is a spatial coordinate system defined by the distance relations among the cognitive objects which are its contents. It has as a minimum the property that objects defined as similar by any individual or culture will be located close to each other in the space, or, more precisely, that the distance between any pair of objects in the space is directly proportional to their perceived dissimilarity. The precise definition of any objects, therefore, is given by its location in V_k, and, as a corollary, any change of definition of any object is represented by its movement through V_k. That V_k has the further systematic characteristics generally attributed to the self-conception in the individual case on the culture in the aggregate can be shown easily. First, and most obvious, since every object in the self (or the culture) is defined in terms of its distance from every other object, the change of definition of any object modifies the definition of every object somewhat. This same property viewed another way shows that V_k serves the same filtering function typically attributed to categories: as can easily be shown, given that any object can be located with regard to k independent object in V_k (or more, of course) then its location in the space vis-a-vis all other objects is completely determined, and the new object will be implicated in the relationships already extent among all other objects in the space.

Furthermore, should the equations presented earlier prove even approximately correct, the self or the culture defined as V_k can also be seen to be hierarchical. Equation (2) argues that the stability of any attitude or cultural belief is a linear function of the congruent amount of information out of which it was originally formed. It follows directly, therefore, that those objects whose locations in V_k have been established on the basis of very large quantities of information + congruent will be quite massive; i.e., their definitions (measured as their locations in V_k) will be relatively hard to change. Those objects whose locations in V_k are established on the basis of limited information, or inconsistent however, would be less massive and consequently easier to move. The most massive objects clearly would be those psychological objects most familiar to the individual or culture and central to his or its activity, and these objects would provide the stable anchoring points against which new information is guaged and in terms of which new objects are located. Changes in these fundamental definitions would require very substantial inputs of $\boxed{\text{energy}}$ information, but once accomplished would bring about major revisions of the self-conception or culture and everyday behavior, whereas

certainty + probability

what about the "crucial experiment" ? mixed metaphor

the shotgun

changes in the more peripheral objects could be accomplished with less effort, but would result in only minor changes. These properties taken together imply a hierarchically organized system which imposes its own structure on new information such that incoming data is processed and understood in terms of relationships already established in the self or the culture. Yet the receipt of new information modifies that structure as well, and so the self and culture as defined here are continually in flux. This flux, however, is itself systematic, with some components highly stable and others quite volatile. This picture corresponds quite well to the descriptions of human mental functioning presented by the most perceptive students of human activity, particularly those of an interactionist perspective (Blumer, Mead, Denzin).

Obtaining the underlying vector space from the matrix $\bar{D}$ is straightforward:* procedurally, the data collection outlined earlier yields a three-dimensional concepts x concepts x person matrix which is averaged across the n persons into a two dimensional concepts x concepts square symmetric matrix $\bar{D}$, where any entry $\bar{d}_{ij}$ represents the average distance between concepts i and j as seen by the respondents. This matrix $\bar{D}$ is transformed routinely into a scalar products matrix B* (Young and Householder, 1938), although it is generally the practice of investigators to "double-center" this matrix by establishing an origin for the space at the centroid of the distribution. This can be done simply during the construction of the scalar products matrix, and the transformation for any cell b_{ij} is given by the equation

$$b^*_{ij} = 1/2 \left(\frac{\sum_{i=1}^{n} d^2_{ij}}{n} + \frac{\sum_{j=1}^{n} d^2_{ij}}{n} - \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} d^2_{ij}}{n} - d^2_{ij} \right)$$

which is a straightforward linear transformation that sacrifices none of the

*The technique outlined in the following pages is based on the classical multidimensional scaling model well known to psychometricians. Other non-metric scaling models are available, but these techniques apply principally to the reduction of matrices which are merely ordinal, and so are not applicable to the continuous, reliable, ratio scaled data provided by the measurement system proposed in this article. See particularly Shephard (1366).

information present in the original matrix D (Torgerson, 1958).

This new centroid scalar products matrix is such that any entry:

$$b^*_{ij} = p_i p_j \cos_{ij} \qquad \text{where} \qquad \begin{array}{l} p_i = \text{the length of vector } \underline{i} \\ p_j = \text{the length of vector } \underline{j} \\ \partial ij = \text{the angle between } \underline{i} \text{ and } \underline{j} \end{array}$$

Consequently, when this matrix B* is reduced to its base by routine factorization (i.e., the application of any standard eigen routine, such as principal axis or jacobi), the result is a factor matrix, F, whose columns F_1, F_2, ... F_k are orthogonal vectors with their origin at the centroid of the vector space spanned by F and where any entry F_{ij} represents the projection (loading) of the ith variable on the jth factor. This matrix has the further properties such that:

$$P_i = \sqrt{\sum_{j=1}^{k} d^2_{ij}}$$

That is, the square root of the sum of squared projections of the ith variable across all the k factors equals the length of the vector of the ith variable, and of central concern:

$$\bar{d}_{ij} = \sqrt{\sum_{f=1}^{k} (d_{if} - d_{jf})^2}$$

This last expression shows that the original distance matrix can be completely recovered from the factor matrix with no loss of information. It is even possible, based on the strength of two additional, but plausible assumptions, to recover still further information as follows:

Almost all scaling techniques, whether uni- or multidimensional, commonly share a single starting assumption; that is, that concepts may be represented as points on a continuum or in a space. This assumption, however, is almost certainly overly rigid in almost all circumstances. What is more likely is that concepts or variables being scaled are representable more accurately by intervals on a scale or <u>regions</u> in space. The color spectrum, for example, does not represent colors as points on a scale, but intervals. Moreover, some colors occupy larger intervals than others; yellow, for example, occupies a smaller interval of the color spectrum than blue. Furthermore, when respondents are asked to estimate the distances between such concepts, it is likely** that the distance <u>between the near boundaries of the regions</u> will be reported. As Figure 1 illustrates, these (reported) surface-to-surface distances are related to the center-to-center distances by the expression:

$$\hat{d} = \bar{d}_{ij} + r_i + r_j \quad \text{where}$$

$\hat{d}_{ij}$ = center-to-center distance
$\bar{d}_{ij}$ = (reported) surface-to-surface distance
r_i = the radius of concept <u>i</u>
r_j = the radius of concept <u>j</u>

