Quality & Quantity 23: 3–20 (1989)

Rotation to simple processes: the effect of alternative rotation rules on observed patterns in time-ordered measurements

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Abstract. Two types of rotation are well known in psychometrics. The first of these is rotation to simple structure, widely known particularly in factor analysis and multidimensional scaling. Rotation to simple structure applies to the case where only a single scaling solution is available, and one wishes to array the configuration in a particular way relative to its axes; usually in such a way that each point projects as much of its variance as possible on a single axis and as little as possible on each of the other axes.

The second type of rotation applies when two scaling solutions are available and one wishes to compare them to each other. In this case, one rotates one or both of the solutions until they fit as well as possible to each other by some criterion. Sometimes rescaling, stretching and shrinking may be involved. This type of rotation is usually referred to as "Procrustes" rotation.

Neither of these rotation schemes is appropriate, however, when time series measurements are available. In the time-series situation, one is not particularly interested in the structure of the solution at any time, nor even in the comparison of any two structures adjacent in time. Rather one is interested in the underlying processes which may be found in the time series.

The kinds of processes which emerge from a time series of multidimensional measurements,, however, are heavily dependent on the rotation rules by which one relates each of the structures in the time series of structures to each of the others.

In the present article, we discuss a weighted least squares rule, and illustrate the situations under which it is appropriate for discovering processes underlying time series measurements for Galileo-type scaling data. An example dealing with social perceptions of time is presented.

The problem

The measurement of process presents difficulties that are somewhat different from those usually encountered in the measurement of structures (Barnett and Woelfel, 1979). Consider, for example, the process which results from a changing set of interpoint distances.

At first glance, it may seem that this scaling problem may be reduced to the more common problem of recovering the structure of the set of points at each time point in the time series. This, however, is not the case.

In fact the problem is one of determing the motions or trajectories of each of the several points across time. Each of the points may move relative to the set of remaining points, or relative to the axes on which the points are projected.

In fact, if the data are assumed to be error free, the motion of each point relative to all others is given by the data, but the motion of each point relative to the axes on which the points are projected can be seen to be a consequence both of the data and the rotation and translation rules by which solutions at each point in the time series are fixed relative to each other point.

If the data are themselves fallible (as, of course, is always the case), then the motion of each point relative to each of the others is also affected by the parameters of the scaling solution.

An example

A substantive example which is appropriate for the study of the measurement of processes is the socio-cultural perception of time (Zerubavel, 1981; 1985). The seven days of the week represent a cycle which, unlike the day, month or year, is independent of natural, seasonal or astronomical cycles. It is rather socially defined, and as such represents the ideal context for examining the perception of time.

The week is a cultural artifact, an artificial rhythm created by human beings which varies among societies. Our seven day week evolved out of ancient astrology and the Judeo-Christian-Islamic tradition of devoting every seventh day for religious activities (Zerubavel, 1985). Different civilizations have other cycles of time longer than the day and shorter than the month. These are typically based on market schedules of alternative calenders.

Clearly, people have perceptions and attitudes toward the days of the week. Among the perceptions people may have of the days of the week are their positions in a temporal order; that is, Monday follows Sunday and precedes Tuesday, which in turn precedes Wednesday, and so forth. Further, people associate each of the days with unique activities. In western societies, people work Monday through Friday and spend the weekend (Saturday and Sunday) in leisure activities. According to Durkheim (1965) the days of the week are separated into the sacred and the profane. Judaism devotes Saturday to religious activities. In Christian cultures, Sunday is reserved for religion, while these functions are performed on Friday in Islamic societies. As a result, ". . . we carry in our minds a sort of 'temporal map' which

Table 1. Distance among days of the week in hours

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
Sunday	0.00					
Monday	0.00	0.00				
Tuesday	24.00	0.00	0.00			
Wednesday	48.00	24.00	0.00	0.00		
Thursday	48.00	48.00	24.00	0.00	0.00	
Friday	24.00	48.00	48.00	24.00	0.00	0.00
Saturday	0.00	24.00	48.00	48.00	24.00	0.00

consists of all our expectations regarding the sequential order, duration, temporal location and rate of recurrence of events in everyday life" (Zerubavel, 1981, p. 14).

If one were to consider only the physiotemporal relations among the days of the week, then each day would be zero distance from its two neighbors, 24 hours from the day before yesterday and the day after tomorrow, and so forth. Table 1 presents the distances one might expect if only temporal relations among days were considered in their inter-day relationships.

The data in Table 1 form a circle in a Riemann space. The figure (a temporal map) is a circle because each day is 0 distance from its nearest neighbor at either side, of course, and it is a Riemannian configuration because the data violate the triangle inequality constraints. (Sunday, for example, is 0 hours from Monday, and 24 hours from Tuesday, but Tuesday is 0 hours from Monday, which results in a 0–24–0 triangle, which cannot lie on an euclidian plane.)

Figure 1 represents the first principle plane of the solution for these data given by the Galielo Version 5.3 computer program at the State University of New York at Albany. The circular configuration is plainly visible, but the flat projection hides the (non-artifactual) third dimension which results from the non-euclidian character of the data.

The data in Table 1 and the picture in Fig. 1, of course, do not take into account the sociotemporal order seen by members of society described by Zerubavel (1985). Clearly, of course, people do not perceive Mondays in the same way as they do Saturdays. We ought to expect, therefore, that actual data taken from normal respondents might show some departure from circularity, but under no circumstances would it be likely that the closed or non-recursive nature of the figure would be lost.

