MULTIDIMENSIONAL REDUCTION OF RATIO JUDGMENTS OF SEPARATION: AN ALTERNATIVE TO THE SEMANTIC DIFFERENTIAL

Jeffrey Danes **Joseph Woelfel**

Scientists acquainted with opinion measurement are becoming more aware of the methodological problems associated with category scales. One very popular category scale is Osgood and Tannenbaum's (1957) semantic differential: A scale that uses bipolar descriptors to measure an unidimensional construct such as activity, potency, or evaluation.

The measurement problems associated with category scales have received considerable discussion in both psychometrics and psychophysics (cf. Lodge, 1981). The limitations and weakness include:

1) Lost information due to limited resolution of categories (imprecise measurement)
2) Ordinal level of measurement
3) Distortion of true response due to: a) a finite number of categories and b) an overanchored, fixed set of responses, i.e., predetermined set of responses along an underlying continuum.

This chapter presents an alternative to the semantic differential: multidimensional reduction of ratio judgments of separation. We first present the "method of ratio judgments" and then present some empirical work comparing the two techniques.

Ratio Judgments of Separation

There are a few properties of objects and stimuli that are so evident, obvious, and pervasive in the observers' daily experience that they become fundamental to their perceptual apparatus. That is, they are "knowns" or "givens" which do not seem to depend upon definition. The concept of distance and time fall into this category. Fundamental ratio measurement of distance is explained by Einstein in the following passage.

> For this purpose [the measurement of distance] we require a "distance" (Rod S) which is to be used once and for all, and which we employ as a standard measure. If, now, A and B are two points on a rigid body, we can construct the line joining them according to the rules of geometry; then, starting from A, we can mark off the distance S time after time until we reach B. The number of these operations requiredis the numerical measure of the distance AB. This is the basis of all measurement of length (Einstein, 1961, p. 5).

Einstein's mode of reasoning is instructive for it illustrates that measurement is a two-staged procedure: First, an arbitrary separation between two points, say the separation between x and

y, is stipulated as the standard of measure, Rod S. Second, this standard of measure is then used for the measurement of the separations between other objects; that is, the distance separating A and B is given as the ratio of this initial standard.

The measurement of distance and time can only be measured in relation to some other standard of measure (Rod S) which itself is defined and measured relative to other distances. A measurement system that employs, at its core, a relative standard is commonly referred to as fundamental measurement (Campbell, 1928; Ellis, 1966; Hays, 1967, Suppes & Zinnes, 1963). On the fundamental measurement of length or distance, Hays (1967) remarks that "Length is measured <u>in terms of</u> length. One need not define length in reference to other quantities" (p. 15).

What is the common principle connecting a fundamental perception such as distance or time? For the perception of both distance and time there exists some type of <u>separation</u> (i.e., an observable distinction, discrepancy, disparity, difference, etc.) among two or more stimuli; thus distance may be viewed as separation in terms of physical location and time as separation in terms of a duration. Viewing the concept of separation as a fundamental perception reveals areas other than physical objects and temporal events for which fundamental measurement is possible, viz., the semantic, symbolic or conceptual domain.

Semantic Separations

Are NIXON and REAGAN separated? Most would answer in the affirmative. How much are they separated? If people are asked to measure the length of their shoes or how much time it takes to tie them without the use of a ruler or clock, they do so quite accurately. However, when they are asked to measure the separation between NIXON and REAGAN they at first are stumped. Why? The answer rests in the existance of a standard metric and rules for its use in the measurement operation. Standard metrics for the measurement of distance and time are highly codified components of the human symbolic repertoire; most everyone is familiar with the mile, yard, foot, and inch and the relationships of one metric to the other. No such metrics or relational rules (other than terms in ordinary language) currently exist for the measurement of semantic separations. Let's suppose for a moment that no standardized metric existed for the measurement of distance, a problem that our ancestors were faced with years ago. Then could the distance between A and B be measured? Yes. But a metric and a rule for doing so must be initially created before measurement could proceed. That is, some arbitrary distance, say the distance between here and there, must first be generated, and the distance between A and B could then be given in relation to that arbitrary distance (Rod S). With respect to the measurement of semantic domain would then be given by the two-staged procedure: the specification of metric and a request to use ratios of that metric for the measurement of the separations among other objects.

The ratio judgment of separation technique discussed here is similar to the family of ratio scaling procedures advocated by Stevens (1951). The logic of this technique is not original, for it was first advocated by Torgerson (1958, p. 293). In the hard sciences, however, the technique dates from antiquity.

