VARIATIONAL PRINCIPLES OF COMMUNICATION[1]

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Many communication scholars believe that modern communication theory should advance (or already has advanced) beyond "simple mechanical models" toward some less simplistic and more sophisticated theory. As I have suggested elsewhere (Woelfel, 1977), however, it is by no means the case that conventional communication theory is or was in any way mechanical, and in fact arguments against the mechanical character of communication theory for that reason have a "straw person" character to them. Quite the contrary, it is evident that most communication theory is predominantly premechanical in character. The development of mechanics itself rests on certain formal logical advancements which communication theorists for the most parts have not yet adopted, and so it is the contention of the present paper that adoption of a specifically mechanical model of communication would be progressive rather than a retrograde step for communication theorists. In this chapter, specifically, it is my intention to attempt to model certain communication phenomena in terms of some of the most powerful of mechanical forms, following what are most frequently called "variational principles," At the same time, I hope to illustrate the logical developments required for such a step and to show how communication theory has as yet failed to pass through these developmental stages.

The development of a variational mechanical model of any phenomena depends specifically on four logical operations. The first of these is what I have called elsewhere (Woelfel, 1977) the Cartesian measurement model. The second is the development of functional representations of variables in terms of the ratios given by measurements. The third requires the recognition and analysis of residual terms in the functions, and the fourth requires the stipulation of minimization of stationary principles. Once these steps have been accomplished, the equations of motion or change for any configuration of the system of variables under consideration can always be written down.

The Cartesian Measurement Model

The first step in the development of a mechanics of any phenomena is the adoption of the Cartesian measurement model. This model consists entirely in arbitrary agreements among participating scholars to measure phenomena as ratios to an arbitrary standard unit. (Descartes, of course, did not invent this method, but we refer to it here as the Cartesian model due to his explicit formal recognition of this model as a complete alternative logic in contradistinction to the categorical logic of Plato, Aristotle and Aquinas which dominated the intellectual arena prior to the Renaissance).

We have described the Cartesian or ratio method of measurement elsewhere in greater detail (Woelfel, 1977; Woelfel & Fink, 1980), but it is useful here to point out the fundamental

difference between this model and the categorical logic. The categorical logic is a logic of inclusion and exclusion based on category membership. Reasoning or deduction is possible only on the basis of certain types of overlaps among categories. Thus it is evident that if Socrates is a man, then he is mortal, because the entire category man is included in the category mortal. However, it is not possible to determine whether or not Shiela is a man on the basis of knowing she is featherless, since only part of the category featherless overlaps with the category man. In a category logic, only three outcomes are possible: yes, no and doubtful, that is, no answer. An "equation" or syllogism in category logic is either correct, wrong or inconclusive.

In Cartesian or comparative logic, however, relationships are not only expressed in terms of inclusion and exclusion, but as infinitely variable numerical proportions or ratios. Thus, we may say that B is 2.6 times as long (or bright, or friendly) as A. Furthermore, if C is 5.2 times as long (or bright, or friendly) as A, then C is twice as long as B.

There are two advantages to this system which are necessary for a competent mechanics. First, this system can carry more information in a quantitative sense than the categorical logic; it is much more informative. Secondly, since it is quantitative rather than categorical, when deductions based on this system are wrong (that is, when they yield outcomes that do not correspond to measurements) they are not only wrong, as would be an erroneous deduction within a categorical syllogism, but they are wrong by a measurable quantity. Deductions within a categorical logic are either wrong or right, but each deduction within a comparative logic leaves a residual term. If the deduction is perfectly correct, the residual term is zero; as errors increase the residual term grows larger. The residual term is important as we shall see, since it provides not only a basis for knowing how accurate the logical system may be in any particular instance, but because it provides at once the basis for correcting errors and developing general principles for dealing with the phenomena in the future.

As we shall see, every principle of mechanical physics depends on comparative measurement and comparative logic, and no principle of physics is stated in any other than comparative terms. Insofar as the measurement model of communication is still categorical, as it is for the most part, it cannot be described as "mechanical."

Functional Representation and Time

Just as the logic of categorical systems is based on the syllogism, the logic of comparative measurement is based on the function, a word introduced in 1694 by Leibnitz. The most elementary function within a comparative logic is the function by which the position of an element is determined. As we suggested earlier, the length of an object (or its friendliness or brightness, or any attribute whatever) is expressed as a ratio to some arbitrary length (or unit of length, unit of brightness, unit of friendliness, etc.). This arbitrary unit may be thought

of with no loss in generality as a coordinate axis, and the position of the element to be measured is then given as a (ratio) function of the original arbitrary element. Thus if the element of length is the meter, and we imagine a coordinate axis laid off in the direction of the meter, the position of the end point of the element to be measured is given by

$$y = f(m)$$

where, y = the length of the unit to be measured
f = the function
m = the length of the unit (one meter).