We might also expect that certain activites are more closely associated with certain days among actual respondents. Work, for example, ought be most closely associated with the weekdays, particularly those at the beginning of the week, while concepts related to relaxation ought be more

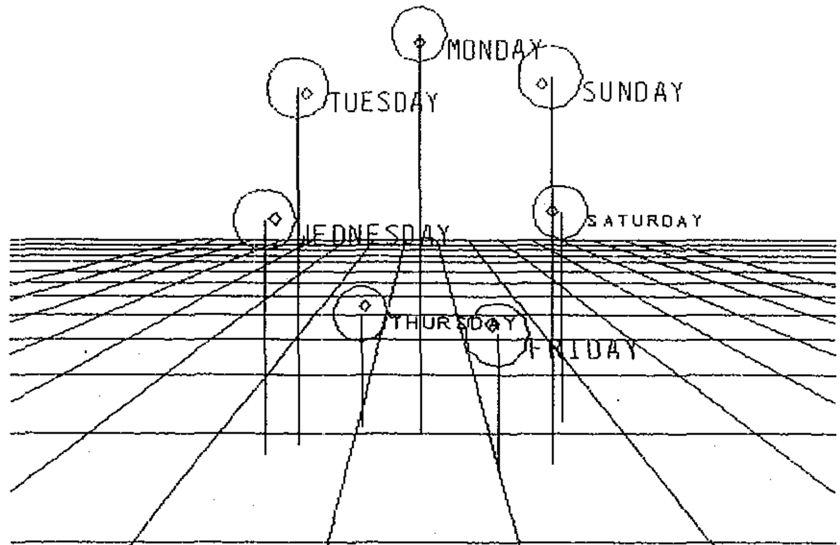


Figure 1: Ideal Week

closely associated with the weekend. In practice, then, we might expect the sociotemporal structure of the days of the week to resemble an elongated circle or ellipse, with certain behavioral stimuli located close to the different days of the week.

Respondents might also perceive themselves to stand in some psychological relationship to each of the days. We might expect, for example, that people who favor relaxation over work might perceive themselves closer to the weekend than to the weekdays.

Significantly, however, this set of relations might be expected to vary throughout the course of a normal week. While one might consider himself or herself closer to, say Saturday than to Monday, he or she might consider himself or herself closer to Monday when it is, in fact, Monday, than he or she does when it is any other day. Moreover, it may well be that one feels closer to Monday in proportion to how far it is to Monday chronologically.

If these assumptions are approximately correct, then dissimilarities collected from respondents over a week's time might be expected to describe a process whereby a person's "self" exhibits a quasi-orbital motion around an elliptical structure of the days of the week.

While these assumptions might well be wrong, it is very unlikely that any scaling algorithms currently in use by psychometricians could recover a process like the one described here even if the assumptions were correct. If this is the case, then we would be led to reject these assumptions on artifactual ground whether they were correct or not.

In the present article, we attempt to determine whether a reasonable set of scaling operations can be found which would yield a pattern resembling this sociotemporal model of the week.

Method

During a period from April 7 to April 30, 1980, researchers at the State University of New York at Albany and at Rensselaer Polytechnique Institue telephoned 430 randomly selected telephone subscribers in the Capital District of New York (Albany, Schenectady, Troy and environs) and asked them to report the interpoint distances among twelve concepts on a numerical scale. The twelve concepts were:

1. Sunday
2. Monday
3. Tuesday
4. Wednesday
5. Thursday
6. Friday
7. Saturday
8. Work
9. Relaxation
10. Alcohol
11. Marijuana
12. Yourself

The numerical scale was a typical "Galileo" type scale, in which respondents were given a "criterion pair" – in this case they were told that Sunday and Monday were 24 units apart – and were then asked to compare all other distances as ratios to this initial distance (Woelfel & Fink, 1980). Since the process expected by theory anticipates changes of magnitude relations among the stimuli that may not involve changes in rank order, some sort of magnitude estimation procedure is required by the theory.

This procedure resulted in about 18 responses per day for a 23-day period. Data from each day of the week were then pooled, so that all data gathered on a Monday were collected into a single file, as were all data collected on a Tuesday, and so forth. This yielded seven samples of approximately 60 cases per sample, representing each of the seven days of the week.

Analysis

As an initial approximation to a solution, data from all respondents within a day (i.e., all respondents interviewed on a Sunday) were averaged into a single mean dissimilarities matrix for that day, resulting in seven mean dissimilarities matrixes.

Examination of the mean dissimilarities matrices as well as the raw data reveals (as is always the case for Galileo-type magnitude estimation paired comparison difference scales) that many of the respondents do not adopt the unit standard required in the instructions. That is, they either forget that they were asked to consider the distance between Sunday and Monday to be 24 hours, or they sometimes argue that this is not correct and that another number ought to be used. Sunday and Monday – or any two adjacent days – are 24 hours apart if the midpoint of the days, 12 noon, is taken to represent the day. On the other hand, if one measures from the end of one day to the beginning of the next, of course the days are zero hours apart.

When very large samples are available, this has a surprisingly small effect on the mean dissimilarities, probably because, in most instances, respondents are about equally likely to overestimate as underestimate the numerical size of the standard ("criterion") pair. Standard errors are inflated by this effect, of course, and particularly in small samples – approximatley 60 per cell in the present study – adjustment of the data is typically required.

In the case of a time-series of measurements, of course, it is imperative that each set of data in the time series be adjusted in the same way, otherwise comparability over time is lost. Several forms of adjustment were attempted. Only the most effective is reported here.

