FOUNDATIONS OF COGNITIVE THEORY

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By the aid of language different individuals can, to a certain extent, compare their experiences. Then it turns out that certain sense perceptions of different individuals correspond to each other, while for other sense perceptions no such correspondence can be established. We are accustomed to regard as real those sense perceptions which are common to different individuals, and which therefore are, in a measure, impersonal .... The only justification for our concepts and system of concepts is that they serve to represent the complex of our experiences; beyond this they have no legitimacy.

A. Einstein, The Meaning of Relativity, Fifth Edition, Princeton University Press, 1956, pp. 1-2.

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INTRODUCTION

The theory presented in this essay is a formal axiomatic theory of the dynamics of cognitive processes. So frequently do we debate the merits of axiomatic versus other types of theory, however, that we often underemphasize the differences among formal axiomatic theories themselves.

This essay views science as an extension of everyday human inquiry, albeit a specialized and highly developed mode of inquiry. Science has thrived at least in part* because it is a superior form of inquiry for some purposes. This superiority, I believe, derives from the great capacity of science as a communication system to carry large amounts of precise information about problems of practical interest to people. Any system of knowledge which enhances the survival ability of a society would be of "practical interest" even if the society or individual did not perceive that utility. A science which does not enhance our ability to make precise statements about important events in human affairs that can be understood unambiguously by others to their practical advantage will not survive in a world where knowledge is power.

Axiomatization is an important step toward this goal since it allows for systematic deductive predictions of empirical outcomes. Failure of these predictions to conform to observed outcomes leads directly to a falsification of inadequate axioms, and thus theories may be improved to fit observations to ever increasing accuracy. But making predictions - or even manipulations - which are correct is not enough. Predictions must also be informative - they must yield information of real use value to humans.

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* No notion of human or organizational "purposefulness" is intended here, but rather a Darwinian "Natural Selection" concept is intended. Those societies which develop scientific coding systems for whatever reasons - including chance - will fare better in a competitive environment if the theory meets the criteria enumerated here.

Theories are informative to the extent that they provide information people want and need and can use to answer some felt interest. A theory, for example, such as

It always rains in summer

This season is summer

.˙. It will rain this season

is formal, axiomatic, empirically correct, but relatively uninformative.

This essay begins with the assumption that an axiomatic theory is a prerequisite to successful, informative science. But it goes further in that it assumes some axiomatic theories are better than others. It examines the axiomatic theory of physics closely to determine why it is so informative. It concludes that the utility of physical theory derives from characteristics of a) the symbol system it uses; b) the measurement rule it adopts, and c) the formal deductive properties of its rules for combining symbols (i.e., its grammar). The language of physical science is capable of conveying more information less ambiguously than other linguistic systems and therefore makes more informative statements about its domain of inquiry. A theory which predicts where, when, how much and for how long it will rain is fundamentally better than a theory which merely predicts that it will rain, even if both theories are "correct."

Part two attempts to describe a theory of cognitive processes self-consciously written to conform to the formal characteristics of the most useful of physical theories. It should be evaluated in comparison to other theories (both in this volume and elsewhere) not only in terms of how closely its predictions are borne out by observations, but also in terms of how useful and informative its predictions might be in furthering some human interest.

PART I

General Considerations

A. Science as a Communication Process

Science is a social process involving the collective work of generations of scientists which has ramifications for the daily lives of all members of society. As an organized and social body of thought, science concerns itself with socially validated knowledge--knowledge which has been confirmed across observers and across time. Observations which are unique to a single observer or a single time point are never included in the body of accepted scientific theory in any science, and we accept as scientifically validated facts only those observations which are invariant across careful observations by multiple observers. Even more stringent is the requirement for scientific laws, which must be invariant across observers and across time. A little reflection quickly reveals the centrality of communication in the process of science, therefore, since invariances among the observations of different observers at different times can only be established by communication.

These observations are themselves deeply dependent upon the language or symbol set by which they are encoded, as is very apparent when we consider how we can communicate them to another observer for confirmation, or even how we recall them for comparison with our own later observations. Whether or not we accept the strong Whorfian hypothesis that our observations themselves are determined by our symbolic repertoire, both communication and recall do seem completely dependent on symbols, and science without inter-observer and over-time correspondence has no real meaning. These considerations warrant a very careful scrutiny of the role of language in science.

B. Science as a Special Descriptive Language

Any language--even a language specifically for scientists--must have at a minimum three elements: (1) a set of symbols, (2) a set of rules for setting the symbols into correspondence with observations made by observers, and (3) a set of rules for combining symbols. In a vernacular language (like English), the set of symbols consists primarily of the set of words and non-verbal gestures which the bulk of the language users recognize. Each of these words or gestures must be individually remembered; there is no rule by which a person who knows some of the words may construct all the others. The set of rules for setting each subset of these words into correspondence with observations is lengthy, unsystematic, inductive and informal--the result of generations of informal social negotiation. Nor do all users of the language share exactly the same rules: one observer may differ considerably from another in the set of observations to which he or she may attach the word "good," for example, or "friendly." Moreover, the set of rules for combining words is similarly unsystematic and often ambiguous.

The combination of the words "good" and "person," for example, has a meaning which is a combination of the meanings of the two. Adding the word "friendly" to this combination transforms the meaning of the phrase in a roughly additive way, that is, the combination "good" and "friendly" is some sort of sum of the meanings of "good" and "friendly." Adding an adverb like "very" changes the meaning in a multiplicative fashion, but how this new meaning is derived is not explicit. In general, for example, the adverb "extremely" has a higher multiplicative value than the adverb "very," but this is not true for every user of English (Cliff, 1959, 1974). These ambiguities cannot be permitted in science, and for a simple reason: if our goal as scientists is to establish invariances among observations across observers via communication, we must rule out the possibility that variability in observations reported by multiple observers is attributable to amgiguities in the use of the language by which the observations are compared. To prevent this, we might set up several criteria that any satisfactory scientific language must meet.

First, all users must agree unequivocally on the set of symbols to be used. Second, the number of symbols in the set must be at least adequate, in all possible permutations, to enumerate all possible observations. Third, all users must share a simple unambiguous set of rules for setting observations into correspondence with symbols, and fourth, the rules for combining symbols must be unambiguous and consensual. To help assure that these four criteria are met, and to assure efficiency in use of the language, we will specify a fifth criterion: all rules--i.e., the rule for defining symbols, the rules for establishing correspondence between symbols and observations, and the rules for combining symbols--must be as "simple" as possible. We might accomplish this by specifying that the most primitive set of such rules be small, and that every additional rule may be derivable from the original set, rather than simply remembered one by one as is by and large the case in vernacular language. Before turning our attention to the way in which science constructs explanations, therefore, we might well examine the process by which science, as a collective social enterprise, describes phenomena.

1. Fundamental Descriptive Variables. Aside from very specialized verbal symbols (like "nucleus" or "force") the primary set of symbols of science consists of the set of mathematical symbols. For the time being we will restrict our discussion to a subset of these--the set of real numbers.

This subset of symbols is of extreme importance in science since measurement (which is used here as a synonym for observation or description) is usually defined as the process of setting observations into correspondence with elements of the set of real numbers according to some rule (Campbell, 1921, 1952; S. S. Stevens, 1951, 1968; Suppes and Zinnes, 1963). As Kramer (1974) puts it:

> Thus a measure is a function whose domain is some class of sets and whose range is an aggregate of non-negative real numbers. It would then seem logical to divide an explanation of measure into two parts, first, a discussion of the domain, the class of sets to be measured, and second, the rules which are to govern the range.
> (pp. 393-394)

For scientific purposes, the real numbers are superior to English words for at least three reasons: first, all the real numbers may be created out of a small set of symbols at once by any experienced user--even those real numbers never encountered before. Second, there is no ambiguity about the "meaning" of any real number among scientists. Once any real number has been written, its relationship to all other real numbers is completely and unambiguously determined for all users. Third, many valuable operations are defined for the real numbers.

