A THEORY OF LINEAR FORCE AGGREGATION IN ATTITUDE FORMATION AND CHANGE

Joseph Woelfel

The importance of interpersonal influence in the formation and change of attitudes and behavior has long been virtually axiomatic among sociologists. Most frequently, this process is described as one in which an other holds expectations for an individual, and it is assumed that the individual's behavior is in large part governed by the expectations for an individual, and it is assumed that the individual's behavior is in large part governed by the expectations of that other (Parsons, 1951; Woelfel and Haller, 1971 a,b). When the set of others holding expectations for the individual becomes multiple, however, and when the set of expectations becomes disparate, the axiomatic character of current theory breaks down and a multiplicity of theories have been generated to account for the response of the individual to the set of multiple and disparate influences to which he/she is exposed (Heider, 1958; Festinger, 1957). For the most part, and in spite of considerable divergences, all such theories have dealt with such cases as situations of discrete choice wherein the individual must choose, among other alternatives, to respond to one of the expectations over the others. The range of choices open to the individual along with the criteria upon which such decisions are made has constituted the main range of variation among such theories (Mettlin, 1970).

In their recent article, Woelfel and Haller (1971a) have presented unusual data along with a mode of analysis considerably at variance with these theoretical notions. Based upon data taken with the Wisconsin Significant Other Battery (Haller and Woelfel, 1969), they present measures of the educational and occupational expectations held by 950 "significant others" for a set of 99 high school students. For each student in the sample, Woelfel and Haller report a variable number of significant others (averaging about 9 per student) each of whom holds expectations for the student for whom he/she is influential which are more or less at odds with the expecations of other significant others for that same individual.

In two important regards, the Woelfel-Haller treatment of this data is unusual. First, the expectations of these significant others are status-level expectations, and are measured on level-type scales. Second, rather than devising a theoretical set of criteria on the basis of which the focal individuals are presumed to choose among the various levels of educational and occupational attainment expected of them by their set of significant others, Woelfel and Haller simply aggregate the expectation levels of the set of significant others for each individual into an arithmetic mean, which they presume to be the aggregate expectation level toward which those focal individuals

adjust their aspirations (controlling, of course, for other important variables in the system).

Because this unique procedure diverges so considerably from conventional social-psychological theory, and because the levels of variance explained by the Woelfel-Haller data are higher than the best data previously reported in the same substantive area (Sewell, et al., 1969), this article proposes to spell out the theoretical implications of this aggregation procedure in considerable detail.

Assuming variables measured on approximate ratio scales (a condition which is approximated quite well for those attitudes or behaviors which may be expressed as a rate over time, as <u>number of cigarettes smoked per day</u>, the Woelfel-Haller procedure implies a theory which states that the attitude of an individual equals the arithmetic mean of all attitudes proposed to that individual from all sources, or:

$$\bar{X} = \frac{\Sigma X_1}{N} \qquad (1)$$

Where,

$\bar{X}$ = The attitude expressed as a proposed rate of behavior

ΣX_1 = The sum of the rates of behavior proposed to the individual

N = The number of messages ego has received about the behavior from all sources.[1]

This way of aggregating incoming information has serious implications for the question of attitude stability and change. Since the attitude of an individual toward a behavior expressed as a rate over time (X) is itself assumed to be a mean score, then the formula for attitude change corresponds to the formula for the change in a mean value given additional individual values. Following the conventional notation,

$$\bar{X}_2 = \frac{\bar{X}_0 N_0 + \bar{X}_1 N_1}{N_0 + N_1} \qquad (2)$$

Where,

$\bar{X}_2$ = The new attitude expressed as a proposed rate of behavior

$\bar{X}_0$ = The original attitude expressed as a proposed rate of behavior

$\bar{X}_1$ = The mean value of the incoming new information provided the individual