In the present analysis, each case was first adjusted so that the distance between Sunday and Tuesday was set to 24 hours. This was done by calculating the number by which the Sunday-Tuesday distance for each individual respondent had to be multiplied to equal 24; then all values for that individual case were multiplied by that ratio. This procedure was chosen in preference to more generalized least-squares central dilation fitting algorithm since the more general method considers each dissimilarity in the data equally likely to be shifted up or down across measurement sessions. The present solution treats the distance between Sunday and Tuesday as "privileged"; that is, it judges on *a priori* grounds that this distnace is certain to remain the same over time, and therefore attributes all differences in the measured values of this distance to be error.

This assumption is almost certainly wrong when stated in absolute terms, but will yield a better solution than least-squares matching to the extent that it is approximately true; even through the overall sum-of-squares goodness of fit measure for the more generalized procedure might be numerically lower.

Secondly, all values exceeding 999 were deleted from each case. These modified data were then averaged across all cases within each day of the week, yielding seven adjusted mean dissimilarities matrices, one for each day of the week.

This simple "clipping" of extreme values is less sophisticated than either a monotonic transformation or a hinged "smoother" which "unweights" extreme values proportional to their distance from the mean or median value, but it is not inconsistent with standard engineering practice, and meets the requirement that the metric remain unchanged across time intervals. Clearly, a monotonic or non-metric adjustment routine is innappropriate when changes in the overall size of the configuration is expected and meaningful over time, since the monotonic transformations in common use will adjust the overall sizes of each configuration in the time series to be the same.

As a more general rule, whenever two or more datasets are to compared, they must be treated identically if differences (or similarities) between them are not to be attributed to the different ways they have been treated. The non-metric monotone transformation is, in fact different every time it is used and thus confounds meaningful comparisons across datasets.

Each of these dissimilarities matrices was entered into the Galileo(tm) Version 5.3 Computer Program at the State University of New York at Albany, which extracted the principle axes of each of the seven configurations.

The Galileo program computes a centroid scalar product matrix following Torgerson (1958), then extracts the principle axes following a method given by Van de Geer (1972). The principle difference between the factoring algorithm in the Galileo program and more conventional programs is that it extracts all eigenvectors, including the imaginary eigenvectors present when the original dissimilarities matrix is indefinite (Woelfel and Barnett, 1982).

Each of the last six sets of eigenvectors were then rotated to a modified least squares best fit on the set immediately preceding it in the time series. The modification to the least squares rotation procedure consisted of leaving several of the stimuli out of the least squares fitting criterion. Specifically, each set of eigenvectors was rotated to its target matrix (the one preceding it in the time series), but the rotation was set to minimize only the sum of the squared distances between the days of the week, that is, the rotation minimized the expression