Once having defined a satisfactory symbol set (a process which, we should recall, is not complete after several thousands of years of pure and applied mathematical study) it remains to form some unambiguous, consensual rule by which linkages between the symbol set and observations may be made.

Here Einstein's conception of the measurement of distance is instructive (Einstein, 1961):

> For this purpose (the measurement of distance) we require a "distance" (Rod S) which is to be used once and for all, and which we employ as a standard measure. If, now, A and B are two points on a rigid body, we can construct the

> line joining them according to the rules of geometry; then, starting from A, we can mark off the distance S time after time until we reach B. The number of these operations required is the numerical measure of the distance AB. This is the basis of all measurement of length. (p. 5)

Einstein's measurement procedure is two-staged: first, an arbitrary distance (or discrepancy, in the general case) is stipulated by the scientist. It is vital to note that rules for the perception or measurement of this initial measurement distance or discrepancy are not stated; rather the scientist must assume the observer shares with him/her a common referent for the ordinary language symbol "distance" or "difference," and that the observer can make this initial recognition unaided by further definition. Ultimately it is this call to common experience as codified in ordinary language symbols that establishes a link between the everyday experience of the observer and the scientific theory.

While the choice of the unit of measure is arbitrary, choice of different standards will have consequences for the patterns of measurements made with the system. Choosing as Rod S some ordinary language symbol whose relation to other such symbols is stable over time might make results of the measurement more clearly interpretable in terms of the ordinary language system than would a Rod S defined by a symbol whose meaning fluctuates in the vernacular system.

Secondly, the scientist specifies a rule by which other instances of distance or discrepancy are to be compared to this unit. In this case, observers are asked to make ratio comparisons of all other distances or discrepancies to this arbitrary standard.

Distance can only be measured in relation to some other standard of measure (Rod S) which itself is undefined and unmeasured. A measurement system that employs, at its core, an unmeasured standard is commonly referred to as fundamental measurement (Campbell, 1928; Ellis, 1966; Hays, 1967; Suppes and Zinnes, 1963). On the fundamental measurement of length or distance, Hays (1967) remarks that "Length is measured in terms of length. One need not define length in reference to other quantities" (p. 15), Danes and Woelfel, 1975).

As communication scientists, however, we may inquire into the underlying character of a fundamental perception such as distance or time. First, fundamental measurements assume for the perception of both distance and time that there exists some sort of separation (i.e., an observed distinction, discrepancy, disparity, difference, etc.) among two or more stimuli; thus distance may be viewed as separation in terms of physical location and time as separation in terms of a durational array. This separation is symbolized by some vernacular word or set of words such that the mention of the word serves as a cue to recall the observation. When the scientist wishes to refer to this observed separation, he or she cites these associated words. The observations called to mind by the words represent the meaning of the words. There is no direct access to the observation not mediated in this way by words or symbols of some sort.

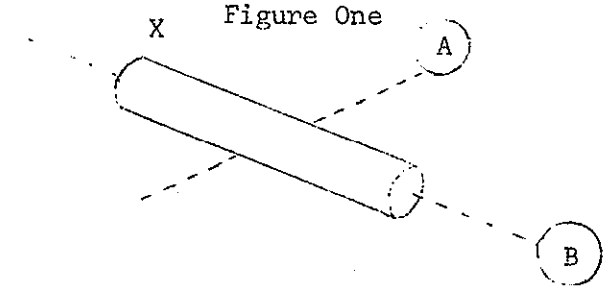
Secondly, when several of these observed separations have been recalled, fundamental measurement requires that all observers apply an unambiguous rule for comparing these observations to each other. The simplest such rule requires comparison of a sequence of observed separations to an arbitrarily chosen observed separation to determine whether or not they are identical. Similarly, a stronger (ordinal) rule would require the respondent to determine whether observed separations are larger or smaller than the original arbitrary standard separation. Einstein's distance rule above requires a ratio judgment of separation; i.e., the respondent is required to specify how many times larger or smaller than the original standard the observed separation may be. The more rigorous this rule, the more precise the resulting measures (given, of course, that the observer understands and can apply the rule). The <u>meaning</u> of the resulting measure is given by the meaning of the vernacular word(s) which call up the separations in the first place. By now, most people in industrialized settings have become quite adept at making ratio judgments of separation in physical location (distance) and temporal arrays (time), as well as in other abstract domains, such as monetary value, although it is clearly true that the general population cannot be expected to use these rules with the precision of trained scientific observers. No rule is required by nature, however, and scientists are free to choose any rule, but different rules will yield different geometries and processes. By a pragmatic argument, some of these will prove more helpful than others for differing purposes.

2. Transformation Rules. Once a consistent symbol system has been consensually defined, and unambiguous rules of correspondence have been established, we may concern ourselves with the set of rules for combinations of symbols. In the case of the real numbers, these rules are relatively unambiguously established in the form of the field axioms, which define the operations of addition and multiplication for the real numbers. Since these symbols have now been set into correspondence with the observations, these operations on the symbols are equivalent to transformations of the observations which correspond to the symbols. Two particular classes of these transformations are of substantial interest. The first of these consists of what we may term cross-observer transformations.

To illustrate the meaning of cross-observer transformations, consider the case of two observers A and B viewing the cylindrical object X as in

Figure One. Even though both observers utilize the same symbols and share unambiguously rules for setting observations into correspondence with these

Figure One



symbols, and even though they both observe the "same" object, through the aid of communication they find their observations do not correspond, since observer A sees the cylinder as an oblong figure, while observer B sees a perfect circle. Clearly, in the example given, this failure of correspondence cannot be attributed to characteristics of the "object" alone, nor to ambiguous use of symbols or rules of correspondence, and must therefore be attributed to characteristics of the observers. The task of the scientist is now straightforward but nevertheless tedious: he or she must find some transformation from among the permissible rules of combinations--or define a new rule of combination--which transforms the observations (now encoded into symbols) of one or both observers until they correspond. The specification of invariants under transformation is generally considered the primary task of scientific theory (Einstein, 1961; Reichenbach, 1958; Kramer, 1970; Pieszko, 1970). (In the present example, of course, a simple rotation through 90 degrees will suffice.) Those characteristics of the observations which are invariant under this transformation--that is, those characteristics which are not dependent on the perspective of the observer--

may be meaningfully attributed to the object observed. If no characteristics of the observations which are invariant across the required transformation can be found, the observations are not consensually validated, and are not considered "real" or "objective," but must be considered, at least for the time being, idiosyncracies of the observers. The first task of scientists, therefore, once symbol systems and rules of correspondence have been established, is to find transformations which render observations by different observers congruent and to isolate properties of the observations which are invariant across those transformations. We are accustomed to call "real" those observations for which such transformations have been identified, and it is revealing to discover how many of the discoveries of modern physical science have been formal mathematical achievements of just this sort. The discovery that all differences in physical perspective could be transformed away by only one class of transformations--i.e., rotations and translations--was a landmark discovery for classical science since it was then a formal mathematical question as to which properties remain invariant across this general class of transformations.