N_0 = The number of messages out of which the old attitude has been constituted

N_1 = The number of messages present in the incoming information

Some of the consequences of this view involve fairly serious divergences from current theory, and it is important that they be pointed out. First, the formula clearly implies that the stability, i.e., resistance to change, of any attitude is wholly dependent upon N_0, or the number of messages out of which the original aggregate attitude was formed. This means, of course, that such obvious variables as the degree of satisfaction or dissatisfaction with which an individual views an attitude he/she holds are unrelated to the likelihood that the attitude will change. While such a notion goes a long way toward a parsimonious theoretical explanation for such persistent yet invidious self conceptions as alcoholic, drug addict, cigarette smoker, etc., nonetheless there are few sociologists who would disregard the role of pleasure and pain in attitude change, and indeed pleasure seeking and pain avoidance are the root concepts of much current psychological theory.

Second, since the formula for the change of a mean score makes no reference to the variance around the original mean, the homogeneity or heterogeneity of the influences to which an individual has been exposed have no consequence for the stability of an attitude within this theory. Consider the two cases following:

	RATE OF SMOKING PROPOSED	
	P_1	P_2
S_1	5	10
S_2	5	4
S_3	5	6
S_4	5	2
S_5	5	3
	$S = 25$	$S = 25$
	$\bar{X}p_1 = 5$	$\bar{X}p_2 = 5$

Person One (P_1) and Person Two (P_2) are each exposed to five messages proposing rates of cigarette smoking appropriate to them from five different significant others (S_1 through S_5). All of P_1's significant others agree while all of P_2's significant others differ. Yet both result in the identical attitude, and each person's attitude is presumed to be equally stable, as can be readily seen from the fact that, if each receives the same new message, e.g., that he/she should smoke 11 cigarettes per day, each person's new attitude will change by the same amount, i.e., from five to six per day $(25 + 11)/6 = 6$. Although not so dramatic a divergence from current theory as the first case, nonetheless it is widely assumed that homogenous influences tend to produce rigid, dogmatic attitudes which resist change (Adorno, et al 1950; Rokeach, 1960), while heterogeneous patterns of influence conduce toward flexibility and change--a prediction clearly contrary to that presented here.

Third, since the formula for the change of a mean assumes a model both continuous and linear, the amount of change in an old attitude will increase without limit as the value of incoming information diverges further from the value of the original

attitude. This means that the maximum amount of attitude change can be induced by proposing new information maximally divergent from what the individual already believes. Clearly current theory does not assume linearity here, since selective preception theory must argue that information gravelyinconsistent with the individual's present belief system tends to be rejected (Bruner, 1958). In the interest of scientific parsimony, it would seem that the simpler linear model ought to be proposed until data force its rejection, and curvilinearity ought not to be assumed in the absence of such evidence.

In spite of its divergence from current theoretical thinking, the assumptions underlying the theory presented here are very appealing, and can be spelled out quite exactly.

First, as expression (2) shows quite clearly, if no new message is introduced, both $\bar{X}_1$ and N_1 will be zero, giving:

$$\bar{X}_2 = \frac{\bar{X}_0 N_0}{N_0} = \bar{X}_0 \qquad (3)$$

Thus, the first assumption of the theory is that, in the absence of any new information introduced into the system, an attitude will remain unchanged indefinitely. Expression (3) may be seen as an existence therom defining a coordinate system in which a state of zero force may exist.

Secondly, again transforming equation (2) we obtain:

$$\bar{X}_2 (N_0 + N_1) = \bar{X}_0 N_0 + \bar{X}_1 N_1 \qquad (4)$$

As we pointed out earlier, N_0 is the only factor retarding the change of X_0. As such it is sensible to consider it an expression of the intertial mass of the attitude. N_1 may be similarly construed for the change message. Given these equivalences, $N_0 + N_1$ can be seen as the inertial mass of the new attitude once the told attitude and the change message have been aggregated. The $\bar{X}$ values (X_0, X_1, X_2) have been defined as rates, and can thus be considered to be analogous to velocities of behavior. Expression (4) may be rewritten in the general case, therefore:

$$\underbrace{M_2 V_2}_{\text{(Total Final Momentum)}} = \underbrace{M_0 V_0 + M_1 V_1}_{\text{(Total Initial Momentum)}} \qquad (5)$$

Expression (5) shows that momentum is conserved in this system.