$$d^2 = \sum_{i=1}^{k} [w(i, t) - w(i, t - 1)]^2,$$

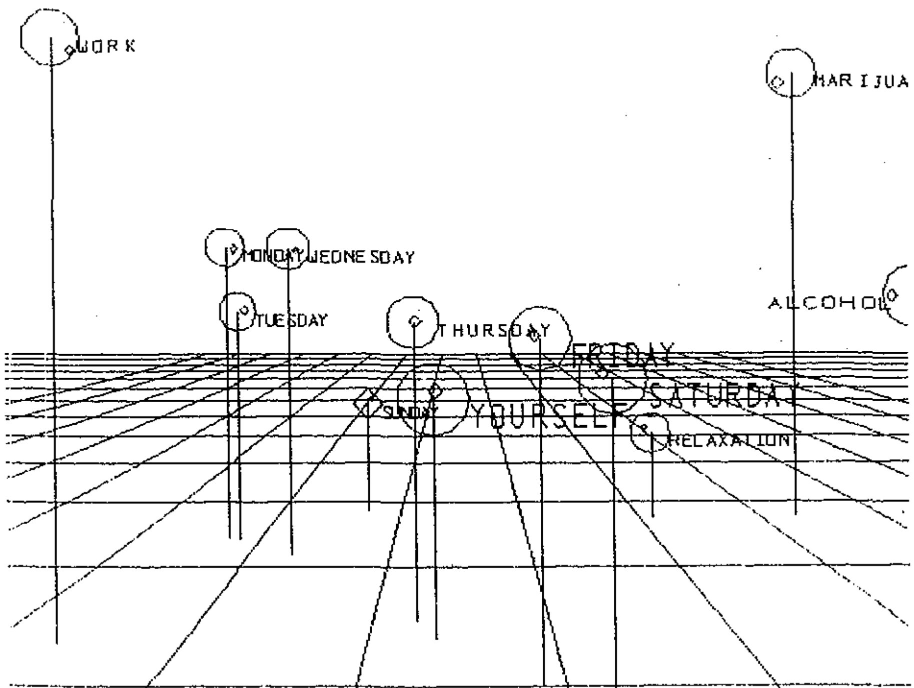


Figure 2 Adjusted Week

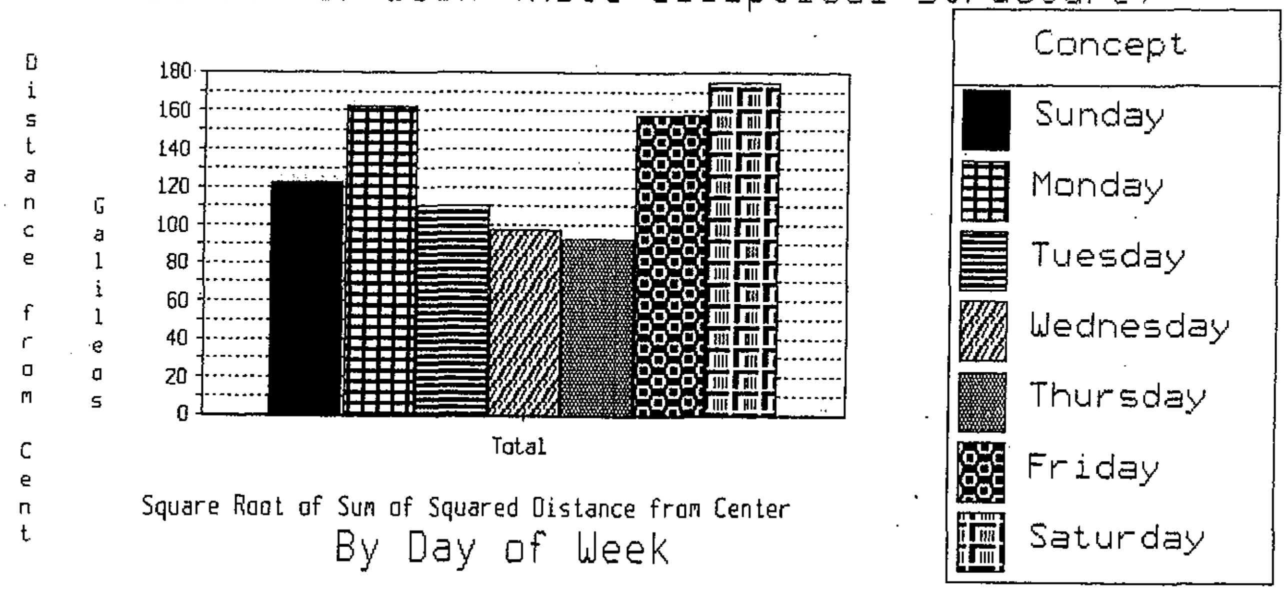
where d^2 = the total squared distance of all days of the week at time t from their positions at $t - 1$, $w(i, t)$ = the ith day at time t, $w(i, t - 1)$ = the ith day at time $t - 1$.

While the other five concepts (work, relaxation, alcohol, marijuana and yourself) were rotated, the distances between their positions at t and at $t - 1$ were not considered in the least squares criterion.

This rotation scheme has the effect of testing the assumption that the days of the week do not change postion over time, but that the remaining concepts exhibit motion relative to themselves and to the days of the week. It is important to understand that this rotation scheme does not guarantee that such a solution will be found, but will reveal it if it is a possible solution. the procedure is described in detail by Woelfel et al., 1988.

Table 2 presents statistics for the set of all seven adjusted datasets averaged together. (Detailed tables of statistical information for each of the days of the week taken separately are not presented here, but are available from the authors.) The resulting configurtions were plotted by the Galileo*STRATEGY plotting program. The left-to-right dimension represents the first principal axis, the vertical axis is the second principal axis, and the third dimension is represented by the depth of the picture.

Figure 2a: Distances from Each Day to the Geometeric Center of Week (Note elliptical structure)



Galileo

Table 2. Statistics for all data adjusted Sunday–Tuesday = 24; Max val. = 999

Row	Col	Mean	Stan. dev.	Std. err.	Skewness	Kurtosis	Count	Min. val.	Max. val.	Error
1	2	15.903	9.669	.531	3.275	18.460	331	.0	96.0	3.3
1	3	23.927	1.321	.073	−18.001	323.018	330	.0	24.0	.3
1	4	33.939	14.016	.773	3.970	26.855	329	.0	160.0	2.3
1	5	43.488	52.280	2.878	13.459	215.619	330	.0	900.0	6.6
1	6	53.468	66.331	3.646	5.050	33.028	331	.0	600.0	6.8
1	7	45.882	76.345	4.203	6.570	65.513	330	.0	960.0	9.2
1	8	104.661	146.347	8.609	2.206	4.799	289	.0	800.0	8.2
1	9	82.845	131.653	7.563	3.164	11.676	303	.0	900.0	9.1
1	10	24.978	71.583	3.983	6.001	42.337	323	.0	640.0	15.9
1	11	59.539	130.365	7.299	3.829	16.021	319	.0	960.0	12.3
1	12	46.980	132.857	7.683	5.005	26.507	299	.0	960.0	16.4
2	3	22.121	54.735	3.009	6.690	52.258	331	.0	600.0	13.6
2	4	32.400	69.610	3.832	6.531	46.504	330	.0	600.0	11.8
2	5	50.752	108.069	5.949	6.299	43.704	330	.0	960.0	11.7
2	6	62.488	87.039	4.836	4.443	24.596	324	.0	756.0	7.7
2	7	70.280	91.542	5.109	4.429	27.604	321	.0	840.0	7.3
2	8	113.168	159.687	9.442	2.484	6.706	286	.0	960.0	8.3
2	9	98.617	142.250	8.282	2.545	6.750	295	.0	800.0	8.4
2	10	84.957	126.537	7.293	3.309	12.735	301	.0	840.0	8.6
2	11	19.205	56.837	3.167	9.222	112.438	322	.0	800.0	16.5
2	12	59.502	108.432	6.356	4.582	26.698	291	.0	960.0	10.7
3	4	20.711	48.027	2.648	7.736	74.218	329	.0	600.0	12.8
3	5	35.133	72.645	3.999	5.792	36.395	330	.0	600.0	11.4
3	6	54.933	93.419	5.166	5.760	40.061	327	.0	864.0	9.4
3	7	66.167	105.751	5.884	5.376	34.239	323	.0	900.0	8.9
3	8	115.101	166.916	9.970	2.427	6.079	286	.0	960.0	8.6
3	9	98.639	147.411	8.568	2.555	7.123	296	.0	900.0	8.7
3	10	70.915	107.516	6.136	3.128	10.368	307	.0	600.0	8.7
3	11	28.645	67.025	3.274	6.063	48.046	324	.0	700.0	13.0
3	12	60.725	132.345	7.705	4.594	23.465	295	.0	960.0	12.7
4	5	25.945	51.199	2.827	4.728	23.991	328	.0	400.0	10.9
4	6	44.823	83.860	4.637	4.794	24.736	327	.0	576.0	10.3
4	7	68.098	130.249	7.225	4.832	24.584	325	1.0	900.0	10.6
4	8	103.150	144.987	8.573	2.484	7.186	286	.0	960.0	8.3