The second form of transformation of interest to us may be termed cross-time transformation, and can be understood as a simple analytic extension of cross-observer transformations. In this case, instead of considering two observers A and B, we may think of a single observer moving from point A to point B in Figure One. (This, of course, is equivalent to a static observer and a rotating object, and so only one of these cases need be considered.) Here we are concerned with finding a transformation which will establish correspondence between observations made by the observer at one time with observations made at a second time. While the logic of these transformations is identical to that of cross-observer transformations, these transformations hold a special place in science insofar as they constitute the set of equations which describe motion and change. Furthermore, cross-time transformations introduce a new complexity into our considerations since they involve the combination--and hence the need for new combination rules--of two or more fundamental variables, one of which is always time. Time, like distance, is a fundamental variable. Following Michels, time is that which is measured by a clock, and a clock is ". . . any device which emits signals such that the interval between any two contiguous signals is the same" (Michels, 1968,p.23). The most convenient combination rule for time and distance is again a ratio rule--the ratio of distance travelled to time yields the derived descriptive variable <u>velocity</u> as

$$V = \frac{\Delta s}{\Delta t},$$

and other such derived variables are well known, such as acceleration:

$$a = \frac{\Delta v}{\Delta t}$$

Following the same reasoning, we may now consider a primary task of science as the establishment of cross-observer invariances in the perceptions of such derived variables as valocity and acceleration for observers themselves in motion relative to each other and to the object observed. This is quite important, since it is generally <u>changes</u> that we seek to explain, and changes are always and only derived variables across time.

C. Explanation in Science--Causal Laws

Once we have established a system for the description of processes observed by observers we can inquire into systems of explanations of these processes. Our starting point for this inquiry is the understanding that explanations always answer some interest that human observers hold, and consequently not all observations require explanations. A careful historical analysis of science shows clearly that scientists are not generally concerned with the explanation of invariances, but rather with changes. Although much more sophisticated formulations might be found, a general cognitive principle shared by scientists might well be the assumption that, left alone, things will remain as they are. When things do "remain as they were" we are not surprised, and require no explanation. If we find the scene along our route to work today is as it was yesterday we do not seek an explanation. If we find instead that a building there yesterday is gone today we ask why. Nor is this only true of static phenomena: we expect processes to continue, and seek explanations only when the processes stop or change--thus we may well ask why a co-worker fails to come to work on a given day, but do not inquire as to why he or she does show up each day. In the dynamic model just described, we do not inquire as to why a static object remains static, or why a moving object continues to move. What we seek to explain are <u>changes</u> in processes, or technically, <u>accelerations</u>. In fact, following our general rule that, left alone things remain as they are, we assume that anything which does not remain as it was was not left alone. And as arbitrary as our original assumption that things left alone remain the same is our assertion that "something" has affected those things which do not remain the same. In general, scientists have created two wholly arbitrary concepts corresponding to the two components of this "inertial" hypothesis: <u>inertial mass</u>, which is the concept acting toward "sameness," and "force," which acts toward change. Neither of these concepts is "real" in the sense of fundamental concepts like distance or time, but are wholly constructs derived from our inertial assumption. We define inertial mass as that quality of matter which resists acceleration,

$$m = \frac{k}{a}$$

where k = a constant, and force as that which produces acceleration, as

$$F = ma$$

The constructed nature of these concepts is clear from the fact that neither of them is ever directly measured in a fundamental way, as time and distance are measured, but both are always derived from the fundamental variables distance and time and the "inertial" assumption.

Mass, for example, is hardly the objectively concrete kind of phenomenon it seems. Mass is defined as the quality of matter which resists acceleration. As such, mass cannot be directly touched, seen, heard, smelled, tasted, or sensed in any way. Measurement of mass requires the theoretical linking of the concept to some observable phenomenon like acceleration, as in the expression:

$$m = \frac{F}{a}$$

If we can somehow control for force, then the theoretical link between mass and acceleration allows us to convert observed acceleration into mass. We might, then, roll a steel ball down an inclined plane until it strikes another ball, measure the valocity of the second ball after an interval of time t (since velocity can be measured) and calculate the average acceleration by dividing the velocity by the interval of time. At this point, however, the mass of the second ball is still unknown. Only after we roll, for example, the same first ball down the same inclined plane into a third ball will we be able to establish a relative measure of mass. Since the force F may be considered the same in both instances, we may write:

$$F_1 = m_1 a_1 = F_2 = m_2 a_2$$

Thus,

$$m_1 a_1 = m_2 a_2$$

or better,

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

Note that the operations of measurement yield exactly the theoretical expression by which mass is defined in theory. Since both a_1 and a_2 are measurable as ratios of distance and time, m_1, the mass of the second ball may be expressed as a ratio to m_2, the mass of the third ball. At this point, we will have established a relative measure of mass. It is crucial to note that the measure, as are all such measures, is a ratio or proportionality and this proportionality is calculated from the theory. We expect this proportionality to remain constant over time. If it does we are satisfied that we have found the correct explanation. We do not seek to explain further, since the ratio is constant or invariant, and in general our inertial assumption does not require us to explain invariances.

These measures, although arbitrary, are of some real heuristic value, since they reduce changes to <u>nameable invariances</u>, and invariances are handy since they allow us to predict (and control) future outcomes with certainty. Regardless of the unsatisfactoriness of this state of affairs philosophically, attaining strong predictability of phenomena is of very great value.

Even so, scientists in the 19th and 20th centuries--like Hertz (1956), Poincare (1946), Einstein (1961), Heisenberg (1928) and others--have been dissatisfied with this arbitrariness, and have sought alternatives. Why they have done so is not difficult to see.

Consider an infinitely long train moving along a track. The track and surface of the earth provide a convenient inertial frame of reference, and the train is assumed to be moving along the track with a constant velocity V_t relative to the surface of the earth. The car may be assumed to be moving in the same direction with a velocity V_c relative to the earth's surface. As long as the car and train do not approach near-light velocities we are safe in establishing their relative velocity as

$$V_{t-c} = V_t - V_c \tag{1}$$

If this result is negative, the train will appear to fall behind at a steady rate; if positive, the train will appear to move ahead of the car at a steady rate. Let us assume that this state of affairs has maintained itself for a very long time, i.e., the value of V_{t-c} has not changed over many generations of train riders and car passengers. Should the value of V_{t-c} suddenly, say, decrease, most riders of both car and train will require an "explanation." But before an explanation can be forthcoming, a satisfactory description of what has happened is needed, since the observers in the car will experience the event differently from observers in the train. Car observers will see the train slow down; train riders will see the car speed up. Who is correct? Clearly we can't hope to explain why the car speeded up if it didn't speed up, and similarly for the train. Perhaps we might call for an "objective" view from a third observer at the roadside. This third observer can resolve the issue directly from within his or her inertial frame, but the objectivity of this view breaks down quickly if we consult yet a fourth observer, say, on the moon. Clearly descriptions of events will differ from perspective to perspective, and therefore clearly what are satisfactory "explanations" in one inertial frame will in general fail in another.

This is really not a problem, as long as the transformations from one inertial frame to another are known, since the "laws of nature" in one reference system can be transformed into the laws of nature in another reference system by means of these transformations. Furthermore, those properties of observations which are invariant across the transformations will be aspects of "reality" about which all observers will agree, regardless of their own reference frame. Those in possession of the standardized language and its transformation rules will find their experiences orderly, comprehensible and comparable with those of other observers. On the other hand, the observer who does not possess the standardized symbols and transformation rules will be strongly individuated, his or her experiences will be confusing and incompatible with the experiences of others; he or she will be continually surprised by events, and most likely will believe that "reality" is not orderly or rule governed, but is a place where events happen spontaneously and unpredictably; experiences will seem vague and indefinite, incapable of precise measure. This is, of course, an apt description of our current understanding of human cognition, and social and cultural processes.

PART II

Theory

Part I described science as a collective process which acts to establish consensus among observers and over time about their observations. To be compared across observers and over time, observations must be encoded into symbols and communicated, a process which introduces additional uncertainty into the measurement/comparison process. To reduce this noise, scientists have developed specialized languages which consist of (a) special sets of symbols, (b) special rules of correspondences, and (c) special rules of combinations or transformations which relate observations across observers and time.