To illustrate this idea, the following kind of question is asked: If NIXON and REAGAN are one hundred (100) units apart, then in relation to NIXON and REAGAN how far apart are REAGAN and HART? Most persons would probably report a number larger than one hundred.

In providing a standard metric (Rod S) for respondents to judge other separations among semantic objects, the separation among objects may be conveniently assessed by a question of the form: "If x and y are u units apart, how far apart are A and B?" A question as such requests a separation judgment from the respondent ("...how far apart are A and B?") which is based upon ratios of the separation between x and y. A and B in this case refer to all of the possible pairs of the semantic objects to be scaled.

The matrix **S** is used to represent the pairwise separation judgments. The diagonal elements of **S** are zeroes ($\mathbf{S}_{ii} = 0$) and **S** is symmetric ($\mathbf{S}_{ij} = \mathbf{S}_{ji}$).

The Geometry of Separation

The concept of a geometry of separation capitalizes on the recognition that physical distance is viewed as a special case of separation in general, and thus is isomorphic to semantic separation in formal structure. Therefore, semantic separations may be presented in geometric format analogous to the depiction of physical distance; the separations in the matrix **S** may be arrayed in a geometric pattern. Consider the null matrix:

$$\mathbf{S} = \begin{array}{ccc} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$$

Here, $\mathbf{S}_{ab} = \mathbf{S}_{ac} = \mathbf{S}_{bc} = 0$, the three objects lie on a point in a zero (0) dimensional space. In the matrix:

$$\mathbf{S} = \begin{array}{ccc} a & b & c \\ 0 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 0 \end{array}$$

The separations form a line segment in a one-dimensional space which may be displayed as:

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a                    b                    c
.--------------------.--------------------.
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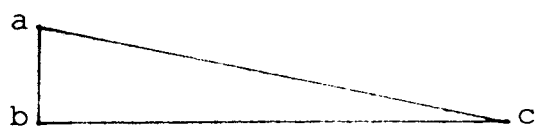
And the matrix:

$$\mathbf{S} = \begin{array}{ccc} a & b & c \\ 0 & 1 & 2.24 \\ 1 & 0 & 2 \\ 2.24 & 2 & 0 \end{array}$$

represents a triangle in a two-dimensional Euclidean space.



And finally consider the matrix **S** that extends outside the domain of real numbers:

$$\mathbf{S} = \begin{array}{ccc} a & b & c \\ 0 & 1 & 4 \\ 1 & 0 & 2 \\ 4 & 2 & 0 \end{array}$$

This geometrical patterns represents a complex, non-Euclidean space of 2 dimensions; one real and one imaginary. The translation of semantic separations will produce a spatial manifold of **K** dimensions, where **K** is at least one less than the number (n) of semantic objects (**K** = n-1). Classical multidimensional (MDS) scaling is used to find the dimensions and corresponding coordinates of **S** (Torgerson, 1958).

An Empirical Comparison

Data were collected to compare the semantic differential and ratio MDS. For the semantic measurements using the semantic differential, three scales polarized with evaluative descriptors and separated by seven steps were used. The objects scaled were: ROCKEFELLER, MCGOVERN, WALLACE, NADER, REAGAN, KENNEDY, FORD, HUMPHREY, ERVIN, NIXON, and MYSELF.

For the ratio judgments of separation, Rod S was created by specifying that the separation between GOOD and NEUTRAL was fifty (50) units apart. Separation judgments were obtained for all of the above semantic objects including the evaluative descriptors used in the semantic scales, i.e., GOOD, BAD, FAVORABLE, UNFAVORABLE, POSITIVE, and NEGATIVE.

Each of the 50 respondents (students) provided separation judgments among the paired combinations of the 17 semantic objects; then using the semantic differentials the 11 public

figures were evaluated. After a five-week period, the respondents were again administered the same instruments.

The two scaling methods are compared in terms of reliability, dispersion/discrimination, and assumed properties.

Table 1 presents the coordinate matrix obtained from the mean **S** matrix. The first dimension corresponds to evaluation as defined by Osgood and Tannenbaum (1957). The second dimension represents political position.

TABLE 1
Coordinate Matrix for Three Dimensions

		1	2	3*
1.	Favorable	56.78	29.62	42.46
2.	Myself	52.18	-24.19	-10.07
3.	Good	46.47	-1.57	-1.64
4.	Positive	46.15	12.36	17.85
5.	McGovern	18.63	-42.00	-27.36
6.	Ervin	17.20	-3.14	0.01
7.	Nader	13.16	-5.27	-11.03
8.	Kennedy	9.60	-0.92	-13.03
9.	Humphrey	2.84	1.97	3.88
10.	Ford	-7.34	19.36	-7.89
11.	Rockefeller	-8.86	3.64	-11.10
12.	Reagan	-12.74	0.72	-10.97
13.	Wallace	-30.25	11.23	-15.78
14.	Nixon	-44.14	47.28	-28.93
15.	Unfavorable	-48.27	-17.84	25.28
16.	Bad	-55.07	-15.89	22.50
17.	Negative	-56.29	-15.36	25.84

*The Eigenvalue for the third dimension is negative; in terms of absolute value it is the third largest. Further, it is the second most reliable dimension.