If the element to be measured is twice as long as the unit, we could write the function more explicitly as

$$y = 2m$$

Less well-known is the fact that the Cartesian measurement model is completely abstract and rests on no empirical foundation, and hence it may be applied without modification to the measurement of any experience whatever, even an experience as abstract and personal, for example, as affection (Woelfel, 1977; Woelfel and Fink, 1980; Hamblin, 1973). We may choose as an arbitrary standard unit, for example, the amount of affection one feels for the average stranger, and represent the amount of affection one feels for any arbitrary person or object as a function of this standard. We might like a friend twice as much, for example, and thus write

$$y = f(x) \qquad (1)$$

where y = the amount of affection we feel for our friend,
f = the function = 2
x = the amount of affection we feel for the average stranger, and so we write again

$$y = 2x \qquad (2)$$

This, of course, is the same functional form we wrote earlier to describe the length of an arbitrary unit, or the position of an arbitrary object. The logical form is independent of the experience it represents.[2] In order to illustrate the development of a mechanics of communication rather than physical experience, we will continue to refer to affection toward a friend as the dependent variable, but it should be recalled that we might just as well refer to a physical point.

So far, by adopting this system of measurement over the categorical system we have gained some advantage, in that it is more informative to say how much we care for the friend than simply to say that we do care. But, we gain a second advantage as important as the first, and that is the ability to define new variables by applying functional reasoning. Suppose, for example, that we measure the attitude again at a later time and obtain a new value, y_1. (We will now designate the old value as

y_0 to avoid confusion). This allows us to create the new variable

$$\Delta y = y_1 - y_0 \qquad (3)$$

While this step is so simple that the reader may lose patience, it is worth trying to see how the same step could be accomplished using categorical logic. Indeed there is not limit to the new variables we can create within this logic. We create the variable <u>time</u>, for example, by the same ratio method[3], and designate the time of the first measurement as t_1 and the second as t_0 so that

$$\Delta t = t_1 - t_0 \qquad (4)$$

Combining these two functions, we may create yet another new variable

$$v = \Delta y / \Delta t \qquad (5)$$

where v is the <u>ratio</u> of the change in attitude to the change in time, or thus the <u>rate</u> of change of the attitude, or velocity of attitude change. In fact, this variable represents the average rate of change of the attitude over the interval Δt, and leaves us ignorant of the rate of change or velocity at any precise moment, but we can solve this difficulty by imagining the time interval Δt growing smaller without limit and, as it does so, the average velocity across the (even smaller) interval can be made as accurate an approximation as we like to the instantaneous velocity. This instantaneous velocity is referred to as the <u>time derivative</u> of the position (or attitude) as is symbolized by

$$\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt} \qquad (6)$$

so that

$$v_t = dy/dt \qquad (7)$$

V_t, therefore, represents the instantaneous velocity or rate of change of the attitude at time t.[4]

Inertial Principles and Residual Terms

It should be clear that equation (7) defines the velocity of the attitude or its rate of change at a specific instant of time, and that we are free to measure the velocity an any number of such instances. This is turn gives rise to the possibility that the velocity might itself change over time, and hence be itself a function of time, or

$$v = f(t) \qquad (8)$$

It is at this point that arbitrary but important stipulations or principles are introduced into a theory if it is to be a mechanical theory. These stipulations are rules in the sense Cushman and Tompkins (1980) speaks of them, but they are rules which scientists agree to honor solely for the purpose of developing a common framework for understanding and communicating about their experiences. When they were originally postulated, their inventors and followers usually thought they were discoveries about the nature of reality, but later theorists realize they are in fact definitions of a frame of reference against which we gauge our experience. The first of these, (usually called Newton's First Law, due to the old way of thinking) says that for any system which is left alone, that is, which is not in communication with its near surround, there will be no changes in velocity. We might write

$$v = f(t) = \text{constant} \qquad (9)$$

to indicate that velocity is a constant of the time, or using the notation of the calculus

$$a_t = dv/dt = 0 \qquad (10)$$

where a_t = the acceleration of the attitude at time t, which means that the change in velocity with regard to the change in time (acceleration) is zero. This principle is not meant, of course, to say that changes in velocity are impossible, but merely that any such change must be accounted for by means of some communication of the moving element with its environment. If we evaluate the derivative of the velocity with regard to time at some point in time (i.e., enter numerical measurements into expression (10)) and find that it does not equal zero, but rather equals

$$a_t = dv/dt = 0 + K \quad , \qquad (11)$$

then the variable K expresses the magnitude of the communication from the environment. The actual value of this variable will depend on the arbitrary units in which v and t are themselves measured. But if, perhaps at some other time or for some other sample of data, we evaluate the same expression and find

$$a_{t1} = dv_1/dt_1 = 0 = K_1 \quad , \qquad (12)$$

then the ratio K/K_1, will not depend on the units of measure, and will represent the ratio of forces impressed on the system from the environment at the two times or for the two samples.

We have written expressions (11) and (12) in what may seem a strange form to illustrate that these forces are measured as residual or "left-over" terms from what our stipulations would have led us to expect had the system been left alone, and we will see again and again that what we call "explanatory variables" are always residual terms. The process of scientific explanations

always requires us to set up our reference frames in such a way that the residual terms are minimized, and then to seek processes involving communication between the changing element and its environment which are correlated with the residual terms.