4	9	85.689	127.597	7.379	2.675	7.841	299	.0	800.0	8.6
4	10	70.206	116.303	6.595	3.892	17.795	311	.0	900.0	9.4
4	11	31.415	78.205	4.338	7.765	77.957	325	.0	960.0	13.8
4	12	53.212	111.944	6.540	5.039	30.000	293	.0	960.0	12.3
5	6	21.141	42.795	2.370	7.534	74.419	326	.0	540.0	11.2
5	7	33.138	62.492	3.461	6.811	54.279	326	.0	600.0	10.4
5	8	93.073	141.962	8.365	2.592	8.046	288	.0	960.0	9.0
5	9	61.880	119.750	6.812	3.180	10.463	309	.0	684.0	11.0
5	10	45.395	82.047	4.652	4.056	20.432	311	.0	600.0	10.2
5	11	50.561	93.357	5.211	4.353	26.309	321	.0	900.0	10.3
5	12	45.930	107.716	6.240	4.991	29.745	298	.0	960.0	13.6
6	7	22.147	63.284	3.500	7.109	60.419	327	.0	720.0	15.8
6	8	85.102	153.903	8.991	2.816	8.817	293	.0	960.0	10.6
6	9	40.495	96.899	5.477	4.283	21.598	313	.0	800.0	13.5
6	10	31.713	76.643	4.258	4.942	28.934	324	.0	600.0	13.4
6	11	58.968	103.065	5.807	3.976	18.017	315	.0	720.0	9.8
6	12	33.936	96.404	5.575	6.412	48.420	299	.0	960.0	16.4
7	8	81.384	150.868	8.799	2.730	7.981	294	.0	960.0	10.8
7	9	35.524	99.275	5.594	5.170	31.810	315	.0	864.0	15.7
7	10	23.910	76.546	4.272	6.630	50.927	321	.0	720.0	17.9
7	11	61.476	102.515	5.776	3.118	10.569	315	.0	686.0	9.4
7	12	28.641	93.487	5.389	6.853	53.672	301	.0	960.0	18.8
8	9	55.295	109.921	6.263	4.416	25.360	308	.0	960.0	11.3
8	10	56.210	107.192	6.241	3.808	20.306	295	.0	960.0	11.1
8	11	111.581	151.923	9.128	2.992	10.727	277	.0	960.0	8.2
8	12	96.403	151.117	8.983	2.805	9.320	283	.0	960.0	9.3
9	10	54.656	121.500	6.890	4.480	24.885	311	.0	960.0	12.6
9	11	116.924	156.873	9.228	2.818	9.590	289	.0	960.0	7.9
9	12	77.575	140.649	8.134	3.545	14.407	299	.0	960.0	10.5
10	11	109.478	147.317	8.636	2.387	6.193	291	.0	860.0	7.9
10	12	34.219	92.963	5.205	6.551	50.816	319	.0	960.0	15.2
11	12	53.483	117.973	6.647	4.818	27.581	315	.0	960.0	12.4

Average observations per cell 311.1970.
Count of all non-zero cells 66.
Mean of all non-zero cells 57.6189
Cell with maximum distance is 11 9 distance is 116.9239.
Cell with minimum distance is 2 1 distance is 15.9033.

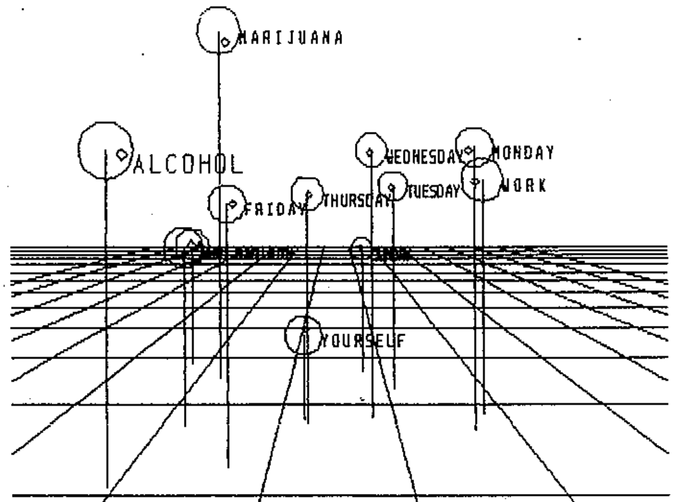


Figure 3: Sunday

The sizes of the spheres in the plots represent the standard errors of the locations of the points, so that there is about a two thirds likelihood that a given point lies within the sphere which represents it.

Eyepoint for the plots is 0, 0, $-W$, where W is the largest absolute coordinate value. Both coordinates of the points and the radii of the spheres which represent them are adjusted for perspective.

Figure 2 shows the first three dimensions of all seven of these datasets averaged together. The configuration of the days does resemble an elongated ellipse, with Saturday and Monday at the opposite ends of the major axis. Each day lies between its two neighbors with the exception of Tuesday, which seems slightly out of position, but nontheless relatively close to where it ought to be expected to appear. As the Graph in Fig. 2a shows, the weekdays are closer to the center of the week than are the weekends (Friday, Saturday and Sunday). Fig. 2a shows, with a slight variation for Sunday, this trend is nearly monotonic as well. While these data are consistent with the hypothesis of a seven day *cycle*, they clearly reject Zerubavel's (1985) hypothesis that the week is perceived as a seven day *circle*.