Part II of this essay will illustrate this process with a theory of cognitive processes.

A. Definitions

We assume that individuals encode observations into symbols, combine and store the symbols in some way and compare them with other persons and across time by means of language. These processes are cognitive processes. Science, by this definition, is a cognitive process, although a collective cognitive process to be sure. Gauging the state of one's health across the years is also a cognitive process, as is determining one's own political position from day to day. Collective cognitive processes (or cultural processes) are those cognitive processes resulting from the coordinated activity of a system of individual cognitive processes, like science or ensemble music or election of government officials.

The primary symbol system underlying cognitive processes is assumed to be the vernacular language. Relatively invariant complexes of observations are symbolized by certain vernacular language words like "red" or "hard" or "disappointed." These several complexes themselves are perceived to differ from each other in some ways; in fact, the minimal comparison between two observations that can be reported is the dichotomous discrimination of difference vs. no difference. The differences or separations among the symbols are considered primitive or fundamental variables in the theory. Any concept in the language has a meaning which is given by its pattern of similarities and differences with the other concepts. Change in these separations over time therefore represents change in meaning or definition of concepts. These changes are cognitive processes.

1. The symbol set. The first step in measurement is the stipulation of a symbol set. We choose the set of positive real numbers (see Suppes and Zinnes, 1973) for several reasons. First, since the set is infinite, there is no minimal interval size as with a finite set like, for example, the semantic differential seven-interval scale. Moreover, the real number system is systematic, forming new symbols by rules; a very large set of transformations in the set (like addition and multiplication, for example) are well known, and a very large set of people are already familiar with elements of the real numbers--far more than are familiar with most other psychometric devices, for example.

2. Rules of correspondence. The second requirement of measurement is the establishment of a clear, consensual and unambiguous rule for establishing correspondences between observations and symbols. We choose, following Einstein (1961) and others (Campbell, 1928; Ellis, 1966; Hays, 1967; Suppes and Zinnes, 1963; Krantz, et al., 1972) a ratio rule. First, an arbitrary element of the set of observations to be measured is designated as a unit standard against which all other observations are to be

compared. As we noted earlier, the primitive observations of cognitive processes are the separations among concepts, and one of these separations is chosen as a standard; other separations are compared to this standard as ratios. (A typical format for the use of this rule in actual practice is given in Appendix One). Formally, the rule is expressed as a conditional statement "if a and b are u units apart, how far apart are x and y?" In the present case, "far apart" is defined to mean "different in meaning" so that increasing numbers represent pairs of concepts of increasingly different meaning. Formally the rule requires that a pair of concepts $S_{a_i b_i}$ whose difference in meaning is perceived to be double that of another pair $S_{a_j b_j}$ should be represented by a separation double that of the second pair, or $S_{a_i b_i} = 2S_{a_j b_j}$. Furthermore, no formal restriction on how small a difference may be reported is established by the scaling procedures: limitations of precision are given by the observational capabilities of the observer and not the scale in which such differences are reported. It is important to understand that the use of a precise scale does not itself guarantee precise measurement, since the actual process by which the scale is employed can add or subtract from precision. Because the length of a bridge, for example, is reported in ratio numbers--say, meters--does not guarantee that the measurements have been carefully made. But the use of an imprecise scale--like a semantic differential scale--is sufficient to limit precision of measure. Whatever imprecision of measure may exist within this system is not a consequence of the imprecision of the scale in which measurements are reported, however, and this is a crucial advantage.

Once accomplished, these procedures make possible a mathematically precise definition of the meaning of any concept: since each concept is defined by its relative similarity to all other concepts, any concept C^i is defined by the 1 x (k-1) vector of separations from the k-1 other concepts. The interrelationships among any subset of k concepts is similarly given by the k x k matrix S of separations among the k concepts. Similarly, the cultural meanings of the n concepts defined by a culture is given by the n x n matrix $\overline{S}$ of separations among the n concepts averaged across members of the culture.

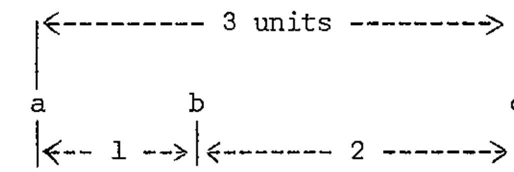
The Geometry of Separation. The concept of a geometry of separation capitalizes on the recognition that physical distance is viewed as a special case of separation in general, and thus is isomorphic to conceptual separation in formal structure. Therefore, conceptual separations may be presented in a geometrical format analogous to the depiction of physical distance; the separations in the matrix $\overline{S}$ may be arrayed in a geometrical pattern. Consider the matrix:

$$\overline{S} = \begin{array}{cccc} & a & b & c \\ a & 0 & 0 & 0 \\ b & 0 & 0 & 0 \\ c & 0 & 0 & 0 \end{array}$$

Here, since $\overline{S}_{ab} = \overline{S}_{ac} = \overline{S}_{bc} = 0$, the three concepts lie on a point in a zero (0) dimensional space. In the matrix:

$$\begin{array}{cccc} & a & b & c \\ a & 0 & 1 & 3 \\ b & 1 & 0 & 2 \\ c & 3 & 2 & 0 \end{array}$$

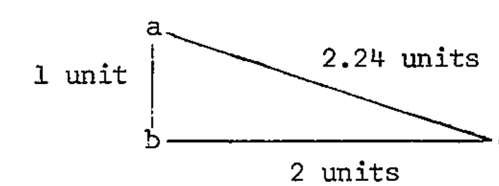
the separations form a line segment in a one-dimensional space which may be geometrically arrayed as the following pattern:



and the matrix:

	a	b	c
a	0	1	2.24
b	1	0	2
c	2.24	2	0

represents a triangle in a two-dimensional Euclidean space.



And finally consider the matrix $\overline{S}$ that extends outside the real number domain:

$$\overline{S} = \begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 1 & 4 \\ b & 1 & 0 & 2 \\ c & 4 & 2 & 0 \end{array}$$

This geometrical pattern represents a complex, non-Euclidean space of 2 dimensions; one real and one imaginary dimension. The translation of conceptual separations into a geometrical configuration will produce a spatial configuration of r dimensions, where r is always one or more less than the number (k) of conceptions judged ($r \geq k-1$).

<u>3. Transformation rules.</u> Among the most important transformation rules are those which describe the symbolic operations by which observations are transformed to correspondence across observers and over time, since

these are the transformations by which information is conveyed among individuals. Since the primitive data of the theory consist in the matrix of reported separations S or $\overline{S}$, we will be particularly interested in transformation rules which preserve these separations. Restricting ourselves to transformations which preserve the raw separations guarantees that the data are never distorted. In this way, data provided by measurements may never be "tampered with" and remain the final arbiter of theory.

a. Frame of reference. Once the observations have been encoded into the symbols of the theory we may begin to compare them across observers and over time to discover invariances. The first step in this comparison process is to transform those observations into a convenient frame of reference (Goffman, 1975; Halliday and Resnick, 1966). While the concept of reference frame has occupied an important place in virtually every social science (and in physics) it has generally resisted precise quantitative treatment in the social sciences. Since this theory is founded on a fundamental variable (separation) which is formally homomorphic with physical distances, it is possible to make use of mathematical procedures developed to establish physical reference systems to generate reference frames for cognitive processes. The procedures used here were developed originally by Jacobi (1846) and subsequently reestablished for psychological data by Young and Householder (1938) and Torgerson (1958) under the name <u>metric multidimensional scaling</u>. First, the matrix of separations S is centered and premultiplied by its transpose to give the scalar product matrix B

$$(1) \qquad b_{ij} = \tfrac{1}{2}\left[\sum_{i=1}^{k} S^2_{ij}/k + \sum_{j=1}^{k} S^2_{ij}/k - \left(\sum_{i=1}^{k}\sum_{j=1}^{k} S^2_{ij}/n\right) - S^2_{ij}\right]$$

which is then reduced by the Jacobi procedure* to an orthogonal matrix of

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* This procedure is formally identical to a complete principle-components factor analysis of the B matrix. It differs from typical factor-analytic procedures in that (1) the input matrix consists of ratio-scaled scalar products rather than correlations, and (2) all the factors are extracted rather than just a subset. This means that the original distances may be regenerated from R with no error.