These equations imply a model of a wholly inelastic collision, wherein an attitude moving along a one-dimensional vector space is overtaken by a change message, resulting in an aggregate of the two which preserves the initial momentum of both. Transforming (2) through the following steps:

$$\bar{X}_2 N_0 + \bar{X}_2 N_1 = \bar{X}_0 N_0 + \bar{X}_1 N_1$$

$$\bar{X}_2 N_0 - \bar{X}_0 N_0 = \bar{X}_1 N_1 - \bar{X}_2 N_2$$

We obtain:

$$N_0(\overline{X}_2 - \overline{X}_0) = (N_1(\overline{X}_1 - \overline{X}_2) \qquad (6)$$

$$N_0(\overline{X}_2 - \overline{X}_0) = -N_1(\overline{X}_2 - \overline{X}_1) \qquad (6')$$

Equation (6′) shows that the product of the change of velocity of the old attitude $(X_2 - X_0)$ and its inertial mass (N_0) equals the inverse of the product of the change of velocity of the change message $(X_2 - X_1)$ and its mass (N_1). Thus equation (6′) may be written more generally:

$$Mv = -mV$$

which is a statement of equality of change of momentum denoting that the action upon the attitude is equal and opposite to the action on the change message.

Equation (6) shows that, as a result of the "collision," X_0 (the old attitude) is accelerated to X_2 (the new attitude) while X_1 (the incoming message) is decelerated to X_2. It is clear from (6) that both the acceleration $(X_2 - X_0)$ and the deceleration $(X_1 - X_2)$ take place within the same interval of time, however long that may be. The duration may be represented by t, as follows:

$$\frac{N_0(\overline{X}_2 - \overline{X}_0)}{t} = \frac{N_1(\overline{X}_1 - \overline{X}_2)}{t} \qquad (7)$$

since $\overline{X}_2$ and $\overline{X}_0$ have been defined as rates or velocities, $(X_2 - X_0)$ can be seen as the change of velocity; dividing by t thus yields average acceleration. Since N_0 has been interpreted as the inertial mass of the attitude, the left side of (7) can be expressed:

$$\frac{N_0(\overline{X}_2 - \overline{X}_0)}{t} = ma$$

The greater the acceleration $(\overline{X}_2 - \overline{X}_0)/t$, and/or the greater the inertial mass to be overcome (N_0), the greater the force impinging on the attitude may be said to be. Since the value of $N_0(X_2 - X_0)/t$ is wholly dependent upon:

$$N_1(\overline{X}_1 - \overline{X}_2)/t$$

We may conveniently define force within the system as:

$$F = N_1(\overline{X}_1 - \overline{X}_2)/t$$

Thus, the following equalities may be seen to hold:

$$N_1(\overline{X}_1 - \overline{X}_2)/t = N_0\ (\overline{X}_2 - \overline{X}_0)/t = F = ma \qquad (8)$$

Thus, the theory expressed in equation (2) can be seen to imply a system whose underlying assumptions are isomorphic to Newtonian mechanics. Equation (3) is analogous to the First Law, defining a coordinate system in which F = 0 is possible. Equations 6 and 6' show that momentum is conserved within the system and that every action implies an equal and opposite reaction, an assumption parallel to Newton's Third Law. Equation (8) of course is analogous to the Second Law, F = ma, which specified a linear unbounded system.

As we suggested earlier, however, expression (2) can be seen to hold only for the one-dimensional case, i.e., in a one-dimensional vector space. Such a space obtains when all data gathered is assumed to vary along only one dimension: When variables not measured on the same vector are introduced, however, the situation changes. Performance in extra-curricular activities, for example, may exert some of its force toward years of schooling attained, but may exert other parts of its force in different directions, e.g., toward greater or lesser interest in dating, athletics, etc. Thus, only a part of the force contained in this new variable is expressed along the original attainment vector.