One of the most important residual terms in mechanics is force, and force may be determined up to arbitrary constant by the method just described as

$$F_1/F_2 = K/K_1 = a_t/a_{t1} \quad . \qquad (13)$$

this means that the ratio of forces is defined as the ratio of the observed accelerations. In this sense, force as a residual term is defined as "that which produces acceleration." As real as the concept of force has come to be to us after centuries of usage, we should nonetheless realize that there is no such entity in nature as "force," but rather force is a logical device which is wholly a product of the arbitrary conceptual system we have created to describe our experiences. It has as its counterpart one of the other most important residual variables in mechanics, inertial mass, which is def'ned up to an arbitrary constant as the inverse ratio of the accelerations of any two elements under a constant force, or

$$m_1/m_2 = A_2/A_1 \quad , \qquad (14)$$

where m_1 = the mass of the 1st element
m_2 = the mass of the 2nd element
A_1 = the acceleration of the 1st element
A_2 = the acceleration of the 2nd element

It is easy enough to deduce from (14) that mass is the reciprocal of force, or "that which resists acceleration." Mass does not mean "quantity of matter" as Newton thought it did, but is instead a residual term which accounts for the differential acceleration of different elements under constant force. There is no way to measure the mass of any body short of pushing it with a known (i.e., previously measured) force and measuring its resistance to that force relative to other elements (or attitudes) subject to the same force. Mass is not a quality of material objects, and it is every bit as sensible to say that some attitudes are harder to accelerate (have more mass) than others as it is to say that some material objects are harder to accelerate than others.

As long as we deal with only one attitude, there is not need for inertial mass to enter the equations, since inertial mass only refers to the resistance an attitude offers to acceleration relative to other attitudes. When we consider more than one attitude, however, we must modify equation (11) to include the mass of the attitude.If the mass does not vary with the time, i.e., if

$$m = f(t) = \text{constant}, \qquad (15)$$

so that

$$dm/dt = 0 \quad ,$$

we may write (11) as

$$m\ (dv/dt) = F$$

or

$$ma = F \quad , \qquad (16)$$

which is Newton's Second Law. We should not consider this a discovery about nature, however, since it is a consequence of our stipulation given in (10).

If the mass does depend on the time, then we must modify expression (14) to make it somewhat more general, and we obtain

$$d/dt\ (mv) = F \qquad (17)$$

Once again, this generalized equation should not be taken as a discovery about the nature of the world, but rather as a statement of principle. In words, equation 17 says that neither the mass nor the velocity nor both of them together ought to be expected to change spontaneously, but rather any change must be considered the result of the application of some force. The quantity in brackets in (17) is called the linear momentum of the attitude, and (17) asserts that any changes in linear momentum must be the result of some communication with the environment, and, in fact, must be equal to the magnitude of that communication.

D'Alembert made this point explicity by suggesting we write (17) as

$$F - d/dt\ (mv) = 0 \quad , \qquad (18)$$

and create the variable

$$I = - d/dt\ (mv) \quad , \qquad (19)$$

which allows us to write

$$F + I = 0 \quad . \qquad (20)$$

We are able to interpret this new variable I as the force due to the motion of the attitude, or what we may call the inertial force, while we may refer to the force resulting from communication with the environment as the impressed force. Thus equation (20) may be interpreted as a principle which says that the impressed forces plus the inertial forces (forces due to the motion) will sum to zero. As Lanczos says (1962, p. 90):

> A given system of impressed forces will generally not be in equilibrium. This requires the fulfilling of special conditions. The total virtual work of the impressed forces will usually be different from zero. In that case, the <u>motion</u> of the system makes up for the deficiency. The body moves in such a way that the additional forces, produced by the motion, bring the balance up to zero. In this way d'Alembert's principle

gives <u>the equations of motion of an arbitrary mechanical system.</u> (Emphasis in original).

So far, by adopting the Cartesian measurement rule we were able to construct two quantitative primitive variables, position (or length) and time. From these and the functional method of comparative logic, we have derived the additional descriptive variables velocity and acceleration, the additional explanatory variables force and mass, and the mixed variable, momentum. None of these variables have any epistemological status other than as creations of the logical system, but nonetheless they are of great utility in constructing consensual and informative statements about our experiences. By saying that these variables have no epistemological status I mean that their form does not depend exclusively on any inherent characteristics of the experiences out of which they are modeled, and so they may be applied to any experiences whatever, whether physical or otherwise. In fact, it may well be that we call physical those experiences which have already been cast successfully into the form of these variables. Two other variables will be of great interest to us. The first of these can be obtained, again through the logic of functional analysis, by multiplying equation (16) by the velocity, dx/dt to obtain

$$vF = vma = m/2\ d/dt\ (vv) = d/dt\ (mv^2/2) \quad , \qquad (21)$$

and, multiply through by dt

$$Fdx = d\ (mv^2/2) \quad . \qquad (22)$$

The quantity on the left side of (22) is the product of the force and an infinitesmal distance, and represents the work done by the force through the interval of distances dx, while the quantity on the right hand side of (22) represents the differential element of the kinetic energy, so that

$$T = mv^2/2 \qquad (23)$$

defines a quantity called the kinetic energy, or the energy that an element (or attitude) has by virtue of its motion. With this in mind, we can understand (22) as a definition which requires that the differential element of the work is equal to the differential element of the kinetic energy. The work done, of course, is to change the magnitude of the kinetic energy. If the attitude moves through no force, there is no change in the kinetic energy.