Figure 1 is a graphic presentation of the first two dimensions.

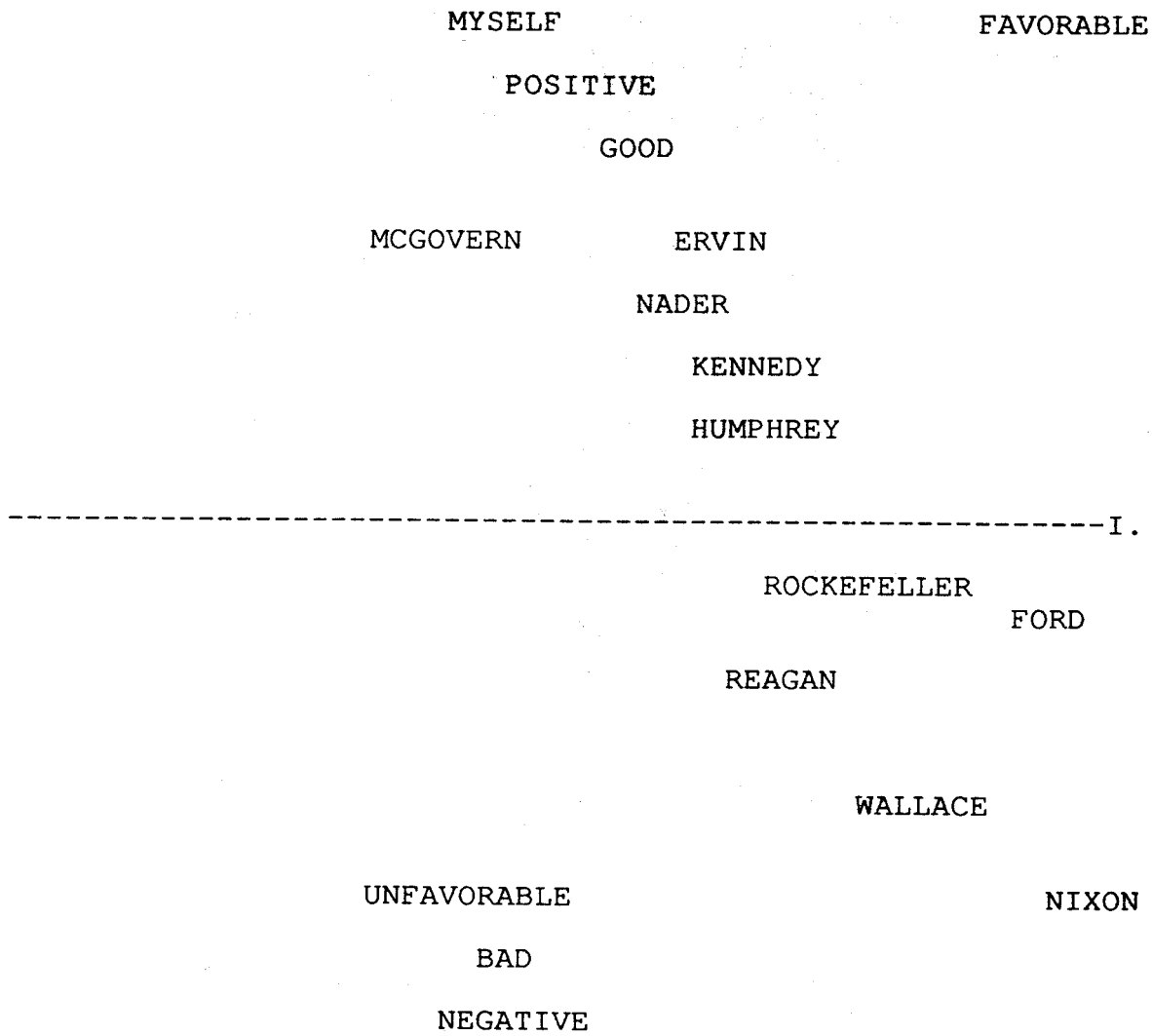


FIGURE 1

Plot of first two dimensions of ratio judgment scale.