Fortunately, expression (2) can be made general by showing that it is a special case of expression (6) the law of cosines:

$$R = M^2{}_1 + M^2{}_2 - 2(M_1M_2) \cos (180^\circ - \Theta) \qquad (6)$$

Where R = the resultant or final momentum and M_1 and M_2 equal two momenta imposed on a body at angle Θ to each other. The resulting momentum R can be shown to be equal to $\bar{X}_2$ (the new attitude) multiplied by its mass, i.e., $N_0 + \underline{N}_1$. Similarly, XN_0 = the momentum of the original attitude, and X_1N_1 = the momentum of the change message. Thus:

$$\bar{X}_2(N_0+N_1) = [(\bar{X}_0N_0)^2 \cdot (\bar{X}_1N_1)^2 - 2(\bar{X}_0N_0\bar{X}_1N_1) \cos(180 - \Theta)]^{1/2}$$

but when all them momenta are projected along the same vector, $\Theta = 0$, 180 and $\cos \Theta = \pm 1$, again depending upon direction. This readily yields:

$$\bar{X}_2(N_0 + N_1) = [(\bar{X}_0N_0)^2 \pm 2 (X_0\bar{N}_0X_1\bar{N}_1) + (X_1\bar{N}_1)^2]^{1/2}$$

Factoring and removing the radical,

$$\bar{X}_2(N_0 + N_1) = \bar{X}_0N_0 \pm X_1\bar{N}_1$$

$$\bar{X}_2 = \frac{\bar{X}_0N_0 \pm X_1\bar{N}_1}{N_0 + N_1} \qquad (7)$$

By inspection, (7) and (2) are identical, showing that (2) is the special case of (6) where $\Theta = 0$, 180. Thus at this point it is possible to state the basic premise of the theory more accurately: <u>an individual attitude equals the vector sum of all information relevant to that behavior an individual receives.</u>

Three key problems are still unresolved at this point, however. First, a procedure for determining the angle Θ has not yet been specified. Second, also at this stage in development, the new vector **R** is likely to be meaningless in terms of its applicant to known behavioral outcomes, since what behaviors lie along the new vector will not be known.[2] Third, in most cases, N_0 and N_1 can only be poorly estimated, and at best represent an aggregate of the amount of information and the massiveness of an individual message. Fortunately these problems can be resolved, since expression (2) can be generalized along its original vector as follows:

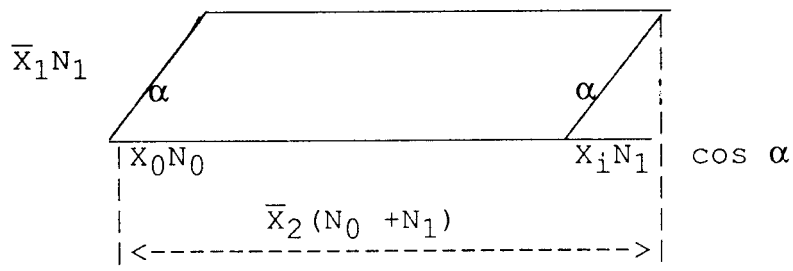

Figure 1
Resolution of Impact of Change Message on the Original Vector of the Dependent Variable.

Figure 1 shows that the projection of the moment of $\overline{X}_1N_1$ on the vector of the original moment X_0N_0 is given by $X_1N_1 \cos \alpha$, where α is the angle included between the two moment vectors. The resultant momentum $(X_2(N_1 + N_0)$ expressed along the original vector is thus given by:

$$\overline{X}_2(N_0 + N_1) = \overline{X}_0N_0 + \overline{X}_1N_1 \cos \alpha \qquad (9)$$

It is convenient to re-express (9)[3] for the general case as:

$$\overline{X}_n = \overline{X}_0 \frac{N_0 \cos \alpha_{0n}}{N_0+N_1+\ldots+N_{n-1}} + \overline{X}_1 \frac{N_1 \cos \alpha_{1n}}{N_0+N_1+\ldots+N_{n-1}} \qquad (10)$$