Even though the mechanical system is not yet complete, it has already become useful as a vehicle for recording and ordering experiences. We may find, for example, after we deflect the attiutde from its original postion x_0 to another position x_1 that, even though subsequently left alone, that is even though no communication between the attitude and its environment takes place, nevertheless the attitude tends to return to its original position. Unlike the relationships we have discussed so far,

this outcome is not required by the internal logic of the mechanical system, and we might just as well find that the attitude is "content" to remain wherever we put it. The extent to which an attitude tends to return to its starting point, therefore, represents an empirical finding. In fact, some research has observed this effect (Woelfel, Holmes, Cody and Fink, 1977, for example among others), although it is outside the scope of this paper to discuss the empirical character of the attitude domain. If this should be the case, however, we can obviously suggest that the work done moving the attitude from x_1 to x_2 should be equal and opposite to the work required to move it back again from x_2 to x_1. Given this fact, along with the relationship between work and kinetic energy established in (22) and (23), we may see that the work required to move the attitude from x_1 to x_2 is given by

$$\int_{x1}^{x2} F dx = T_{x2} - T_{x1} = - \int_{x2}^{x1} F dx = T_{x1} - T_{x2} , \qquad (24)$$

where T_{x2} = the kinetic energy of the attitude of x_2
T_{x1} = the kinetic energy of the attitude of x_1
and $\int_{x1}^{x2} F dx$ = the total work exerted through the interval from x_1 to x_2.

Equation (24) means that the total work exerted through the distance from x_1 to x_2 equals the change in the kinetic energy. Once again, if the force is zero (or perpendicular to the direction from x_1 to x_2) no work is done and the kinetic energy remains constant.

From our definition (16) we know that the attitude will not accelerate back towards its starting point unless exposed to some force, but, since there is some point at which the attitude is stable, we know that the restoring force is itself a function of the position of the attitude, or

$$F = F(x) ,$$

then we can see that the sum of these forces acting through differential elements of distance between any two points will behave similarly to (24). We introduce the new quantity, V(x), which is given by

$$V(x) = \int_{x}^{xs} F(x) dx = - \int_{xs}^{x} F(x) dx , \qquad (25)$$

where V(x) is called the <u>potential energy</u>, and s is selected as a standard reference point. Potential energy is that energy an element (or attitude) possesses by virtue of its position, and as long as the forces acting on the attitude depend only on its position, we may write following (24) and (25)

$$\begin{aligned} T_{x2} - T_{x1} &= \int_{x1}^{xs} F(x) dx + \int_{xs}^{x2} F(x) dx \\ &= \int_{xs}^{x1} F(x) dx + \int_{xs}^{x2} F(x) dx \\ &= V_{x1} - V_{x2} . \qquad (26) \end{aligned}$$

From this we can see that

$$T_{x1} + V_{x1} = T_{x2} + V_{x2} = T + V = E \quad , \qquad (27)$$

where E is a constant called the total mechanical energy. As long as no forces from the environment impinge on the attitude system, this quantity, the total energy, remains a constant, or is conserved.

Neither kinetic nor potential energy exist in nature other than as a consequence of the comparative logic used and the stipulations or principles laid down, but they provide extremely useful conceptual structures against which we may gauge or experiences, particularly when defined in such a way as to remain invariant under conditions of no communication with the environment. Both kinetic and potential energy are built up of several orders of ratios from the original ratios on which measured values are established, and so are completely consequences of the character of the comparative logic. They could not be established in a category logic.

Another such variable, linear momentum, which was defined in (17) is also conserved during conditions of isolation from the surround. If we multiply (16) by dt we obtain

$$Fdt = m(dv/dt)\, dt = d(mv) = dp \quad , \qquad (28)$$

where p = the linear momentum,
Integrating between two times, t_1 and t_2 yields

$$\int_{t1}^{t2} Fdt = P_{t2} - P_{t1} \quad , \qquad (29)$$

where the quantity on the left hand side of (29) is called the impulse delivered by the force F during the interval from t_1 to t_2, Equation (29) defines the change in linear momentum over the interval of time to be equal to the impulse delivered by the force, and simultaneously asserts that the linear momentum will remain invariant if no force from the environment is present.

This system of variables as well as the interrelationships they obey constitute a pattern against which we may express our experiences. The definitions we have presented of conserved quantities within the system should not be taken as arguments that such conservative systems exist, therefore, since no system that actually exists is absolutely conservative insofar as we know. This is equivalent to saying that no empirical system is absolutely isolated from its environment, and, furthermore, is an acknowledgement that the relationships any system has with its near surround are likely to be too complicated to be expressed in minute detail. To the extent that quantities like energy and momentum are not found to be conserved in an empirical system, non-conservative forces must be postulated. These departures from the conservative ideal are therefore attributed to "non-conservative forces," which may in turn be characterized as forces that cannot be ascribed to some specific action of the environment. As Triffet (1968) suggests, "...nonconservative

forces can usually be recognized by the fact that they are not gradients of time-independent scalar potential functions." Lanczos (1962) calls them "polygenic" for the same reason. In any event, insofar as they are residuals or "left-overs" they are nonetheless subject to calculation. In general, they will be found to be functions of time or velocity, and so we may modify (16) to yield

$$ma - F^C = 0 \qquad (28)$$

where F^C = conservative forces
to show that the inertial forces (ma) minus the conservative forces equal zero when the system is not under the influence of any nonconservative forces. This makes it easy to take account of time-dependent nonconservative forces by means of the expression

$$ma - F^C = F(t) \quad , \qquad (29)$$

and similarly, we may take account of velocity-dependent nonconservative forces by writing

$$ma - F^C = F(v) \quad , \qquad (30)$$

There is, of course, no limit to the complexity of the situation which may be dealt with in this way, and we may consider nonconservative forces which depend on both the velocities and the time by setting

$$ma - F^C = F(v,t) \quad , \qquad (31)$$

where the nonconservative function F can take on any form whatever.