It may be argued, then, that all original distance estimates are systematically too small by a variable amount. Furthermore, attempts to fit these truncated distances into a real space will be thwarted. By definition, a real space is one in which any three points <u>i</u>, <u>j</u>, and <u>k</u> must satisfy the relation.*

$$d_{ij} + d_{ik} \geq d_{jk}$$
$$d_{ij} + d_{jk} \geq d_{ik}$$
$$d_{ik} + d_{jk} \geq d_{ij}$$

When the point assumption is violated, as in the matrix D, however, attempts to represent the distances among the surfaces of the hyperspheres as distances among points will generally fail to satisfy the "triangle inequalities" constraints described above. Such a matrix will not be positive, and factorization will yield negative eigenroots signifying the projections of at least some of the variable vectors into

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**It is, at least, <u>more</u> likely than the assumption that the concepts are points.

*If one of these expressions is satisfied as an equality, the points are collinear; if all are satisfied as equalities, the points are coterminus.

imaginary space. Since we have attributed this failure of the real space assumption to a shortening of the distances in the space by a function of the sizes of the concepts scaled, what is called for is a reduction of the imaginary space to zero by an expansion of the real space. This can be done conveniently by subtracting the largest negative eigenroot (i.e., the smallest root algebraically or λmin from every entry of the diagonal of the centroid scalar products matric B^*, since λmin equals the sum of the squared projections of all the concepts scaled on the largest negative factor and hence represents the squared vector length of the longest imaginary factor, while the diagonal entries of the matrix B^* represent the lengths of the vectors of all the scaled concepts in the space). This operation:

$$\hat{B}^* = B^* - I\lambda min$$

will yield an adjusted scalar products matrix B^* which is just positive semidefinite (i.e., contains no negative latent roots). Since the off diagonal cells of $\hat{B}^*$ are the same as those of B^*, and since they further represent:

$$\hat{B}^*_{ij} = p_i p_j \cos a_{ij}$$

where p_i and p_j have been increased, this operation <u>reduces</u> $\cos a_{ij}$, thus <u>increasing</u> d_{ij} and consequently every distance d_{ij} in the original distance matrix D will be increased by a function of the cosine of the angle ij. If the original distance matrix D is subtracted from the distance matrix D corresponding to the matrix B^*, the resulting matrix R can be seen to be a matrix of sums of radii corresponding to the scalar equation:

$$\hat{d}_{ij} - d_{ij} = r_i + r_j$$

or, in matrix form:

$$\hat{D} - D = R$$

This matrix R is overidentified and easily solved for the individual radii.

The advantages of such a technique are dramatic. First, it enables fully continuous ratio scaling of any level of precision desired (accuracy equivalent to typical physical science measures are not unrealistic), secondly, no information contained in the data need be lost, and in fact much latent information is uncovered. Third, the solution arrived at is fully graphic and, particularly when the dimensionality of the resulting space is three or fewer, as is very frequently the case, even visual. Of perhaps even greater importance, given the application of a suitable rotation and translation routine*, is the clearcut advantage of this metric multidimensional scaling technique for studies involving time-ordered observations over a set of known time periods. By the simple subtraction of coordinates over time, motions through the spatial manifold over time may be expressed as velocities, as given by:

$$V_1 = \frac{d_i}{t} = \sqrt{\frac{\sum_{j=1}^{m} (a_{ij} - b_{ij})^2}{t_1 = t_0}}$$

where

V_i = the velocity of concept $\underline{i}$

d_i = the distance concept $\underline{i}$ has moved across the interval of time $\underline{t}$

t = time

a_j = the coordinate value of concept $\underline{i}$ on the $\underline{jth}$ factor of the t_0 space

b_j = the coordinate value of concept $\underline{i}$ on the $\underline{jth}$ factor of the t_1 space

and given multiple time periods, as accelerations:

$$\bar{A} = \frac{\Delta V_i}{\Delta t}$$

These velocities and accelerations are unmistakably measures of cultural change of very high precision. This is so, since the culturally shared definition of any object is given by its location in the manifold V_k, and changes in location represent changes in definition.*

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*A Fortran IV computer program which accomplishes the principle calculations described in this article is available from the author on request.

Implications:

In the limited space available, it has been necessary to gloss over fundamental questions, such, for example, as the well known propensity of persons to "foreshorten" long psychological distances, the problem of rotation to congruence which does not eliminate "true change," the variability of psychological distances due to the perspective of the observer, and so forth. It is not the point to minimize the gravity of these problems, but rather to suggest that these procedures present a new and potentially fruitful avenue of approach. Since the processes under investigation are reduced to motions through a space, the researcher's endeavor may be seen basically as an attempt to discover "laws of motion" in the space. Should it prove to be the case that original distance estimates are systematically distorted, this will be manifested as warpages in the space and consequently, perturbations in the functions describing the motions of the contents of the manifold. Similarly, the problem of rotation to congruence becomes equivalent to the older physical question of the choice of reference systems and absolute versus relative motion. While these changes in perspective may not make solutions to these problems philosophically easier, they do, however, bring large prior literatures in physical science on anologous or identical questions, along with well developed mathematical and other analytical procedures to bear on them. How fruitful such tools might prove is still conjectural, but the potential benefits seem well worth the pains of investigation.