Or more simply:

$$\bar{X}_n = \sum_{i=1}^{n} [\bar{X}_i \left[\frac{N_i \cos \alpha_{in}}{\sum_{i=1}^{n-1} N_i} \right]] \qquad (11)$$

When expressed as in (11), it is clear that the expression:

$$\frac{N_i \cos \alpha_{in}}{\sum_{i=1}^{n-1} N_i}$$

represents a multiplier which gives the proportion of any variable X_i which is projected on the dependent variable vector. As such it is the formal equivalent of a partial slope, and we may write:

$$\bar{X}_n = \bar{X}_i \left(\frac{N_i \cos \alpha_{in}}{\sum_{i=1}^{n-1} N_i} \right) = b_{in}\bar{X}_i$$

thus,

$$bn = \frac{N_i \cos \alpha_{in}}{\sum_{i=1}^{n-1} Ni}$$

This means, quite simply, that the value to the right of the equal sign is recoverable by simple linear partial regression techniques, even though all its component measures are unknown. From this point it is a simple matter to recover both N_i and α_{in}.

First it is convenient to determine the total net "effectiveness" of each of the variables in the regression equation. This can be done by taking a ratio of the unadjusted partial slopes. If one such partial slope is arbitrarily designated as the denominator of all such ratios then its mass will be set at unit, and all other variables will be expressed as proportions of that unit.

$$\frac{b_{in}}{b_{i+1,n}} = \frac{N_i \cos\alpha_{in}}{\sum_{i=1}^{n-1} N_i} \qquad (12)$$

$$\frac{\dfrac{N_{i+1}\cos\alpha_{i+1,n}}{\sum_{i=1}^{n-1} N_i}}{} = \frac{N_i \cos\alpha_{in}}{N_{i+1}\cos\alpha_{i+1,n}}$$

It can eaily be shown, now, that the $\cos\alpha_{in}$ is equal to the partical correclation between the dependent variable n and the independent variable i controlling for all the other independent variables. Given a vector **u** where any element u = the standard score ofthe ith case of the residual variable of the (k-1)th order multiple correlation, where k = the number of independent variables in the system, then:

$$u'X_i/N = r$$

where r = a scalar which represents the zero-order correlation between variables u and i. But u represents that component of the dependent variable unexplained by all the other independent variables in the system, hence r_{uxi} is equivalent to the (k-1)th order partial between X_i and X_n where k equals the number of independent variables in the system.

But the inner product of any two vectors, or scalar product is given by:

$$X'_i X_j = \rho_i \rho_j \cos\alpha_{ij}$$

where ρ_i and ρ_j are the lengths of the vectors i and j. In the standardized case, however, all variable vectors are standardized to unit length, so that $\rho_i = \rho_j = 1$. Thus, $r_{ui} = r_{in\,.\,k-1)} = \cos\alpha_{in}$ thus the cosine of α_{in} is given by the appropriate order partial correlation.[4] Substituting the partial correlation for $\cos\alpha_{in}$ in expression (P), therefore, allows the determination of the ratio of inertial masses of all the variables in the system to any arbitrary variable chosen as a standard unit of inertial mass. The masses thus estimated should not be specific to that regression equation, and the ratio of masses of any two variables should remain constant even when they are used in a different regression equation with a different dependent variable.

Since the relative inertial masses of a series of messages from a number of different sources can be determined by partial regression techniques, it becomes possible, once a suitable unit standard is agreed upon, to determine inertial masses empirically by controlling for number of messages in the question format. Thus, presuming the number of messages from several sources is known or held constant, the inertial mass of a unit message from each source can be estimated. This would allow replacement of

the cumbersome variable "number of messages" by a more meaningful measure (see note 1 above).