Equation (29) does not constitute so much a discovery as a means of searching, for it indicates that some anamoly has been detected in the system's performance. What is actually accomplished is simply the naming of the anamoly (force) and then its expression as a function of time. In this, time serves as a surrogate variable against which other processes may be calibrated so that they might be compared to each other.

Time-dependent forces are commonplace in communication phenomena, and particularly when they are cyclical, as they often are (as, for example, in daily or seasonal fluctuations) they can easily be controlled, even if unexplained.

Velocity dependent nonconservative forces are also common, and frequently depend on the velocity in a fairly simple way. Forces of friction or the resistance due to a viscous medium are examples of velocity-dependent forces which are relatively simple function of the velocity for physical systems, as is random error in message transmission within human communication, or random forgetting in cultural processes.

As a simple example of nonconservative forces within a communication system, consider a single attitude which has been dislodged from its equilibrium position by some message. The

notion that an equilibrium position exists means that there is a potential function, that is, that the force acting on the attitude depends on it's position. We further assume that random forces proportional to the velocity of attitude change must be considered (although it is useful for purposes of the example to consider them completely unknown) and so we may write

$$ma + Cv + Kx = 0 \quad , \qquad (32)$$

which means that the system will be in equilibrium under the forces due to inertia (ma), the nonconservative velocity-dependent forces (Cv) and the restoring forces due to position (Kx), of course assuming there is no other communication with the environment. The constant C represents the magnitude of the resistant force per unit of verlocity, and the constant K represents the force per unit of deflection from the equilibrium point (assuming such forces are linear).

Equation (32) shows clearly that the acceleration of the attitude will be zero at equilibrium, but that the further the attitude is deflected from equilibrium, the stronger will be the forces which attempt to restore it to equilibrium. The more massive the attitude, the less quickly it will respond to those restoring forces. The higher the velocity of the attitude change, the greater will be the resistive forces slowing the change.

There is good reason to believe this equation or one much like it describes (at least an order of approximation) the process of attitude change in humans and cultures. The restoring force is necessitated by the commonplace assumption in communication theory that their exists an equilibrium position for beliefs, attitudes or indeed whole cultures. This requires position dependent potentials. The mass term recognizes the empirical facat that some attitudes, beliefs and cultural elements are harder to change than others, and we simply specify some scalar value, based on measured resistance to change, which quantifies that differential resistance. The velocity-dependent force is required since, as a little thought will show, otherwise the system, once deflected from the equilibrium position, would oscillate endlessly around the equilibrium point.

This equation (32) models the system when it is isolated from its environment, that is when no communications are being received from the surround. It is easy, using the comparatiave logic, to extend (32) to the case where communication from the surround is present, so that

$$ma + Cv + Kx = F \qquad (33)$$

where F represents the force of the external communication. If the external communication varies with the time, we may write

$$ma + Cv + Kx = F(t) \quad . \qquad (34)$$

Equation (34), therefore, will express the state of motion of the attitude at any time as a function of the communication received from the surround.

Systems of Attitudes and Beliefs

Up to this point we have dealt only with a single attitude, which may be interpreted as a single "object" (a friend) whose position varies on a single dimension (or attribute, in this case, "affection"). The comparative logic, however, makes it possible to generalize this cse to any degree of complexity. As a first step, we may consider the case of several objects varying independently along a single attribute. (By "independently", we mean that the change in position of any one of the objects has no effect over the position of any of the other objects).

In this case we may generalize (32) to the set of n independent equations

$$m_i a_i + C_i v_i + K_i x_i = 0 \quad . \qquad (35)$$

$i = 1,2,\ldots,n$
where n = the number of objects.

Similarly, equations (33) and (34) can be generalized by adding the index i to the force term. If the motions of the set of n objects are not independent of each other, however, that is, if the motion of each object influences the motion of each of the others, then the motion of any object j depends on the characteristics of the others, and we may write[5]

$$m_{ij} a_j + C_{ij} v_j + K_{ij} x_j = 0 \quad , \qquad (36)$$

$i,j = 1,2,\ldots,n,$

or, for the case in which the system is in communication with its surround,

$$m_{ij} a_j + C_{ij} v_j + K_{ij} x_j = F_j \quad , \qquad (37)$$

or, when the communication with the surround it time-dependent,

$$m_{ij} a_j + C_{ij} v_j + K_{ij} x_j = F_j(t) \quad , \qquad (38)$$

While equations represented in (38) are quite compact, they describe a very complex system of interrelationships. Briefly the equations in (38) say that the state of motion of any of the attitudes j is determined by the masses of the jth attitude and all n-1 of the others, by the resistive forces due to the velocities of all n particles, by the n(n-1)/2 pairwise forces expressing the mutual interactions of their potential energies due to their positions, and by the forces from outside the system working directly on the jth attitude.

As complicated as they are, equations (38) deal only with motions along a single dimension or attribute, as we mentioned

earlier. In a realistic individual or cultural belief system, the definition of each object involves positions and motions with regard to many attributes, and so the equations in (38) remain incomplete. If each of the attributes is perpendicular (independent) to each of the others, then we may proceed simply by adding another index $\underline{1}$ to (38), but it is possible to show a more general procedure that will work regardless of whether the attributes are orthogonal or not, or even in the case of general curvilinear coordinates.