Again as expected, the concept "work" appears toward the "Monday" side of the ellipse, while the three relaxation concepts (relaxation, alcohol and marijuana) appear nearest to the weekend.

Figures 3 through 9 show the first three dimensions of the configuration for Sunday through Saturday. In each of these spaces the quasi-elliptical figure is visible, although Tuesday is consistently displaced from its expected position.

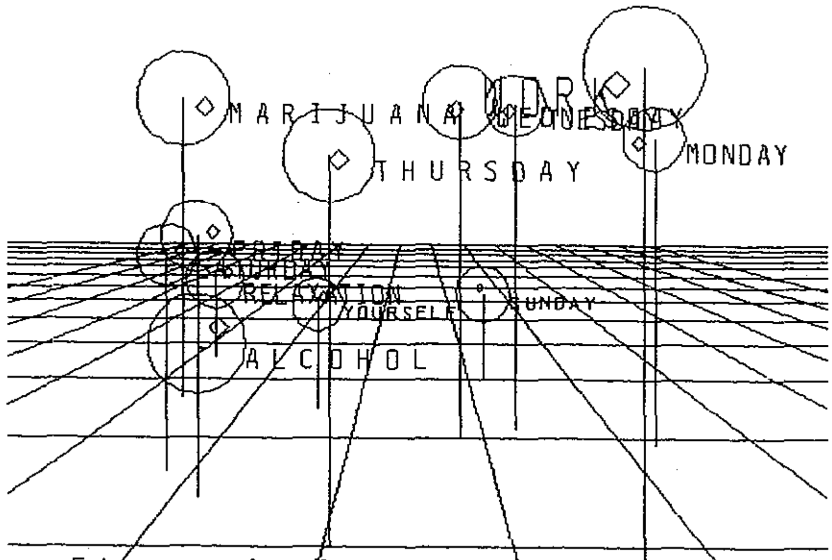


Figure 4: Monday

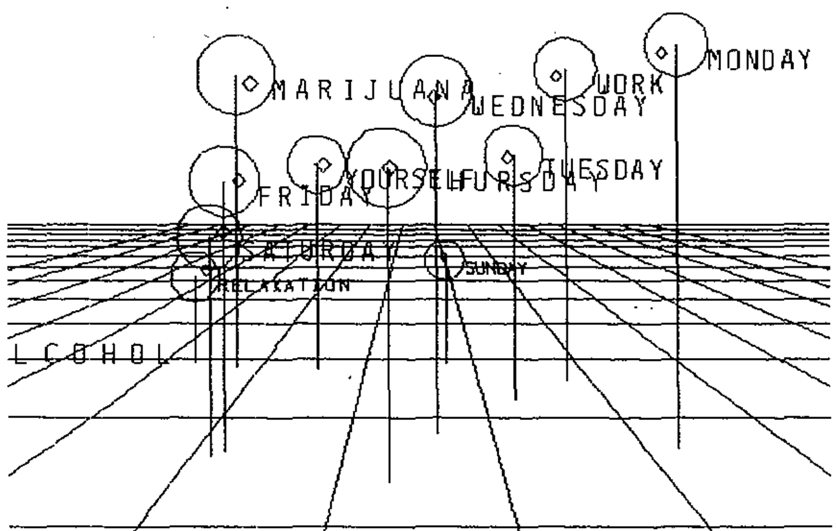


Figure 5: Tuesday

Relaxation concepts continue to lie closer to the weekend, although they move closer to the center of the configuration (and the week) as we move chronologically toward midweek and later. The concept "work" lies closest to Monday on Monday, closest to Tuesday on Tuesday, and closest to Wednesday on Wednesday, but by Thursday, has started to move back