Eigenvectors R. The matrix R represents a rectilinear coordinate system upon which the concepts are projected as vectors. For k concepts, the matrix R is always k x r where $r \leq k-1$. Each column* vector R_j of R represents one dimension of the space and is orthogonal to all other columns. Each row of R represents the position vector R^i of the <u>ith</u> concept in the space (Davis and Snider, 1975).

No information is lost by this transformation, nor, of course, is any created. Since the set of reference vectors upon which the concepts are now projected is orthonormal, however, mathematical treatment of processes among the concepts is substantially simplified, since vector equations defined on rectilinear coordinates take on a very convenient algebraic form.

While this rectilinear coordinate system shares important characteristics with the familiar three-dimensional rectilinear coordinate system of classical mechanics, it differs in two important ways, both consequences of the empirically-derived structure of the concepts measured to date. First, the rank or dimensionality of the space is higher than 3, although the exact rank may vary across concept domains and across time, as well as across individuals. Second, the space is almost always found to be non-Euclidean. In spatial terms, non-Euclidean spaces are warped or bent; in cognitive terms, non-Euclidean separation patterns represent inconsistencies among conceptions.

Non-Euclidean geometric structure is represented in the Galileo configuration by negative characteristic roots (eigenvalues) in the R matrix; negative eigenvalues indicate imaginary components of the eigenvectors

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* It will be convenient later to denote column vectors of R with subscripts and rowvectors of R with superscripts.

corresponding to these roots, since the eigenvalue is the sum of the squared components, as

$$\lambda_j = \Sigma_{i=1}^{k} (R_j^i)^2 \tag{2}$$

While these imaginary components and negative roots were initially considered by many psychometricians to be artifactual or indications of error, their consistent recurrence, stability over time and generally lawful behavior (e.g., they are generally larger in absolute magnitude for domains not clearly understood by or unfamiliar to respondents) seem to indicate they should not be disregarded. Furthermore, they add no essential mathematical difficulties as long as care is taken to preserve their signs during numerical computations.

b. <u>Cross-observer transformations</u>. For any observer, these operations performed across k concepts will yield the k x r matrix R representing a (non-Euclidean) rectilinear coordinate system upon which are projected the k position vectors $R^1, R^2, \ldots, R^k$. The end points of these vectors, as has been shown, constitute a geometric pattern which corresponds to the interrelations among the concepts as seen by the <u>ith</u> individual.

Comparisons of the observations of two or more observers, once those observations have been encoded into this system, constitute a two step procedure. First, a transformation on one or both of the reference frames must be identified which minimizes the discrepancy among the two or more spaces, while preserving the separations within each. Once this has been accomplished, the resulting matrices may simply be compared by subtraction. These distance-preserving transformations are called rigid motions, and consist of rotations and translations on the coordinates.

Translations within the Galileo reference frame are straightforward extensions of translations in the 3 space common to ordinary physical conception. First, some arbitrary concept* R^i is chosen, and its position vector is subtracted from the position vectors of all concepts in the space such that

$$R^{j'} = R^j - R^i \tag{3}$$

Since $R_i - R_i = 0$ (the null vector) this has the effect of placing the <u>ith</u> concept on the origin of the reference frame. This procedure is carried out for the reference frame of each person in the comparison, so that the reference frames of each observer are centered on the same concept.

Next, the two coordinate frames are rigidly rotated to a least-squares best fit on each other. This rotation is accomplished by successive pairwise rotation of the eigenvectors until the total squared distance of concepts from their counterparts across observers is minimized (Woelfel <u>et al.</u>, 1975). Since we are concerned only with those transformations which preserve the original separations, rotations must be carried out separately for the positive eigenvectors and the negative eigenvectors. This is <u>required</u> since distance is not invariant under rotation of complex numbers, and is <u>permitted</u> since each of the positive eigenvectors is orthogonal to each of the negative eigenvectors.

Once these operations have been carried out for any two persons they yield the transformed matrices R_i' and R_j' for the <u>ith</u> and <u>jth</u> individuals. Comparison of spaces is now given straightforwardly by the subtraction

$$R_i' - R_j' = \Delta \tag{4}$$

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* A very useful procedure is the inclusion of the concept "me" or "myself" in the concept set, since the translation described here can then result in a coordinate system centered on the individual's concept of self or the cultural self for the aggregate.

where the matrix Δ represents the difference between the cognitive structures of the ith and jth individuals. Any row δ^i of Δ represents the difference between the definition of the kth concept as seen by the ith and jth persons within a now common reference frame.* The length $|\delta^k|$ of any row vector of Δ represents the distance between or difference in meaning between the same word as used by the ith and jth person.

c. Over-time transformations. The description of process in the Galileo framework involves essentially the comparison of a time-ordered series of individual coordinate frames R_{t0}, R_{t1}, . . . , R_{tn} or aggregate coordinate frames $\overline{R}_{t0}$, $\overline{R}_{t1}$, . . . , $\overline{R}_{tn}$. As is well known in physical science, there exists no single "privileged" coordinate system against which absolute changes may be measured, and the situation is no different in cognitive space. As is clear from the nature of the procedure by which the Galileo coordinate frames are constructed, the orientations of the eigenvectors of any time frame are functions of the state of the configuration at that time, and therefore any change in the configuration over time will result in an artifactual reorientation of the reference axes(eigenvectors). This is equivalent to comparing motions across reference frames which may be "tumbling" (i.e., in non-uniform rotation and translation) relative to each other. The first step in making comparisons, therefore, is a series of rotations and translations as described above to bring the time-series of coordinate systems into best-fit with each other (Woelfel et al., 1975). Several such procedures are possible. First, if no information

* For an interesting alternative procedure for the comparison of individual cognitive structures, see Marlier, 1974, 1976. Marlier's procedure involves the projection of the individual cognitive spaces of a series of individuals into an aggregate space based on the average separation matrix $\overline{S}$, after which individual differences can be estimated by near regression techniques. Marlier is able to account for over 72% of the differences in individual perceptions with this model.

other than that contained within the matrices at each time period is available, rotation and translation to simple least-squares best fit across the time series is appropriate. If additional constraints can be determined on other grounds (as, for example, might be the case if the observer were to know that some of the concepts had been implicated in messages across a time interval and others had not) some of the concepts might be differentially weighted into the minimization procedure or even left as free parameters, as is described in detail elsewhere (Woelfel et al., 1975). One such strategy might be to translate the origin of the reference frame onto the concept of self (the "me") at each time interval, then rotate the spaces serially to a least-squares best fit on those concepts the individual herself or himself reports as relatively unchanging across the time interval measured. For an equivalent cultural solution, the aggregate "me" might be set at the origin of the collective space, and least-squares criteria applied to those concepts collectively judged stable over time. The resulting process would represent the individual cognitive processes or collective cultural processes as seen respectively by the individual himself or herself, or by the culture as a whole. What is most important, however, is the understanding that the description of the processes--and hence the "laws of nature"--within the spaces will be altered by different choices of a rotation scheme, and that there exists no "correct" choice. Once a choice has been made, however, processes will be wholly determined by observations (data) within that framework, and will be the same for all observers who utilize the same rotation scheme. Within this consensus it makes sense to say the processes are observed and laws are discovered; the consensus itself, however, is created by the observers and not discovered.