As we have seen, the state of motion of a system of belief may be described completely in terms of its potential energy, kinetic energy and outside (impressed) forces. This as we have seen is not a description of "reality" but rather a consequence of the logic of the descriptive system which is call "mechanical." When a belief changes within more than one dimension, the comparative logic system, based as it is on ratios, allows us to establish the <u>proportions</u> of the kinetic energy, potential energy and forces which are projected on each of these dimenstions. For the case of kinetic energy these variables may be projected on the two generalized coordinates axes (dimensions) q_1 and q_2. The vector V represents the (arbitrary) velocity of an arbitrary belief of mass m. The component V_1 of V projected on q_1 is given by the ratio V/V, or, by an elementary result of geometry, $V_1 = \cos \alpha$. Working in the other direction, V itself may be given in terms of its components along the axes q_1 and q_2 by the generalized theorem of Pythagoras

$$V^2 = V_1^2 + V_2^2 - 2V_1V_2 \cos \Theta \quad . \qquad (39)$$

Multiplying through by 1/2m gives the kinetic energy as a function of its projections on the generalized coordinates. When the generalized coordinates are linear and orthogonal (i.e., independent), $\Theta = 0$ and the scalar product term $2V_1V_2 \cos \Theta$ in (39) vanishes and the components of the kinetic energy become linearly additive. Even if the coordinates P_1 and P_2 are curved in an arbitrary way, (39) will still be true for arbitrarily small elements, and may write

$$dV^2 = dV_1^2 + dV_2^2 - 2dV_1V_2 \cos \Theta \quad , \qquad (40)$$

where the d's refer, as previously, to the differential element. The component of the velocity projected on the ith axes is actually a ratio if infinitesmal elements, controlling for the projections on the other axes, which is represented by the partial derivative

$$\partial V / \partial q_i \quad , \qquad (41)$$

and similarly, the component of the kinetic energy with regard to each of the generalized coordinates will be given by the same formalism as

$$\partial T / \partial q_i \quad , \qquad (42)$$

as will the components of the potential energy

$$\partial V / \partial q_i \quad . \qquad (43)$$

These expressions (41), (42) and (43) are ratios of infinitesmal arcs of the curved coordinates, controlling for all the other such ratios, and are called therefore partial differential slopes. They are analogous to partial linear regression coefficients, except that they are generalized for the non-linear case.

Now that we have established the way in which the kinetic and potential energies will project on any arbitrary set of generalized coordinates, it only remains to establish in a precise way how these same energies are related to the motion of a system on one dimension and a generalization to any number of arbitrary coordinates is possible.

Although the chain of reasoning is lengthly it is not difficult. To simplify the notation somewhat, we will define the derivative with regard to time according to Newton's notation, so that

$$dq_i/dt = \dot{q}_i = v_{qi} \quad , \qquad (44)$$

where q_i = the position of the belief projected onto the ith generalized coordinate

v_{qi} = the velocity of change of the belief projected onto the ith generalized coordinate.

Similarly, the second derivative

$$d/dt \; dq_i/dt = d\dot{q}_i/dt = d^2q_i/dt^2 = \ddot{q}_i = a_{qi} \quad , \qquad (45)$$

where a_{qi} = the acceleration of the belief projected on the ith generalized coordinate.

Using this notation, and considering the one dimensional case where the belief or attitude is varying only along a single attribute x we may write

$$\dot{T} = d/dt \; 1/2m \; \dot{x}^2 = m \; \dot{x} \; \ddot{x} \quad , \qquad (46)$$

which says, in words, that the time-derivative of the kinetic energy equals the mass times the velocity times the acceleration. This suggests that we take the derivative of (46) to eliminate the velocity term (recalling that a derivative is a ratio), so that we obtain

$$d/dx \; \dot{T} = m\ddot{x} = ma \quad , \qquad (47)$$

which says, in words, that the derivative of the time derivative of the kinetic energy with respect to velocity equals the mass times the acceleration. This means that due to the comparative method we are able to express the mass and acceleration of a

belief or attitude (or any object whatever) in terms of its knietic energy alone.

As it turns out, we can express the force acting on a belief (at least conservative forces in the sense which we discussed them earlier) in terms of its potential energy. As we say in expression (25), the increment in potential energy equals the negative of the work done. But work is force through distance, so the derivative of the potential energy with regard to the distance is the force acting across an arbitrarily small distance. Formally, as we saw in (25) which we rewrite here without the subscript desginating the x dimension, the potential energy V is equal to the integral of force over distance, or

$$V = \int_1^2 F dx \quad , \qquad (48)$$

and so the potential energy at a point will be

$$dV = F dx \quad . \qquad (49)$$

Dividing both sides by an arbitrary small distance dx gives

$$dV/dx = F \quad , \qquad (50)$$

This, of course, is the ratio of the change in potential energy to a change in position of the belief for very small changes in position.