Although the data originally presented by Woelfel and Haller have some important restrictions (Woelfel and Haller, 1971a, b), nonetheless the implications of this analysis, if borne out by further data, are compelling. First, the theory is strikingly simple and parsimonious in its basic assumptions: it assumes that no information is rejected (thus dispensing with the need for assumptions about differential acceptance and rejection of information) and it assumes that the aggregation of all information is simply linear (thus requiring no assumptions about differential effect of differing information). In spite of this simplicity, the Woelfel-Haller aggregate expectations exhibit the highest zero order correlations with aspiration levels yet measured ($r = .64$ for occupational aspirations and mean occupational expectations and $r = .66$ for educational aspirations and mean educational expectations). The simplicity of underlying structure is important not only because of the general preferability of parsimony, but also because it responds extremely well to strightforward and powerful mathematical treatment. Should the underlying linear unbounded structure of the theory prove a reasonable approximation to observed social phenomena, then such tools as the calculus, linear equation theory, vector analysis, and the like can be made very day tools for the social sciences. It is well worth noting, too, that the theoretical structure is ideally compatible with the simultaneous linear equation models like path analysis and econometric analysis that are now rapidly gaining favor among social scientists, particularly economists and sociologists.

It is, of course, premature at this stage to make too much of the homomorphism between the implications of the Woelfel-Haller procedures and Newtonian Theory, but should it prove to be the case that basic mechanical theory provides an approximation of the processes involved in human social behavior within they typical limits of accuracy of sociological research, much theory could be brought over prewritten from the physical sciences. This suggestion will no doubt be met with alarm by those social sciences who prefer to think of human behavior as an emergent phenomenon, but the exten to which human social processes can be approximated by such simple models is a question which can--and should--be resolved empirically rather than by argument.

NOTES

1. We do not seriously propose that all messages from all sources should be weighted equally, as N presumes. This is a simplifying assumption which will be modified shortly.

2. Since the theory presented here is fundamentally Newtonian, by implication all these processes can be represented in a Euclidean real space of finite dimension. The multidimensional scaling techniques required for the generation of such a space have already been detailed (Young & Householder, 1938; Torgerson, 1958; Wisan, 1971) and should make the resolution of the second problem straightforward.

3. $\cos\alpha_m$ has been added to the first term on the right hand side of (10) for generality. When the first term represents an error-free measure of the old attitude, **a** will, of course, be zero, and $\cos \Theta = 1$.

4. Where $r_{in} - (k-1)$ indicates the partial correlation between the ith variable and the dependent variable i + 1 through k for k predictor variables.

REFERENCES

Adorno, T.W., E. Frendel-Brunswick, D.J. Levinson & R.W. Sanford. The Authoritarian Personality, New York: Harper, 1950.

Bruner, J.S. "Social Psychology and Perception," in N. Maccoby, T. Newcomb & Hartley (eds.), Readings in Social Psychology. New York: Holt, Rinehart, Winston, 1958.

Festiner, L., A Theory of Cognitive Dissonance. Evanston, IL: Row, Peterson, and Company, 1957.

Haller, A. O., J. Woelfel & E.L. Fink, The Wisconsin Significant Other Battery. U.S. Office of Education, 1969.

Heider, F., The Psychology of Interpersonal Relations, New York: Wiley, 1958.

Mettlin, C., Unpublished Ph.D. dissertation, University of Illinois, 1970.

Parsons, T., The Social System. Glencoe, IL: The Free Press, 1951.

Rokeach, M., The Open and Closed Mind. New York: Basic Books. 1960.

Sewell, W.H., A.O. Haller & A. Portes. The Educational and Occupational Attainment Process: Wisconsin Farm-Reared Men (1957-69) American Sociological Review, 1969.

Togerson, W. S., Theory and Methods of Scaling. New York: Wiley, 1958.

Wisan, G., The Galileo System - A metric technique for multidimensional scaling. Unpublished University of Ph.D. dissertation, 1971.

Woelfel, J. & A. O. Haller, Significant Others, the Self Reflexive Act and the Attitude Formation Process. American Sociological Review (February), 1971a.

Woelfel, J. & A. O. Haller, A Note on the Identification and Proper Estimation of the Woelfel-Haller Model. American Sociological Review, December 1971b.

Young, G. & A.S. Householder, Discussion of a Set of Points in Terms of Their Mutual Distance. Psychometrika, 3, 19-22, 1938.