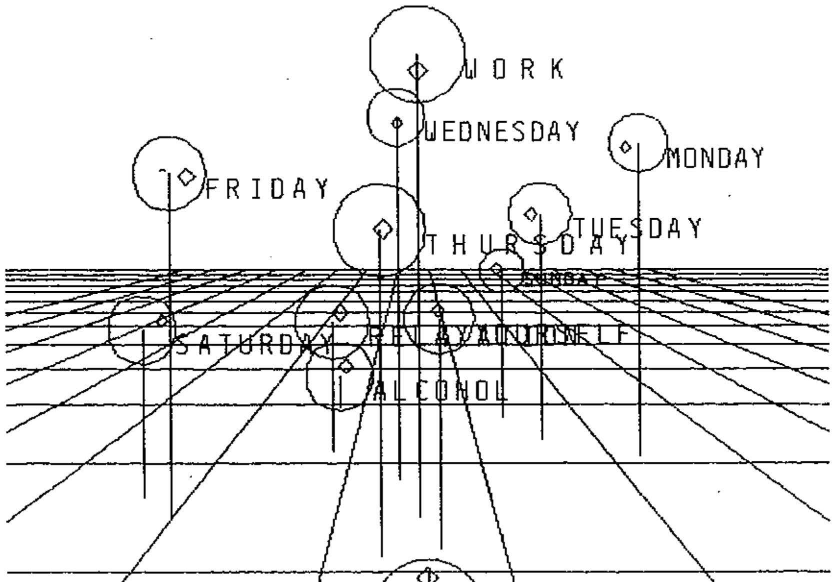


Figure 6: Wednesday

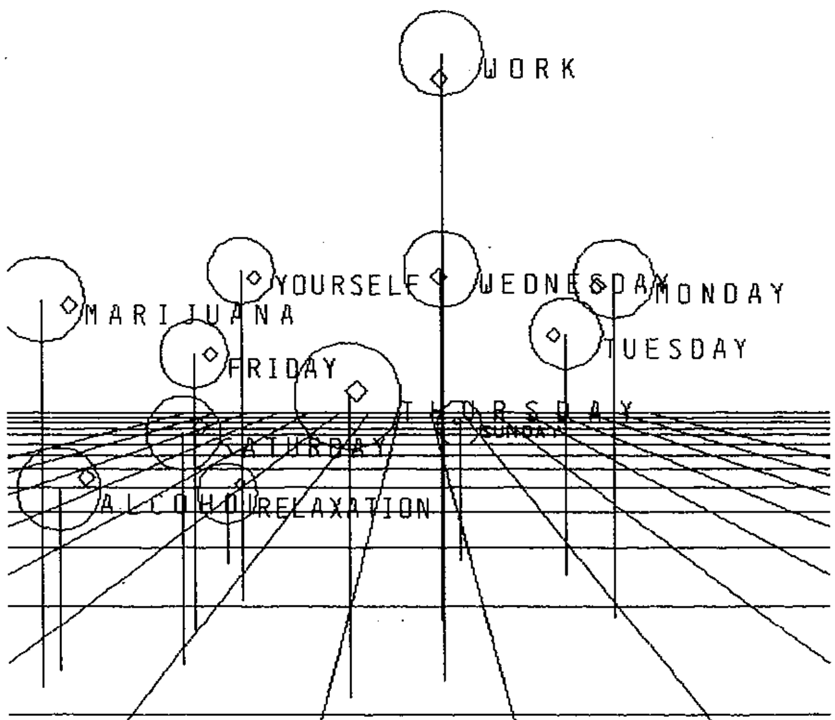


Figure 7: Thursday

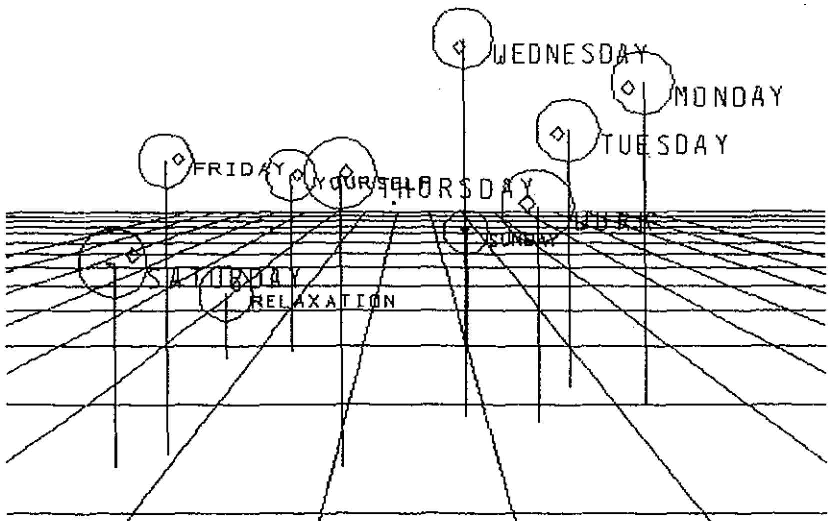


Figure 8: Friday

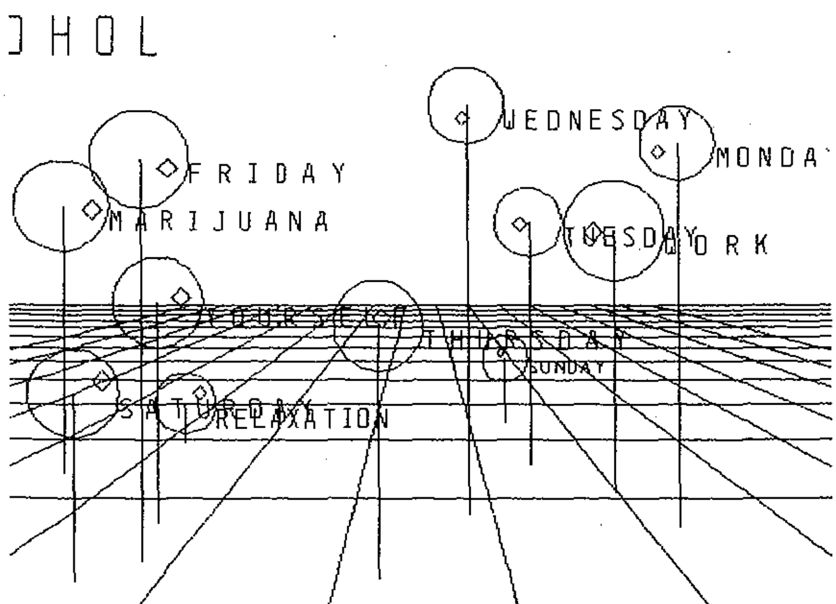


Figure 9: Saturday

toward the beginning of the week. By Friday, work has moved completely back to Monday and Tuesday.