Since we know that, for a conservative system, F = ma, we may write (using (47) and (50))

$$d/d\dot{x}\ \dot{T} = d/dx\ V \quad , \qquad (50)$$

or, equivalently,

$$d/d\dot{x}\ \dot{T} - d/dx\ V = 0 \quad . \qquad (52)$$

This is equivalent to

$$d/dt\ (dT/d\dot{x}) - dV/dx = 0 \quad . \qquad (53)$$

When working in a curvilinear non-orthogonal system, we need to perform these operations relative to the components of the kinetic and potential energies along the generalized coordinates q_i, as we saw earlier. We learned how to do this in (42) and (43), and so we may write

$$d/dt\ (\partial T/\partial q_i) - \partial V/\partial q_i = 0 \qquad (54)$$

or, what is the same thing,

$$d/dt\ (\partial(T-V)/\partial q_i) - \partial(T-V)/\partial q_i = 0 \quad . \qquad (55)$$

Setting L = T - V, we may write (55) as

$$d/dt\ (\partial L/\partial q_i) - \partial L/\partial q_i = 0 \quad . \qquad (56)$$

Expression (56) is the usual formulation of the equations of Lagrange for a conservative system not in communication with its environment. If forces from the surround are impressed on the system, we may write

$$d/dt\ (\partial L/\partial q_i) - \partial L/\partial q_i = Q_i \qquad (57)$$

where the Q_i refer to the generalized force components directed along the generalized coordinates.

Although very compact, the Lagrangian equations can encompass virtually infinite complexity, and the ramifications of these equations is far beyond the scope of this paper. Nevertheless we can hint at some of the range of communication systems to which these equations may refer by rewriting (57) as

$$d/dt(\partial T/\partial q_i - \partial V/\partial q_i) - (\partial T/\partial q_i - \partial V/\partial q_i) = Q_i \qquad (58)$$

and then

$$d/dt\ \partial T/\partial q_i - d/dt\ \partial V/\partial q_i + \partial V/\partial q_i - \partial T/\partial q_i = Q_i \quad . \qquad (59)$$

As we have already noted, if the system is isolated and conservative, the right hand side Q_i vanishes. If the potential energy does not depend on the velocity (as it usually does not) the second term on the left vanishes; if the kinetic energy does not depend on position (as it usually does not), the the fourth term on the left-hand side vanishes. In this case, the equations reduce to Newton's familiar equations (14). This will also be true if there is no potential function, in which only the first term will remain, but, in the absence of a generalized force term Q_i on the right-hand side, this term will also vanish and there will be no acceleration.

If the knietic energy does depend on the velocities (as it usually does), and if a potential function can be defined as a function of position (which is a condition for a system having an equilibrium value, whether or not it is at or near such a value at any time, and if there are dissipative forces dependent on the velocities, then the second term on the left hand side will not vanish and we will find a linearly resistive term like the second term in (34). While this is only the briefest of sketches of the potential of the Lagrangian form, it should suffice to show that, while the Lagrangian equations are capable of expressing or describing virtually any communication system or system of beliefs, attitudes and their changes, it does not impose itself on the system modeled, but rather adapts its shape to the system observed. Moreover, it is a language and logic capable of a much fuller description of these complexities than is a verbal and categorical language.

Moreover, the treatment is greatly simplifying. This is by no means to suggest that the process of working through the many equations presented in this chapter, or the many more required to achieve actual solutions for interesting human belief systems is

easy, but it is to say that, without such a system of thought, such has proven to be impossible. With some thought it is possible to show, for example, that to define the equations for any system of beliefs or attitudes whatever it is only necessary for the theorist to specify the equilibrium state of the system so that a potential function may be defined. Once the potential energy function has been defined, the remainder of the equations may be written down at once.

While it is true that the Lagrangian form does not impose itself on the system under scrutiny, it is also the case that the Lagrangian formulation allows the theorist to constrain the system with particular ease. This is due entirely to the fact that the Lagrangian equations may be expressed in terms of any generalized coordinates whatever, rather than exclusively rectilinear Cartesian coordinates. To give an example of such constraints for an interesting communication situation, assume two cultural belief systems, A and B, which might represent the pattern of beliefs of two countries, groups, persons, or any social unit at all, about n concepts. Assume further that these groups are placed in communication with each other. If on theoretical grounds (or prior measurement experience) we may stipulate that, within each group, beliefs and opinions will not change relative to other beliefs and opinions will not change relative to other beliefs and opinions, but solely with regard to those in the other group, then we have specified each system as a rigid body, i.e., a body in which distances among any pair of points (beliefs or attitudes) remains invariant over time. Under these constraints, each structure A and B may move closer to or further from the other, and/or either A or B may rotate relative to the other. Assuming the internal structure of each culture has been measured with a Galileo-type procedure (Woelfel & Fink, 1980), the cultural systems may be represented by $(n_a + n_b)$ concepts in an r dimensional space, where r is the number of dimensions in the larger space. Ordinarily, $r(n_a + n_b)$ equations would result from the Lagrangian formulation (or would be required in the Newtonian mode) but the constraints established by requiring each structure A and B to be rigid bodies reduces the number of parameters that need to be estimated substantially. Specifically, only the position of the center of one of the structures relative to the other is needed to specify the distance of A from B overall, while r angles are needed to specify the orientation of each body relative to the other, since we need only specify the orientation of the axes of the first body to the axes of the second. (These angles are usually referred to as the Euler angles, after their discoverer L. Euler). It requires r variables (which may be interpreted as the r rectilinear coordinates of the center of one of the cultural rigid bodies, $x_1, x_2, \ldots, x_r$) to describe the location of the center of one of the structures in the space, and r additional variables which may be interpreted as the r position or Euler angles $\alpha_1, \alpha_2, \ldots, \alpha_r$ to specify the state of orientation between the two bodies, so only 2r parameters are required. Since these themselves may depend on the time, we may write both