The mean ratings of the semantic differential are reported in the second column of Table 2. The third column of Table 2 gives the corresponding evaluation scores as captured by the coordinates on the first dimension of the multidimensional space. The two methods of measuring evaluation are nearly identical (r = .97).

TABLE 2
Mean Scale Values: a) Semantic Differential, b) MDS, c) Transformed Semantic Differential

Semantic Object	Semantic Differential	MDS	Transformed Semantic Differential**
Good	3.00*	46.47	54.19
Favorable	3.00*	56.78	54.19
Positive	3.00*	46.15	54.19
Myself	1.73	52.18	30.66
Nader	0.78	13.16	13.05
Kennedy	0.68	09.60	11.20
Ervin	0.52	17.20	08.24
McGovern	0.31	18.63	04.34
Ford	0.28	-07.34	03.79
Humphrey	0.10	02.84	00.45
Rockefeller	-0.12	-08.86	-03.62
Reagan	-0.25	-12.74	-06.03
Wallace	-1.35	-30.25	-26.42
Nixon	-1.39	-44.14	-27.16
Bad	-3.00*	-55.07	-56.99
Unfavorable	-3.00*	-48.27	-56.99
Negative	-3.00*	-56.29	-56.99

*Fixed by Convention

**Transformed semantic differential scale values given by the regression equation:
-1.40 + 18.53 (differential): R = .97; F = 218; df = 1, 15; p < .0001.

The linear regression of the evaluative (MDS) dimension onto the scores given by the semantic differential yield the equation: transformed semantic differential = -1.4 + 18.53 (differential); the transformed values are reported in the fourth column of Table 2. The high correlation (.97) between the scales represents a high degree of convergent validity, i.e., both scales are measuring the same quantity.

Reliability

Test-retest correlations were used to estimate reliability. The test-retest correlation for the elements in **S** is .86.

The separation matrix **S** only represents the dimensions in the coordinate matrix. Some of the dimensions, however, may be more reliable than others. To check for differential reliability, a least-squares rotation (see Chapter 12) was used to match the pretest and postest coordinate matrices. The values on each pretest dimension were then correlated with their corresponding posttest values. A plot of the correlations is given in Figure 2.

Figure 2 shows the first three dimensions to be the most reliable (r = .97, .81 and .87). Visual inspection of Figure 2 also shows that reliability (test-retest) is highly correlated with size (absolute) of the eigenvalue: Large dimensions are more reliable than small dimensions. Here it is interesting to note that the third dimension has a corresponding negative eigenvalue--indicating that the coordinates on this dimension are imaginary numbers, i.e., the three dimensions define a complex space.

The test-retest reliability coefficient for the mean semantic differential scores is .99.

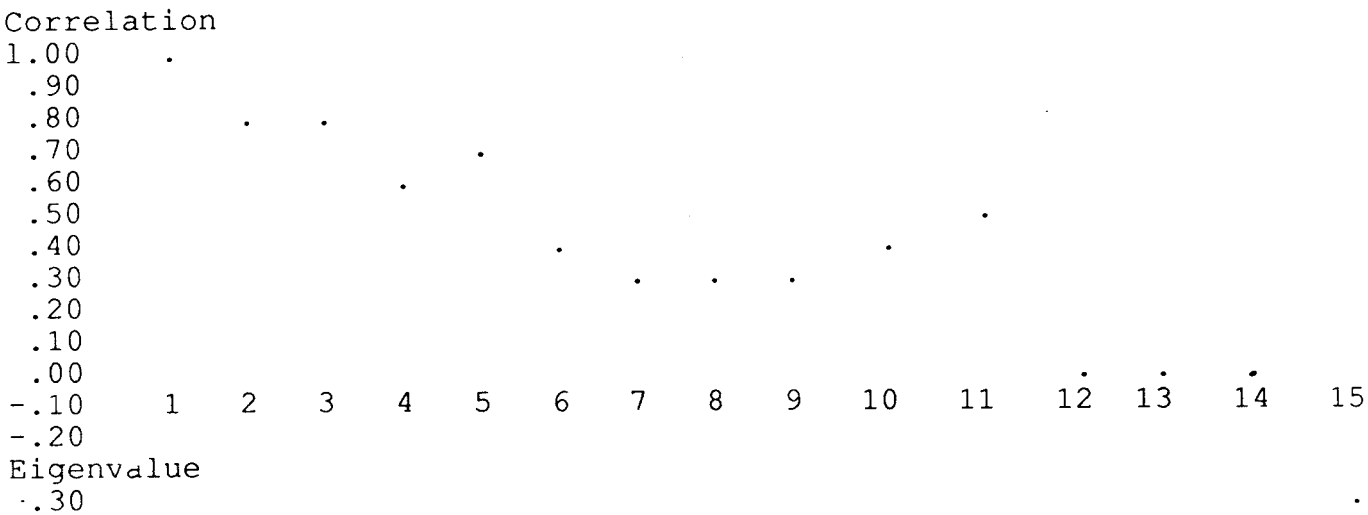


FIGURE 2
Plot of test-retest correlation between dimensions. Dimensions are rank ordered in terms of size (absolute eigenvalue).