The self point is located closer to the weekend, on the average, than to the beginning or middle of the week. Averaging the distance between each day

Table 3. Distances among concepts by day of measurement[1]

Concept	Day of measurement						
	S	M	T	W	T	F	S
SUNDAY	98.00	25.00	25.00	29.00	65.00	21.00	54.00
MONDAY	93.00	62.00	68.00	45.00	58.00	53.00	46.00
TUESDAY	84.00	44.00	48.00	37.00	113.00	44.00	44.00
WEDNESDAY	87.00	51.00	45.00	49.00	74.00	39.00	48.00
THURSDAY	78.00	41.00	37.00	41.00	55.00	27.00	38.00
FRIDAY	50.00	53.00	32.00	23.00	27.00	16.00	39.00
SATURDAY	65.00	10.00	26.00	24.00	33.00	15.00	24.00
MARIJUA	143.00	118.00	70.00	123.00	87.00	78.00	62.00
ALCOHOL	99.00	77.00	57.00	75.00	99.00	87.00	46.00
WORK	82.00	71.00	29.00	50.00	49.00	48.00	40.00
RELAXAT	64.00	19.00	31.00	36.00	44.00	24.00	17.00
Total	85.73	51.91	42.55	4.36	64.00	41.09	41.64
Sunday	12.27	−26.91	−17.55	−19.36	1.00	−20.09	12.36
Monday	7.27	10.09	25.45	−3.36	−6.00	11.91	4.36
Tuesday	−1.73	−7.91	5.45	−11.36	49.00	2.91	2.36
Wednesday	1.27	−0.91	2.45	0.64	10.00	−2.09	6.36
Thursday	−7.73	−10.91	−5.55	−7.36	−9.00	−14.09	−3.64
Friday	−35.73	1.09	−10.55	−25.36	−37.00	−25.09	−2.64
Saturday	−20.73	−41.91	−16.55	−24.36	−31.00	−26.09	−17.64
Marijua	57.27	66.09	27.45	74.64	23.00	36.91	20.36
Alcohol	13.27	25.09	14.45	26.64	35.00	45.91	4.36
Work	−3.73	19.09	−13.55	1.64	−15.00	6.91	−1.64
Relaxat	−21.73	−32.91	−11.55	−12.36	−20.00	−17.09	−24.64

[1] Numbers rounded to nearest whole digit. Concepts in Caps are measured values; lowercase have trend information removed by subtracting column means of measured values from each cell.

and the self across all data sets shows the self point closest to Saturday (28), then Friday (34), Thursday (45), Wednesday (56), Tuesday (59), and Monday (61). Sunday is the same distance from the self as Thursday (45).

The motion of the self-point relative to the other concepts may show a general tendency to move toward the weekend as the weekend grows closer, but there is no mean difference between the distance between the days and the self when interviews are conducted on those days ($\mu s = 50.29$, $\acute{o}\mu s = 26.8$) and the mean distance between the days and the self when interviews are conducted on different days ($\mu d = 46.45$, $\acute{o}\mu d = 22.15$).

Table 3 gives the distances among the key concepts and the self point for each of the seven datasets. Clearly, for these data, the self concept does not "orbit" around the week in a simple way. This is not a failure of the rotation rule, since no clear cyclical pattern of dissimilarities is evident in the mean dissimilarities. Nonetheless, as both Figs. 2 through 8 and Table 3 shows, the distances between the self concept and relaxation concepts are smaller on weekends. Work is closest to the self on Tuesdays and furthest from the self on Sundays, but, once again, a clear monotone pattern throughout the week is not evident.

Summary

The measurement of processes presents special problems not encountered in the measurement of static structures. The expectation that dissimilarities may change in magnitude but not necessarily rank order over time rules out simply monotone scaling solutions, and requires that some stable metric be maintained across measurement sessions. Direct paired comparison magnitude estimation scales, such as the ones applied in the present research, show theoretical promise for maintaining a standard metric across measurement sessions, but, in samples as small as those in the present analysis, require filtering and adjustment.

In the present analysis, procedures which pick a single distance which, on theoretical grounds may be considered stable across the time span of the measurements, and adjust all data to leave that distance invariant, clarified the resulting processes quite considerably, and made visible certain time-invariant structures not otherwise obvious.

On statistical grounds, however, this method of adjustment is quite likely less than optimal, since it is unlikely that any single distance selected from a matrix of dissimilarities should be thought of as particularly privileged. A better solution might renormalise a set of distances to a given average distance, although one must be careful not to artifactually eliminate the possibility of measuring changes in the overall size of configurations over time.

Given that the problem of a stable, invariant metric can be dealt with, it is necessary in time-series rotation schemes to understand the problem of selection of a time-invariant reference frame against which stimuli may be arrayed. It is seldom likely to be the case that all stimuli measured are equally likely to exhibit motion over time, and so it is necessary to consider selecting some subset of stimuli in the configuration as a stable set against which the motions of the others can be arrayed.

In the present case, the simplest of such rules was illustrated, in which some of the stimuli are simply left out of the least-squares matching criterion when degree of fit is assessed. The procedure is theoretically satisfactory to the extent that one is confident on a priori grounds that the stable set (that is, those stimuli which are counted in the least squares criterion) are indeed a "rigid body"; that is, that their interpoint distances do not change.

Although the sample sizes in the present study are too small to allow robust estimation of the distances for each of the 23 days in the time series, it is probably not unreasonable to expect an additional 400 or so cases taken on another week to show a pattern at least grossly similar to the one shown here. If this were to be the case, the movement of the self-point relative to the days of the week would certainly be cyclical even if not an obvious elliptical orbit. They would thus fit the model of a relatively invariant rigid body (the days of the week) relative to which another concept (the self) exhibited an oscillatory motion.

The procedures presented in this paper should by no means be considered generally satisfactory, even though they can yield significant and even dramatic improvements over conventional monotone scaling solutions coupled with unweighted least-squares procrustes solutions for data of this type.

As improvements in data-collection technologies produce increasingly large volumes of data on time series of dissimilarities, increasing attention to such frame-of-reference problems as discussed here would seem warranted.

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