the kinetic energies and the potential energies of the system as functions of 2r generalized coordinates

$$T = T(\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_{2r}) \qquad (60)$$

and

$$V = V(q_1, q_2, \ldots, q_{2n}) \quad . \qquad (61)$$

In general, the Lagrangian form will yield nr-k equations where n is the number of concepts, r is the number of dimensions within which the process takes place, and k is the numbr of constraints which may be established on the system a priori. While it is too early in the development of communication as a mechanical science to establish many solid constraints over the behaviors of beliefs and attitudes under general conditions, the constraints just discussed may be seen as a useful first approximation to the likely conditions which may prevail during the first stages of initial encounters between recently introduced cultures, since they will describe the rotations by means of which purely artifactual differences of opinion due solely to differences in orientation or viewpoint are transformed away. This problem has been studied from a more technical point of view by Woelfel, Holmes and Kincaid (1979), where the potential energy function has been described as a least-squares minimum problem.

Conclusion

This paper has by no means established a complete mechanical science of communication, and indeed the implication that such would be possible in a single short monograph runs contrary to the spirit of this work. Rather we have tried to recognize the great empirical complexity of human communication systems, and on this basis have argued that the complexity is so great that the task of understanding such systems by means of a pre-mechanical categorical model is hopeless. What we have tried to establish is not that a mechanical representation of human communication systems can be complete, since the essence of scientific method consists in ignoring those aspects of any empirical situation which can be ignored without important loss. Any mechanical representation of any empirical system will always be to some extent an idealization. On the other hand, it does not follow that, because a mechanical representation is incomplete it should be rejected, since prior to such a decision we must consider the available alternative models. As I have tried to show, the available alternative model, the categorical verbal model, is dramatically less complete. To a large extent communication scholars have been prevented from realizing the advantages of a mechanical treatment because they have been misled as to what a mechanical model implies. I have tried to show that a mechanical model does not imply machines, or clockwork or wires and springs, but rather implies simply a wholly abstract logical form of argument. If it aids the imagination of any scholar to conceive of interacting beliefs as a set of points connected to each other

by elastic strings moving through a volume of fluid, then he or she ought by all means make use of such an analogy, and the Lagrangian equations will model the system effectively. But the strings and fluids are only a picture in the mind of the investigator, and the equations have no need of them -- they simply describe observed changes in a wholly abstract way.

The essence of the mechanical model, rather, is the notion of proportionality. Mechanical theory makes statements about ratios of abstract quantities which have important counterparts in experience only insofar as measurements of observations of experience have been made with a proportional measurement system. Attempts to cast categorical experiences into the logic of mechanical systems are bound to fail by formal logical reasons alone. Overwhelmingly, relatively casual efforts to fit categorical data (such as Likert-type or semantic differential-type or even ordinal type measures) into a mechanical format, and of other logically indefensible practices such as substituting partial linear regression coefficients for partial differential regression coefficients into arbitrary functional relations have made it impossible to develop meaningful definitions of force, mass, velocity, acceleration, work, momentum, potential and kinetic energy within the human communication disciplines. As a consequence, these terms are used imprecisely and analogously if at all. One may not simply use the words referring to concepts from mechanics and assume therefore that one makes use of the concepts themselves.

Of course, it may well turn out that mechanical models of communication fail to produce the clarity and power that is implied in this essay. No scientist would ever foreclose any possibility on the basis of reasoning alone. It should be clear from this essay, however, that there is no present basis in evidence for saying that mechanical models do not fit human communication processes to useful tolerances. Such evidence, one way or the other, can only be provided by trying. Hopefully, insofar as it describes in some detail how a truly mechanical model of communication might be developed, this essay might provide some impetus toward a rational answer to the question.

NOTES

1. I am grateful to D.L. Kincaid and the Communication Institute of the East-West Center, Honolulu, Hawaii for support in the preparation of this manuscript.

2. Most contempory communication researchers balk at the idea that ratio level measurement can be used as effectively for human data as other, cruder measures, such as Likert-type or semantic differential type scales, or even simple ordinal measures. We have discussed these matters in detail elsewhere (Woelfel & Fink, 1980) as have others (Hamblin, 1976; Shinn, 1976; Stevens, 1951) and will not say more here, except to suggest that among those who are familiar with the empirical evidence about ratio measurement or "magnitude estimation" as it is often called, there remains little but enthusiasm.

3. It is worth noting that time is measured as ratios to a standard interval of time (usually the second), and time, of course, is at least as abstract as affection, yet time is the most precisely measured variable in science.

4. This notation was first published by Leibnitz in 1677, and the same procedure was independently developed about the same time by Newton precisely to deal with experiences of a processual nature, because they both believed that ordinary language was inadequate for the descriptioon of processes. To this day, the calculus remains the most powerful language known for describing processes. Although many communication scholars lay heavy emphasis on the processual nature of communication phennomena, and criticize contemporary communication theory and method for dealing with processes clumsily, the calcular is virtualaly non-existant in Communication journals and books, and very few human communication theorists have any knowledge of it at all. Thus, while we may agree that contemporary human communication theory deals with processes in a clumsy fashion, we cannot agree that it is because communication theory is mechanical, since a mechanics without ratio level measurement and the calcular could not rise much beyond the Greeks on logical grounds alone.

5. We adopt here the summation convention,so that repeated indices are to be summed over, and the summation sign may be omitted.

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