Velocity and acceleration: Once a stable reference frame has been defined (by whatever means) it becomes a simple matter to describe cognitive processes relative to that frame. At any instant, the definition of a concept $\underline{i}$ is given by its location in the reference frame, which in turn is given by its position vector R. Changes in the meaning of any concept will be given by a <u>change</u> in location, or a change in the position vector ΔR. For any interval of time Δt, therefore, the average rate of change of meaning or average velocity is given by $\Delta R/\Delta t$. At any instant in time this velocity will be given by the derivative $V_t = \frac{dR}{dt}$. In the space of reference, R is given by its r components

$$(5) \qquad R = R_1 + R_2 + R_3 + \ldots + R_r$$

Since the reference vectors are orthogonal in the Galileo reference frame, the partial derivatives are linearly additive, giving

$$(6) \qquad V_t = \frac{dR_1}{dt} + \frac{dR_2}{dt} + \ldots + \frac{dR_r}{dt} = \sum_{i=1}^{r} \frac{dR_i}{dt}$$

Equation (6) represents the direction and rate at which a given concept is changing in meaning at an instant $\underline{t}$. This rate itself may change over time, and this change in the rate of change is formally an acceleration, which is given by the second derivative

$$(7) \qquad \alpha_t = \frac{d^2R_1}{dt^2} + \frac{d^2R_2}{dt^2} + \ldots + \frac{d^2R_r}{dt^2} = \sum_{i=1}^{r} \frac{d^2R_i}{dt^2}$$

It is these accelerations that require explanation according to the discussion in Part One, and so they are of particular importance. Nevertheless it is important to understand that the accelerations will turn out differently if different rotation and translation strategies are employed earlier in the analysis, and so also, therefore, will the laws which account for them. This suggests an additional strategy for such transformation decisions: for completely practical reasons, those distance-preserving transformations should be chosen which produce the simplest laws of motion within the cognitive reference frame.

4. Explanations of cognitive processes. The equations developed in the previous section are powerful descriptive tools, and many even more powerful descriptive equations can be found in physics, engineering and mathematics books dealing with mechanics and vector and tensor analysis, as long as one is careful to generalize those equations to r dimensions while paying careful attention to the signs of the roots corresponding to the dimensions. The implication that equations for cognitive processes may be found in physics books has generally been viewed with a combination of suspicion and alarm by social scientists on the grounds that psychological or cultural processes are not analogous to physical processes. These arguments are not germane here, since the equations listed do not predict or require any specific processes in the cognitive reference frames, but simply describe those processes whatever they may be. That such equations can describe processes within this system is not an empirical question, but simply a formal consequence of the arbitrary distance rule chosen. The question at issue is not whether equations which describe the processes observed in the system can be found, but rather whether those equations, once found, are sufficiently simple to allow predictability greater than that obtainable with ordinary language. To the extent that such equations yield patterned regularities they will yield such increased predictive power. As we suggested earlier, such patterned regularities, or invariances, once named, constitute scientific laws valid within the reference system. To

illustrate what such laws might look like in this system, consider the following example:

Figure Three represents the (hypothetical) outcome of measurements of the dissimilarities or separations among six persons as they appear to some sample of observers. The separations among the persons which the respondents observe have been translated into numbers by the ratio distance rule (p.) resulting in the separation matrix S_k for each observer, then averaged across observers to yield the matrix $\overline{S}$ with elements

$$\overline{S}_{ij} = \sum_{k=1}^{n} S_{ijk}/n$$

which represents the average separation between the ith and jth person as perceived by the n observers. This matrix $\overline{S}$ has been factored completely* to yield the rectilinear spatial coordinate system represented by the matrix $\underset{k x r}{R}$ where

$$B = R\delta R'$$

$$\text{where } \delta^i_j = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$$

The position of each individual in the space R is given by the position vector R^i whose magnitude $\rho_i = \sqrt{\sum_{j=1}^{r} (R^i_j)^2}$ where R^i_j = the jth component of the ith position vector. Each column vector R_j represents a reference vector orthogonal to each other reference vector (eigenvector) whose length is given by

$$\rho_j = \sqrt{\lambda_j} = \sqrt{\sum_{i=1}^{k} R_{ij}^2}$$

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* i.e., all $r \leq k-1$ roots have been extracted.

where λ_j is the jth root of the characteristic equation for B.

After several repeated measures, assume we have established that the concepts (people in this example) signified by the R^i are not in motion relative to one another, i.e., $R^i_{t_1} - R^i_{t_0} = 0$ for all R^i across all time intervals Δt_i measured. Assume further that, at this point in time, all n observers receive a message which says, in English,

S_1 "Sally and Charles are similar."

Subsequently a series of additional measures across time are taken. We now must make several assumptions, each of which may be falsified by the observations if they fail. First, we may assume that the message will result in some change in the configuration of vectors R_i. If this is so, the eigenvalues and eigenvectors of B will be different for the post-message measurements than for the pre-message measures. These differences may be certified within probability parameters by standard statistical procedures; correlations of corresponding eigenvectors across time may be statistically non-unity; cannonical correlations of the R_i across Δt may be statistically non-unity by Chi-Square criteria; mean differences between position vectors may be statistically significant by ANOVA procedures, etc. Row interactions and row x column interactions in N way repeated-measures analysis of variance may be performed on either the coordinates of R or the distance matrix $\overline{S}$ to determine whether specific concepts or specific pairs of concepts are differentially affected by the message. See Gillham and Woelfel, 1975; Woelfel et al., 1975.

Secondly, we might assume that only the concepts (persons) referred to in the message will be directly affected by the message. If this is true, then a rotation and translation* of the coordinates across any interval of time could be found for which all differences $R^i_{jt_1} - R^i_{jt_0} = 0$ where neither i nor j are concepts implicated in the message, but where $R^i_{jt_1} - R^i_{jt_0}$ does not equal zero if both i and j are mentioned in the message. ($R^i_{jt_1} - R^i_{jt_0}$ may or may not be zero if either i or j but not both are mentioned in the message.) This transformation is given by translating both R_{t_0} and R_{t_1} to an origin at the centroid of those concepts thought to be unaffected by the message (or on one of those concepts itself), and rotating about this origin until the squared distances among the hypothetically stable concepts are at a minimum. If the hypothesis is correct, these differences will be zero by statistical criteria, while the distances between the manipulated concepts will be non-zero by the same criteria. If this hypothesis is false, no such rotation can be found.

A stronger version of the hypothesis would predict not only motion vs. stability, but also the direction and magnitude of such motion. Mature, trustworthy hypotheses about the direction and magnitude of resultant motion can only be made after many careful observations within the system, but initial guesses based on our understanding of the meanings of English words and their effects can provide useful starting points.

The meaning of the English words in statement S_1 imply that the observer has overestimated the separation between Sally and Ralph. If, in general, people attempt to comply with the meaning of the message--i.e., adjust their view in the direction of the view expressed in the message--then, in

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* We are restricted to rotations and translations since these "rigid motions" preserve distances (separations) within time periods.

general, the distance between Sally and Ralph should be reduced by receipt of the message. This relative motion may be differentially attributed to R_1 (Sally) and R_2 (Ralph) in Figure 3. By convention, the force of this message may be defined as the sum of the vectors F_1 and F_2 where $F_1 = -F_2$. Since by definition the force F is equally attributed to each concept R_i, differential displacement along the $R^i - R^i$ vector must therefore be attributed to characteristics of the R^i. That quality of the R^i which differentially resists acceleration (or displacement) is called <u>inertial</u> <u>mass</u>, which is given by

$$\frac{m_i}{m_j} = \frac{|\Delta R^j|}{|\Delta R^i|}$$

We seek now to determine some distance-preserving transformation such that the ratios of the respective $|\Delta R^i|$'s remains invariant across repeated messages and over time or in which the ratios of the $|\Delta R^i|$'s are known functions of some measurable events. Such an outcome would be an inertial reference frame, and within this frame, the known values of the ratios of the $|\Delta R^i|$'s constitutes valuable information about the differential <u>magnitude</u> of the response of the R^i's to messages.