Dispersion and Precision of Measure

The two measurement methods yield compariable reliability coefficients (.99 versus .97); the correlation between the scales is .97. Inspection of Table 3 shows greater dispersion for the dimensional (MDS) scale. In every case (except for Kennedy), the obtained dimensional (MDS) scale shows greater dispersion than the transformed semantic differential scale. This analysis supports the popular contention that category scales "compress" or "distort" scale response. In general, there is a nonlinear relationship between category scales and those based upon magnitude estimation methods (Lodge, 1981, p. 15).

TABLE 3
Dispersion Comparison of the Evaluative Dimension (MDS) and Transformed Semantic Differential

Semantic Object	Evaluative (MDS) Dimension	Transformed Semantic Differential
Myself	52.18	30.66
Nader	13.16	13.05
Kennedy	09.60	11.20
Ervin	17.20	08.24
McGovern	18.63	04.34
Ford	-07.34	03.79
Humphrey	02.84	00.45
Rockefeller	08.86	-03.62
Reagan	-12.74	-06.03
Wallace	-30.25	-26.42
Nixon	-44.14	-27.16
Standard Deviation	24.84	15.99
Variance	617.05	255.96

Measurement Assumptions of the Semantic Differential

Figure 3a gives a graphic presentation of two measurement properties assumed by the semantic differential: (1) the assumption that each of the two bipolar discriptors are the same distance from the origin (equi-separation), and (2) that the angle relative to the origin is 180° (collinearity). Figures 3b and 3d are graphic presentations of the failure of each of these assumptions.

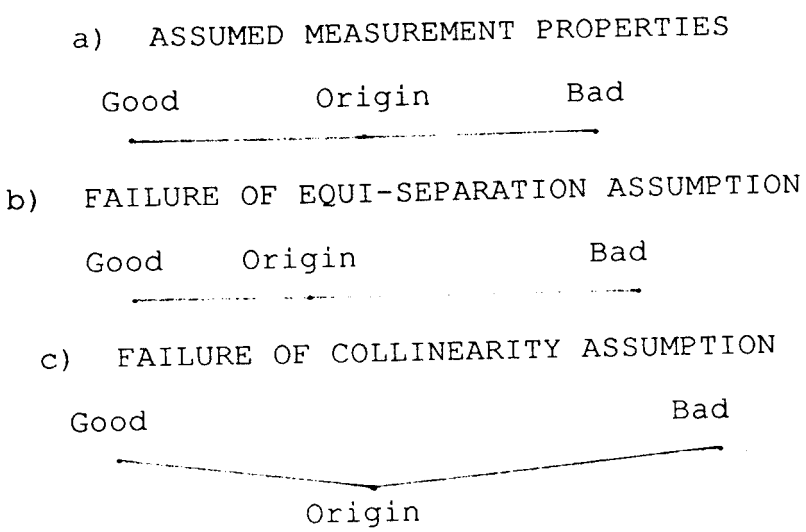


FIGURE 3
Assumed measurement properties of semantic differential.

Ratio judgments of separation do not make the assumptions given in figure 3a.

From the data in **S**, one can determine the measured length of bipolar discriptors from the origin as well as the angle with respect to the origin. Using the mean matrix **S**, this was done in the following way. First **S** was partitioned into a submatrix whose elements represent separations among the six semantic objects: GOOD, BAD, POSITIVE, NEGATIVE, FAVORABLE AND UNFAVORABLE.

The **B*** double-centered scalar products matrix (see Torgerson, 1958, p. 258) was formed and used to find the required lengths and angles. The diagonal elements of **B*** represent squared length from the origin; the off-diagonal elements of **B*** represent the product of two lengths and the cosine of the angle relative to the origin.

The obtained lengths (separations) from the origin are: GOOD (45.39), BAD (57.14), FAVORABLE (66.32), UNFAVORABLE (51.98), POSITIVE (51.73), and NEGATIVE (57.68). The obtained angles are: GOOD-BAD 168), FAVORABLE-UNFAVORABLE (152), and POSITIVE-NEGATIVE (138). No significance testing was done, so it was unknown whether these statistics are within the bounds of sampling error.

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