Strict confirmation of the hypothesis that the message may be represented as a force vector on a line through the two concepts in the message by an observed angle of

$$180^\circ = \cos^{-1}\left(\frac{R^j \cdot R^i}{|\Delta R^j||\Delta R^i|}\right)$$

to within statistical criteria. Strict confirmation of the inertial hypothesis is given by the criterion

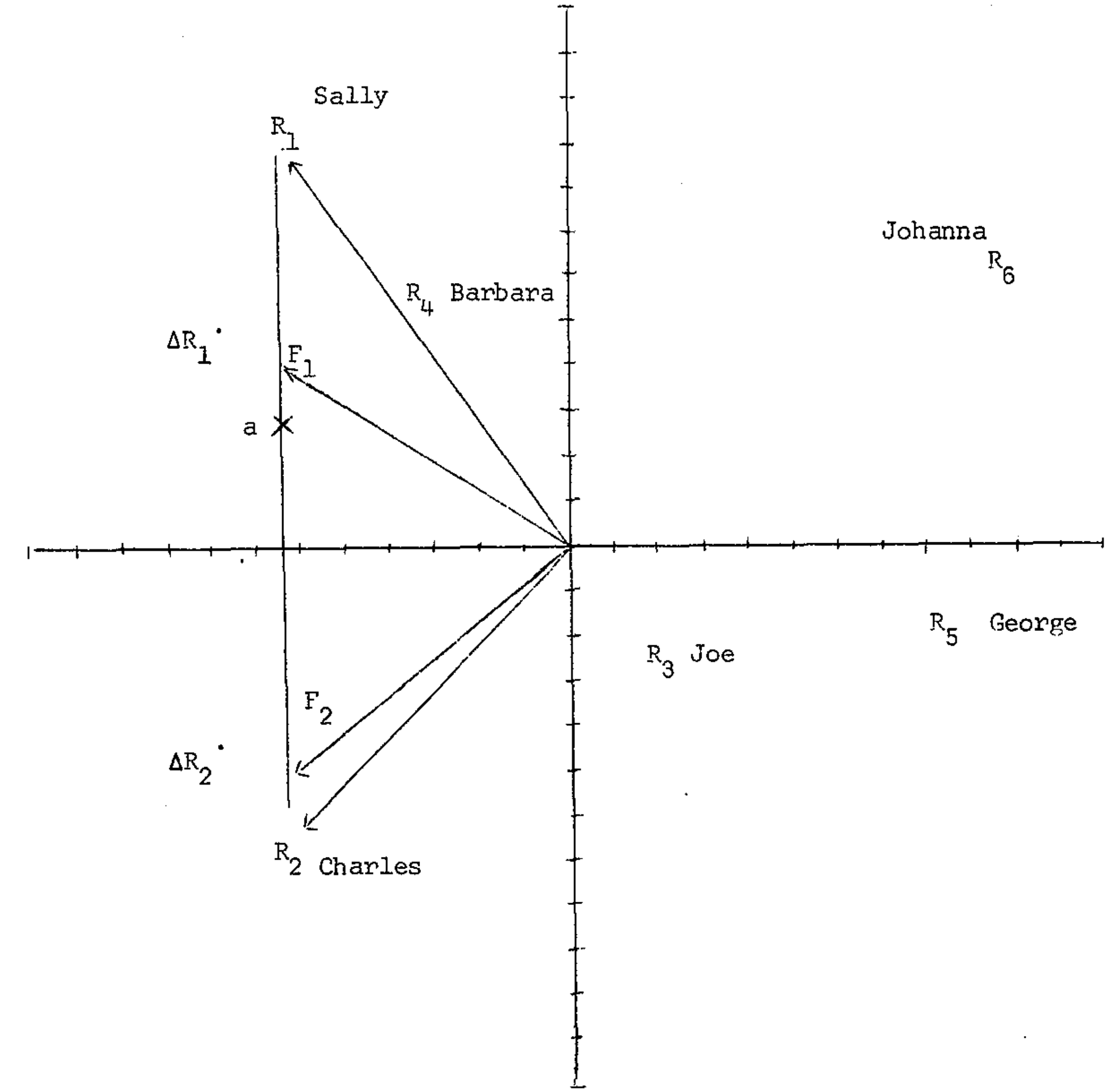


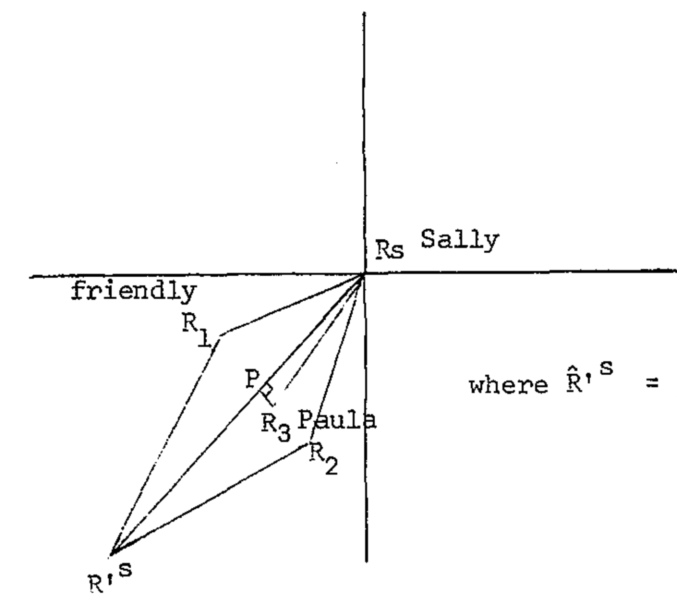
Figure 3. Hypothetical two-dimensional representation of the separations among 6 people.

$$\frac{\dfrac{|\Delta R^i|}{|\Delta R^k|}}{\dfrac{|\Delta R^j|}{|\Delta R^k|}} = \frac{|\Delta R^i|}{|\Delta R^j|}$$

for all values of i, j and k.

A yet more complex hypothesis might suggest a useful combination rule. We might hypothesize, for example, that English sentences add like vectors, i.e., the meaning of the English sentences "Sally is friendly" and "Sally is helpful" is given by

$$\hat{R}'^s = R^1 + R^2$$



where $\hat{R}'^s$ = predicted motion vector for Rs

If sentences add like vectors, then the resultant vector $R'^s = R^1 - R^2$ can be considered a single message vector R'^s resulting in Rs moving along the R'^s vector with an acceleration <u>a</u> inversely proportional to Ms; at time t_p Sally will be closest in meaning to Paula, i.e., $|R^s - R^3| =$ min at p. This minimum distance is given by

$$(10) \qquad |R^3| \sin \left(\cos^{-1} \left[\frac{R^3 \cdot R'^s}{|R^3|\ |R'^s|} \right] \right) = S_3^s \text{ min.}$$

and the distance $|PR^3|$ is given by

$$(11) \qquad \frac{R^3 \cdot R'^s}{|R'^s|} = |R^s - R^3| \text{ min.}$$

These hypotheses can be tested unambiguously given the regression $R'^s = \beta_1 R^1 + \beta_2 R^2 + e$ where $\beta_1 R^1 + \beta_2 R^2$ is the $\hat{R}'^s$ predicted value of the combination of the component messages R^1 and R^2 weighted to a best fit against the observed result R'^s. The $_i$'s represent those characteristics of the R^i which influence their resistance to change of meaning, and the inertial hypothesis suggests that the ratios of these $_i$'s to each other should remain invariant within the inertial frame. These results generalize immediately to the n-vector resultants of combinations of n sentence and the nth order multiple regression check

$$(12) \qquad R'^s = \sum_{i=1}^{n} \beta_i R^i + e$$

These hypotheses also are easily falsified, requiring yet more complexities to be allowed in the theory. The β_i's, for example, may be dependent on factors like distance, mass, etc., which will require more complex hypotheses. The important point, however, is to illustrate that the rejection of hypotheses leads directly to the development of successively more accurate if perhaps more complicated descriptions of processes, and correspondingly more complicated hypotheses which correspond to observations to within increasingly better approximations.

Once the system has been set into motion, it iteratively improves its fit to observations while providing a consensus among observers within which this increasing pool of comparable observations may be interchanged. The result is a tendency toward individually and collectively enhanced observational capacities, reasoning ability and access to information for those who use the system.

Once an inertial reference frame has been stipulated, hypotheses consist of statements about the forces generated by different events in the inertial frame. Failure of these hypotheses (e.g., the hypothesis which suggests a message-like S^1 will result in forces along the vector connecting the concepts linked in the message, which in turn results in motion only along this vector) requires stipulation of an additional force (in this case, acting to produce motion out of the anticipated vector represented by e in (12)). Research must then uncover observed events in the frame which correspond to the residual force vector inferred by the motion out of the predicted vector.

PART III

A Current Assessment

Theories are traditionally evaluated in the social sciences on two grounds: the reliability of their measures and the extent to which outcomes predicted by the theory conform to observed outcomes (validity). In terms of these criteria, this theory has proven quite satisfactory. Many careful studies have shown reliabilities above those considered requisite by most social scientists (Barnett, 1976; Marlier, 1976; Cody, 1976; Gilham and Woelfel, 1976). Moreover, outcomes predicted by the theory have been in good conformity with observation. Barnett, Serota and Taylor (1975), interviewed by telephone a small sample of registered voters in a U.S. congressional district to determine the set of concepts they mentioned most frequently while describing an upcoming congressional race. Sixteen of these concepts were included in a "Galileo" questionnaire which was administered to a larger sample and the results entered into an early version of the Galileo computer program. Based on the solution resulting, they advised a little known candidate in his first attempt at public office as to the optimal set of messages he should send to the electorate, to move himself closer to the location of the "me" or average voter's position in the space. Two subsequent measures showed that this message had the desired cognitive effects--i.e., the candidate moved as predicted. As a consequence, this political newcomer defeated his experienced opponent (the incumbant congressman) with nearly 60% of the vote (Serota, et al., 1977).

Similarly, in a later, more sophisticated laboratory experiment, Cody (1976) entered similar data into the Galileo 3.9 computer program which utilized equations (8) through (12) to determine the optimal message strategy to increase successfully the credibility of two moderately well-known political candidates. Similar procedures have been used commercially to aid in the diffusion of educational innovations; the formation of a state-wide organization for special education; to aid in the reformation of a state educational system and to aid in the sale of commercial products and services. In each of these and other cases, the results haveebeen more precise and informative than those yielded by already proven existing procedures, and their dollar value has greatly exceeded the costs of the research.

While the extent to which this system will prove useful in basic attitude-behavior research is still open, Gillham and Woelfel (1976) have shown that it may be used in lieu of much more tedious conventional methods to determine the attributes along which persons are perceived by groups. Barnett (1976) showed that these procedures were able to detect effects of bilingualism on cognitive processing too small to be detected by the most sensitive of conventional scaling methods. Danes (1976) has shown in laboratory experiments that the "inertial mass" hypothesis expressed in the theory (see Saltiel and Woelfel, 1975) accounts for resistance to attitude change far more accurately than plausible conventional models. Marlier (1974; 1976) showed in a laboratory experiment that the set of transformations designated by the theory account very accurately for differences in individual perspectives about railroad nationalization. Brophy (1976) showed that a sizable portion of the variance in perceptions of members of an academic department, as measured by these techniques, could be accounted for by their positions in a communication network. Wakshlag and Edison (1976) showed that these procedures produced mesaures of the credibility of message sources more precise than conventional semantic-differential and factor-analytic models. Fink et al. showed that these procedures provide precise measures of the differential perceptions of the U.S. power structure across levels of socio-economic status. Danes and Woelfel (1975) showed that these techniques produce more reliable information for a given sample size than do traditional ordinal scaling methods. Craig (1975) showed the system produced extremely stable measures of the perceptions persons held about nations, although ambiguities in the persuasive messages he generated from the theory precluded unambiguous tests of its dynamic assumptions in his experiment. Mistretta (1975) showed that the system made accurate predictions about the perceptions of crimes and their penalties consistent with Durkheim's predictions. Barnett (1972) showed that the system yields stable and reliable outcomes even under adverse conditions like cross-domain scaling and across politically turbulent circumstances. Gordon (1976a) showed that these procedures provide accurate measures of the perceptions of radio stations and their program formats precise enough to predict observed listening patterns, and further showed (Gordon, 1976b) that changes in the metric established by the experimenter yield ratio-level changes in scaling outcomes.

This evidence shows that the theory compares quite favorably with other social science theories in terms of traditional reliability and predictability figures. But such data can be seriously misleading, if one considers only the extent to which the measured data provided by the theory are reliably (reproducibly) measured and the outcomes predicted by the theory are confirmed by these observations. Although the measures yielded by the theory are in the range of the reliabilities of traditional theories (or usually somewhat higher) the fineness of gradation of the measures is usually two or more orders of magnitude better, and the quantity of information yielded is proportionately higher. Clearly if one measure provides 100 units of information at 90% reliability and a second provides 10 units of information at 90% reliability, the former measure is preferable by an order of magnitude difference.

This same reasoning applies to the confirmation of predicted outcomes. A proper evaluation of the theory in contrast to others should note that, not only are the outcomes predicted by the theory confirmed to smaller tolerances (usually by about a factor of two or more), but the predicted outcomes are themselves far more complicated than those derived from earlier theory. The theory presented here, in other words, predicts outcomes about which earlier theories are generally mute or indecisive, and finds these predictions confirmed within smaller tolerances than the cruder predictions of earlier theories are confirmed by methods appropriate to them.

While these experiments support the key premises of the theory, it should be clear from the discussion above that the construction of a useful theory is a lengthy collective social process which requires not only causal hypotheses, but the development of symbol systems, logical roles of combination, measurement rules and a relatively large cadre of trained users even before information substantial enough to warrant hypothesis formulation can be collected.

Ultimately, any theory is to be judged on the extent to which it makes correct, useful and informative statements about problems of real human interest on the basis of observations which can be made at a cost commensurate with their use value. A good theory, therefore, must make the solution of some class of human problems easier. The more important the problems and the easier and more certain the solutions, the better the theory.

On first reading, it may be difficult to see how the tedious equations of the preceding pages can make the solution of human problems easy. In fact, however, once mastered, this system does vastly simplify important human activities. Although the derivation of the equations presented earlier was strenuous work, once derived they need not be derived again for each use. In fact, all of them have been encoded into computer software which makes the tedious logical manipulations they entail quite automatic. It will be the pragmatic ease with which this theory can enable us to solve difficult and important problems that determines its ultimate acceptability.

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